TRANSACTIONS

OF THE

ROYAL SOCIETY OF EDINBURGH.

TRANSACTIONS

OF THE

ROYAL SOCIETY

OF

EDINBURGH.

VOL. XXIV.

EDINBURGH:
PUBLISHED BY ROBERT GRANT & SON, 82 PRINCES STREET.
AND WILLIAMS & NORGATE, 14 HENRIETTA STREET, COVENT GARDEN, LONDON.

MDCCCLXVII.

THE KEITH, BRISBANE, AND NEILL PRIZES.

The above Prizes will be awarded by the Council in the following manner:—

I. KEITH PRIZE.

The Keith Prize, consisting of a Gold Medal and from £40 to £50 in Money, will be awarded in the Session 1867–68, for the "best communication on a scientific subject, communicated, in the first instance, to the Royal Society during the Sessions 1865–66 and 1866–67." Preference will be given to a paper containing a discovery.

II. MAKDOUGALL BRISBANE PRIZE.

This Prize is to be awarded biennially by the Council of the Royal Society of Edinburgh to such person, for such purposes, for such objects, and in such manner as shall appear to them the most conducive to the promotion of the interests of science; with the *proviso* that the Council shall not be compelled to award the Prize unless there shall be some individual engaged in scientific pursuit, or some paper written on a scientific subject, or some discovery in science made during the biennial period, of sufficient merit or importance in the opinion of the Council to be entitled to the Prize.

1. The Prize, consisting of a Gold Medal and a sum of Money, will be awarded at the commencement of the Session 1868–69, for an Essay or Paper having reference to any branch of scientific inquiry, whether Material or Mental.

2. Competing Essays to be addressed to the Secretary of the Society, and transmitted not later than 1st June 1868.

3. The competition is open to all men of science.

4. The Essays may be either anonymous or otherwise. In the former case, they must be distinguished by mottoes, with corresponding sealed billets superscribed with the same motto, and containing the name of the Author.

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5. The Council impose no restriction as to the length of the Essays, which may be, at the discretion of the Council, read at the Ordinary Meetings of the Society. They wish also to leave the property and free disposal of the manuscripts to the Authors; a copy, however, being deposited in the Archives of the Society, unless the Paper shall be published in the Transactions.

6. In awarding the Prize, the Council will also take into consideration any scientific papers presented to the Society during the Sessions 1866–67 and 1867–68, whether they may have been given in with a view to the Prize or not.

III. NEILL PRIZE.

The Council of the Royal Society of Edinburgh having received the bequest of the late Dr PATRICK NEILL of the sum of £500, for the purpose of "the interest thereof being applied in furnishing a Medal or other reward every second or third year to any distinguished Scottish Naturalist, according as such Medal or reward shall be voted by the Council of the said Society," hereby intimate,

1. The NEILL PRIZE, consisting of a Gold Medal and a sum of Money, will be awarded at the commencement of the Session 1868–69.

2. The Prize will be given for a Paper of distinguished merit, on a subject of Natural History, by a Scottish Naturalist, which shall have been presented to the Society during the three years preceding the 1st May 1868,—or failing presentation of a Paper sufficiently meritorious, it will be awarded for a work or publication by some distinguished Scottish Naturalist, on some branch of Natural History, bearing date within five years of the time of award.

AWARDS OF THE KEITH, MAKDOUGALL BRISBANE, AND NEILL PRIZES, SINCE 1862.

AWARD OF THE KEITH PRIZE.

19th Biennial Period, 1863-65.	Principal **Forbes**, St Andrews, for his "Experimental Inquiry into the Laws of Conduction of Heat in Iron Bars." published in the Transactions of the Society.

MAKDOUGALL BRISBANE PRIZE.

4th Biennial Period, 1864–66.	Not awarded.

AWARD OF THE NEILL PRIZE.

3d Triennial Period, 1862–65.	**Andrew Crombie Ramsay**, F.R.S., Professor of Geology in the Government School of Mines, and Local Director of the Geological Survey of Great Britain, for his various works and memoirs published during the last five years, in which he has applied the large experience acquired by him in the Direction of the arduous work of the Geological Survey of Great Britain to the elucidation of important questions bearing on Geological Science.

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ERRATA.

Page 21, line 4 from bottom, *for* value v *read* value of v.

... 23, line 7 from top, *for* $\cos t + \sin t$ *read* $\sin t - \cos t$.

... — line 4 from bottom, *for* e^{-s} *read* e^{-2s}.

... — line 3 from bottom, the first *minus* sign in the line should be *plus*.

... 26, line 2 from bottom, *for* 23° 23′ *read* 66° 37′.

LAWS

OF THE

ROYAL SOCIETY OF EDINBURGH.

AS REVISED 5TH JANUARY 1857.

[illegible]

LAWS.

[By the Charter of the Society (printed in the *Transactions*, Vol. VI. p. 5), the Laws cannot be altered, except at a Meeting held one month after that at which the Motion for alteration shall have been proposed.]

I.

Title.

THE ROYAL SOCIETY OF EDINBURGH shall consist of Ordinary and Honorary Fellows.

II.

The fees of Ordinary Fellows residing in Scotland.

Every Ordinary Fellow, within three months after his election, shall pay Two Guineas as the fee of admission, and Three Guineas as his contribution for the Session in which he has been elected; and annually at the commencement of every Session, Three Guineas into the hands of the Treasurer. This annual contribution shall continue for ten years after his admission, and it shall be limited to Two Guineas for fifteen years thereafter.*

III.

Payment to cease after 25 years.

All Fellows who shall have paid Twenty-five years' annual contribution shall be exempt from farther payment.

IV.

Fees of Non-Resident Ordinary Fellows.

The fees of admission of an Ordinary Non-Resident Fellow shall be £26, 5s., payable on his admission; and in case of any Non-Resident Fellow coming to reside at any time in Scotland, he shall, during each year of his residence, pay the usual annual contribution of £3, 3s., payable by each Resident Fellow; but after payment of such annual contribution for eight years, he shall be exempt from any farther payment. In the case of any Resident Fellow ceasing to reside in Scot-

Case of Fellows becoming Non-Resident.

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* At the Meeting of the Society, on the 5th January 1857, when the reduction of the Contributions from £3, 3s. to £2, 2s., from the 11th to the 25th year of membership, was adopted, it was resolved that the existing Members shall share in this reduction, so far as regards their future Annual Contributions.

A modification of this rule, in certain cases, was agreed to 3d January 1831.

land, and wishing to continue a Fellow of the Society, it shall be in the power of the Council to determine on what terms, in the circumstances of each case, the privilege of remaining a Fellow of the Society shall be continued to such Fellow while out of Scotland.

V.

Defaulters.

Members failing to pay their contribution for three successive years (due application having been made to them by the Treasurer) shall be reported to the Council, and, if they see fit, shall be declared from that period to be no longer Fellows, and the legal means for recovering such arrears shall be employed.

VI.

Privileges of Ordinary Fellows.

None but Ordinary Fellows shall bear any office in the Society, or vote in the choice of Fellows or Office-Bearers, or interfere in the patrimonial interests of the Society.

VII.

Numbers Unlimited.

The number of Ordinary Fellows shall be unlimited.

VIII.

Fellows entitled to Transactions.

The Ordinary Fellows, upon producing an order from the TREASURER, shall be entitled to receive from the Publisher, gratis, the Parts of the Society's Transactions which shall be published subsequent to their admission.

IX.

Mode of Recommending Ordinary Fellows.

No person shall be proposed as an Ordinary Fellow without a recommendation subscribed by *One* Ordinary Fellow, to the purport below.* This recommendation shall be delivered to the Secretary, and by him laid before the Council, and shall afterwards be printed in the circulars for three Ordinary Meetings of the Society, previous to the day of the election, and shall lie upon the table during that time.

X.

Honorary Fellows, British and Foreign.

Honorary Fellows shall not be subject to any contribution. This class shall consist of persons eminently distinguished for science or literature. Its number shall not exceed Fifty-six, of whom Twenty may be British subjects, and Thirty-six may be subjects of foreign states.

---

* "A. B., a gentleman well skilled in several branches of Science (*or Polite Literature, as the* "*case may be*), being to my knowledge desirous of becoming a Fellow of the Royal Society of Edin-"burgh, I hereby recommend him as deserving of that honour, and as likely to prove a useful and "valuable Member."

This recommendation to be accompanied by a request of admission signed by the Candidate.

XI.

Personages of Royal blood may be elected Honorary Fellows, without regard to the limitation of numbers specified in Law X. Royal Personages.

XII.

Honorary Fellows may be proposed by the Council, or by a recommendation (in the form given below*) subscribed by three Ordinary Fellows; and in case the Council shall decline to bring this recommendation before the Society, it shall be competent for the proposers to bring the same before a General Meeting. The election shall be by ballot, after the proposal has been communicated *viva voce* from the Chair at one meeting, and printed in the circular for the meeting at which the Ballot is to take place. Recommendation of Honorary Fellows. Mode of Election.

XIII.

The election of Ordinary Fellows shall take place at the Ordinary Meetings of the Society. The election shall be by ballot, and shall be determined by a majority of at least two-thirds of the votes, provided Twenty-four Fellows be present and vote. Election of Ordinary Fellows.

XIV.

The Ordinary Meetings shall be held on the first and third Mondays of every month from November to June inclusive. Regular Minutes shall be kept of the proceedings, and the Secretaries shall do the duty alternately, or according to such agreement as they may find it convenient to make. Ordinary Meetings.

XV.

The Society shall from time to time publish its Transactions and Proceedings. For this purpose the Council shall select and arrange the papers which they shall The Transactions.

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* We hereby recommend ________________________________
for the distinction of being made an Honorary Fellow of this Society, declaring that each of us, from our own knowledge of his services to (*Literature or Science, as the case may be*), believe him to be worthy of that honour.

(To be signed by three Ordinary Fellows.)

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To the President and Council of the Royal Society
of Edinburgh.

deem it expedient to publish in the *Transactions* of the Society, and shall superintend the printing of the same.

XVI.

How Published.

The Transactions shall be published in Parts or *Fasciculi* at the close of each Session, and the expense shall be defrayed by the Society.

The Council.

There shall be elected annually, for conducting the publications and regulating the private business of the Society, a Council, consisting of a President; Six Vice-Presidents, two at least of whom shall be resident; Twelve Councillors, a General Secretary, Two Secretaries to the Ordinary Meetings, a Treasurer, and a Curator of the Museum and Library.

XVII.

Retiring Councillors.

Four Councillors shall go out annually, to be taken according to the order in which they stand on the list of the Council.

XVIII.

Election of Office-Bearers.

An Extraordinary Meeting for the Election of Office-Bearers shall be held on the fourth Monday of November annually.

XIX.

Special Meetings; how called.

Special Meetings of the Society may be called by the Secretary, by direction of the Council; or on a requisition signed by six or more Ordinary Fellows. Notice of not less than two days must be given of such Meetings.

XX.

Treasurer's Duties.

The Treasurer shall receive and disburse the money belonging to the Society, granting the necessary receipts, and collecting the money when due.

He shall keep regular accounts of all the cash received and expended, which shall be made up and balanced annually; and at the last Ordinary Meeting in January he shall present the accounts for the preceding year, duly audited. At this Meeting, the Treasurer shall also lay before the Council a list of all arrears due above two years, and the Council shall thereupon give such directions as they may deem necessary for recovery thereof.

XXI.

Auditors.

At the Extraordinary Meeting in November, a Committee of three Fellows shall be chosen to audit the Treasurer's accounts, and give the necessary discharge of his intromissions.

The report of the examination and discharge shall be laid before the Society at the last Ordinary Meeting in January, and inserted in the records.

XXII.

General Secretary's Duties.

The General Secretary shall keep Minutes of the Extraordinary Meetings of the Society, and of the Meetings of the Council, in two distinct books. He shall, under the direction of the Council, conduct the correspondence of the Society, and superintend its publications. For these purposes, he shall, when necessary, employ a clerk, to be paid by the Society.

Secretaries to Ordinary Meetings.

The Secretaries to the Ordinary Meetings shall keep a regular Minute-book, in which a full account of the proceedings of these Meetings shall be entered; they shall specify all the Donations received, and furnish a list of them, and of the donors' names, to the Curator of the Library and Museum: they shall likewise furnish the Treasurer with notes of all admissions of Ordinary Fellows. They shall assist the General Secretary in superintending the publications, and in his absence shall take his duty.

XXIII.

Curator of Museum and Library.

The Curator of the Museum and Library shall have the custody and charge of all the Books, Manuscripts, objects of Natural History, Scientific productions, and other articles of a similar description belonging to the Society; he shall take an account of these when received, and keep a regular catalogue of the whole, which shall lie in the Hall, for the inspection of the Fellows.

XXIV.

Use of Museum and Library.

All articles of the above description shall be open to the inspection of the Fellows, at the Hall of the Society, at such times, and under such regulations, as the Council from time to time shall appoint.

XXV.

Register Book.

A Register shall be kept, in which the names of the Fellows shall be enrolled at their admission, with the date.

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ERRATUM.

Page 408, *Foot-note*,* *line* 3.

An erroneous definition of the word *Mycology* in M'Nicoll's "Dictionary of Natural History Terms," p. 317, wherein he confounds the older and better known term *Myology* with *Mycology*, has given rise to an inadvertent confusion of the two terms here. The use of the term *Fungology* would render such confusion of words so similar in their spelling impossible. The term *Myology* is properly defined in Noah Webster's, and other standard *English* Dictionaries; but M'Nicoll omits the word altogether, though he includes its definition under the term *Mycology;* an error both of omission and commission scarcely pardonable in a Dictionary exclusively of *scientific terms!*

LAWS

OF THE

ROYAL SOCIETY OF EDINBURGH,

AS REVISED 31st OCTOBER 1866.

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He shall keep regular accounts of all the cash received and expended, which shall be made up and balanced annually; and at the Extraordinary Meeting in November, he shall present the accounts for the preceding year, duly audited. At this Meeting, the Treasurer shall also lay before the Council a list of all arrears due above two years, and the Council shall thereupon give such directions as they may deem necessary for recovery thereof.

XXI.

Auditor.

At the Extraordinary Meeting in November, a professional accountant shall be chosen to audit the Treasurer's accounts for that year, and to give the necessary discharge of his intromissions.

XXII.

General Secretary's Duties.

The General Secretary shall keep Minutes of the Extraordinary Meetings of the Society, and of the Meetings of the Council, in two distinct books. He shall, under the direction of the Council, conduct the correspondence of the Society, and superintend its publications. For these purposes, he shall, when necessary, employ a clerk, to be paid by the Society.

Secretaries to Ordinary Meetings.

The Secretaries to the Ordinary Meetings shall keep a regular Minute-book, in which a full account of the proceedings of these Meetings shall be entered; they shall specify all the Donations received, and furnish a list of them, and of the donors' names, to the Curator of the Library and Museum: they shall likewise furnish the Treasurer with notes of all admissions of Ordinary Fellows. They shall assist the General Secretary in superintending the publications, and in his absence shall take his duty.

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The Curator of the Museum and Library shall have the custody and charge of all the Books, Manuscripts, objects of Natural History, Scientific Productions, and other articles of a similar description belonging to the Society; he shall take an account of these when received, and keep a regular catalogue of the whole, which shall lie in the Hall, for the inspection of the Fellows.

XXIV.

Use of Museum and Library.

All Articles of the above description shall be open to the inspection of the Fellows at the Hall of the Society, at such times and under such regulations, as the Council from time to time shall appoint.

XXV.

Register Book

A Register shall be kept, in which the names of the Fellows shall be enrolled at their admission, with the date.

LIST OF THE ORDINARY FELLOWS OF THE SOCIETY.

N.B.—*Those marked * are Annual Contributors.*

1846 *Alex. J. Adie, Esq., Rockville, Linlithgow
1866 *Col. Sir James E. Alexander of Westerton
1867 *Rev. Dr W. Lindsay Alexander, 17 Brown Square
1848 Dr James Allan, Inspector of Hospitals, Portsmouth
1856 *Dr G. J. Allman (SECRETARY), Professor of Natural History, 21 Manor Place
1849 *David Anderson, Esq., Moredun, Edinburgh
1845 *Dr Thomas Anderson, Prof. Chemistry, Univ., Glasgow
1823 Warren Hastings Anderson, Esq., Isle of Wight
1867 *Thomas Annandale, Esq., 3 Hope Street
1840 James Anstruther, Esq., W.S. 10
1862 *T. C. Archer, Esq., Director of the Museum of Science and Art, 9 Argyle Square
1849 *His Grace the Duke of Argyll (HON. VICE-PRESIDENT), Inverary Castle
1822 Dr G. Walker Arnott, Prof. Botany, Univ., Glasgow
1820 Charles Babbage, K.H., London
1843 David Balfour, Esq., Trenaby
1835 Dr J. H. Balfour (GENERAL SECRETARY), Professor of Medicine and Botany, 27 Inverleith Row
1867 *George F. Barbour, Esq., 11 George Square
1862 *Hon. Lord Barcaple, 3 Ainslie Place
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TRANSACTIONS.

I.—*On the Principle of Onomatopœia in Language.* By Professor BLACKIE.

(Read 19th December 1864.)

By ὀνοματοποιία the Greek grammarians understood that principle, or tendency in the growth of language, according to which certain words are formed by an imitation of the sounds which they signify. Thus, ὀγκ, the root of the Greek word ὀγκᾶσθαι, to *bray*, may be considered to have been formed of a human mimicry of that animal to which human beings of the lowest cerebral capacity are peculiarly compared; and in the same way, *laogh*, the Gaelic for a *calf*, seems to contain a sound to which only the throats of Highland calves, Highland chieftains, and Highland crofters are competent. The word *onomatopœia*, like some other technical terms of the old grammarians, is not particularly happy, for it means only and generally *word-making*, or rather *name-making*, and says nothing of the principle by which the special class of words in question is made. Instead of this term, therefore, I should prefer to speak of the imitative or pictorial principle in the formation of human speech; and I should contrast the whole class of words in which the operation of this principle can be traced, with another class, derived from ideas or notions about the thing to be named in the mind of the word-maker. Thus, the modern Greeks call a cock πετεινό, that is, the *fowl*, or *flying* animal, from πέτομαι, to fly; and the Latin word, *equus*, a horse, if it comes, as Professor MÜLLER says, from the Sanscrit root *á'su*, *swift*, will be another word formed on the same principle. The roots of these words I propose to call notional roots, as contrasted with the onomatopœtic class of roots, which I propose to call pictorial roots, or roots formed by *phonic imitation*.

Professor MÜLLER, in his valuable work on the Science of Language, has, in both volumes, either denied altogether the existence of this class of words, or treated them with such marked disfavour, that in his system they do not appear at all as effective agents in the formation of reasonable speech, but merely play a subordinate and scarcely human part in the precincts of the poultry-yard and the pig-sty. If, in the central table-land of Asia, before the divarication of the great Aryan races, a Persian pig gave a grunt, the learned Professor might perhaps be willing to admit, or might be forced to admit, that there was some connection in the way of mimetic reproduction between the sound uttered by that animal and the words γρύζω in Greek, *grunnio* in Latin, *grunt* in English, and *grumphie* in Scotch. If, when the sacred chickens were observed by the Roman augurs in their cages to give forth an attenuated indication of the approaching fates, according to their vocal capacity, and if the speakers of the Latin dialect of the Aryan family agreed to designate the sound then emitted by the root *pipi*, familiarly known as a verb of the fourth conjugation, *pipire*, with the variety *pipilare*, applied to sparrows—in this case also, we presume, those who disown the pictorial principle would be inclined to concede some pretty mimicry of the small unreasoning by the great reasoning animal. Or, to take an example from an altogether different quarter, in the word "*chirumvurumvuru*, used by the Africans on the Zambesi river, to designate a sudden violent tornado, with lightning, thunder, and rain, who can refuse to recognise a beautiful imitation of the long-continued roll of peals of thunder in a mountain district?"* But then they would say that in forming such words a man acts as a parrot and not as a man; and in the philosophy of human speech we can take no account of an element which denies the distinctive character—namely, reason—of the being who forms it. It is against this view of the part played by the imitative principle of our nature in the formation of language that I now submit a few observations.

In treating this matter, I shall first state the arguments in favour of the extensive operation of this principle, which appear to me conclusive, and then shortly consider the nature of the objections that have been brought against it. But, before making a regular muster of the arguments for or against any position, it appears to me to be of the utmost consequence to see how the presumptions lie. When a man is tried before a jury for a special act of felonious appropriation, the fact that he is *habit* and *repute* a *thief*, although no part of the evidence on which he can be convicted, will certainly operate against him to some extent in the minds of the most impartial jury. In the same way, it must have been observed that in the discussion of the most famous literary, scientific, and philosophical questions, there is an under-current of presumption of some kind or other, which secretly determines which side the reasoner will take, more powerfully than all the arguments that are articulately brought forward,—a presumption of which these arguments are sometimes only the servile satellites. So, in the present case, I ask, first, is there any presumption why words should not be formed by the human voice, in imitation of certain sounds emitted by or connected with objects in the external world? Man has, no doubt, been well defined a reasonable, or at least a reasoning animal; but he is no less truly, and no less largely, an imitative animal. It may be said that there are more persons in the world who can give true pictures of things by word or line, than there are who can argue about them soundly; for one instance of false portraiture in common conversation, you shall have a hundred exhibitions of bad logic. From the earliest words and actions of the child to the ripest productions of dramatic genius, you have the principle of imitation constantly and intensely at work. Many a literary reputation, exercising a powerful sway over thousands and tens of thousands of delighted readers, rests in a great measure on mimicry, on what may be called a sort of *parrot* work, in the service of reason, no doubt, but not at all dependent upon any high function of reason for its potency or its popularity. It has seldom been heard that the most effective mimics are the most profound reasoners; and, on the other hand, a profound reasoner is often found deficient in that vivid power of imitating the striking points of detail which is the strength of the popular novelist, and the best spice of convivial conversation. There is therefore no presumption against the action of this so universal principle in the formation of language, but rather the contrary. And if the element by which sounds in the external world are signified in human speech is itself sound, how should we more naturally expect the one to express the other, than by some sort of imitation, more or less complete, according to the character of the vocal organ? I go, then, to nature, prepared to expect imitative phenomena in human speech; and I find them, not one here and one there, but everywhere in the richest abundance. Can any one hear the English words *smash*, *dash*, *thump*, *dumb*, *squeak*, *creep*, *clatter*, *chatter*, *click-clack*, *ding-dong*, *sigh*, *sob*, *moan*, *groan*, *hurry-skurry*, *skimble-skamble*, *wiggle-waggle*, and not believe that these words were framed by the human voice, with the express intention, more or less successfully realised, of giving a dramatic representation of the thing signified? This is so obvious, that, as already stated, Professor Müller has been forced to admit it, to a certain extent; but, at the same time, watches with the sternest jealousy that the action of such a principle shall not be allowed to travel beyond the narrow precincts of the poultry-yard and the pig-sty. But, however he may wish to circumscribe the operation of the principle, it is quite certain that it acts not only most powerfully in the low region here indicated, but that this pictorial power of words is one of the most powerful instruments by which human speech is made to affect the human imagination, and becomes an instrument in the skilful wielding of which one of the great merits of a great poet has always been felt to consist. When, for instance, Homer says:—

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* On the Zambesi, Notes of a long Journey. By JAMES STEWART. (Good Words, Feb. 1865.)

> Δουπῆσεν τε πεσὼν ἀραβῆσε δὲ τεύχεα ἐπ αὐτῷ.
>
> "*With a hollow sound he smote the ground, and his armour rattled o'er him;*"

or Göthe—

> "*Aus dem hohlen dunklen Thor*
> *Drängt sich ein buntes Gewimmel hervor,*"

every one feels that the poet, under the influence of the rhythmical instinct which is an expression of reason, is only using the materials of language for producing an æsthetical effect, on the same imitative principle by which these materials themselves were originally framed. And we can prove the actual making of words on this principle from observation. A happy father calls his child "*little goo-goo!*" Why? Because the little creasy-armed, chubby-faced Hopeful has a throat, and *g* is a guttural letter; and, therefore, as naturally as a chicken cries *pip, pip*, the baby sends forth *goo, goo*, as the first notice of its march into the realm of articulate speech; and the delighted parent, by the exercise of the parrot faculty, immediately forms a name for his son, which might have remained for ever, as the only name it should get, did not the conventional rights of baptism interfere, not to mention the long prescriptive claim in favour of *baby* and *boy*, which the labial letters from old Greek and Roman days have succeeded in establishing against the guttural. For I certainly do believe, whatever may be said to the contrary, that the Hebrew word *em*, the Greek μαῖα and μήτηρ, the German *Amme*, and the common English *ma'*, *pa'*, and *baby*, have something to do with the use of the labial letters, so natural to the toothless gums of children, and so obvious in the cries of certain animals. Of the consonants indeed, which brutes use to modify their vocal cries, of which the vowel is always the grand element, the labials and gutturals, along with the snarling R, the rolling L, and the sibilant S, seem to be the most common. We shall not therefore be surprised to find an ox called *Bo* in Latin, Greek, and Gaelic, or to hear the bellow of oxen called μυκᾶσθαι in Greek, while the bleat of sheep is called μηκᾶσθαι, and the cry of goats, in German *meckern*, for which I do not know that we have a specific word in English. And if the Greeks say ὑλακτῶ for the bark of a dog, it is not because their language is not mimetic in this case, while ours certainly is, but because ὑλακτῶ is merely a lengthened derivative form of the root ὑλ, which is only a feebler form of our English *howl*, German *heulen*. In the same way that the letter R in the Greek κορώνη, the Latin *corvus*, the Hebrew ערב, and the English *crow*, has something to do with the sound uttered by that class of animals, I shall continue to believe, without any reference to GRIMM's Law, so long as in the world of animated sound neither swallows shall have been heard to grunt on the eaves, nor pigs to twitter in the sty, nor bulls to mew in Bashan, nor cats to bellow at the fireside.

Let so much therefore be allowed,—be held as admitted,—though not without manifest unwillingness by those who disown the principle we now advocate. But now comes the more important question, for the sake of which alone the preceding examples have been given, as a sort of postulate, rather than as demanding proof. Is this all? If only a few names of animals, and certain phenomena in nature always accompanied by sound, are to be explained by the principle of pictured articulation, we are advanced but a very short way, and the great body of the roots of a language, expressing not *sounds* but *notions*, remains unexplained. When I express the idea of thinking in Latin by the root *med*, in Greek by μητ, and in English by *think*, what possible connection can such words have with screaming, or grunting, or twittering, or with the cry of any unreasoning animal? For man, as a reasoning animal, must have a method of proceeding in forming his language, altogether different from the procedure which would suffice for unreasoning brutes; his discourse is not only φωνή, mere *voice*, but it is λόγος, that is simply the outside of reason, and expressed in Greek (as all the world knows) by the word which likewise signifies reason. Depend upon it, all the important roots of a language must be *notional;* otherwise, we suppose man acting without reason, and our philosophy sinks into the lowest sensationalism of the French school of the last century.

Now, before answering this argument, I must again protest distinctly against the presumption here implied, that the assertion that we do any thing without the intervention of conscious notions and ideas is degrading to man, and ignores that reason which is his characteristic. We eat, drink, sleep, love, hate, dance, fly into sublime passions, and write lofty poetry, not without reason, indeed, but certainly in nowise by virtue of consciously worked out products of reason, called abstract ideas. If it should be found, therefore, that certain words denoting mental action are only a secondary application of words originally painting an outward mechanical action or position, or even a mere sound, I see nothing to be ashamed of in the matter. A man may make himself a pig, or worse than a pig in many ways, but certainly not merely by painting a pig-sty, or by ventriloquizing a grunt, or even by borrowing a grunt, for the expression of some moral or metaphysical idea. The degradation to a reasonable being in the matter of language consists, not in the borrowing from physical sources, but in not submitting the borrowed physical material to a native metaphysical treatment.

This premised, we remark that it is a known tendency of language to grow, not by the creation of new roots, when they are not necessary, but by a dexterous use of the stock already acquired. In harmony with this fact, we have a right to suppose that the original framers of language having succeeded, by the principle of phonic imitation, in making a vocabulary to express the sounds made by animals or sounding bodies, and the related names by which these should be known, would not stop here, but would proceed to apply the same principle to a much wider and more important range of ideas. Nor was the stepping-stone far to seek, by which they soon learned to pass from the domain of single imitated sounds to the domain of actions generally, and of all sorts of ideas. For if we attend to the process of nature in such cases, we shall observe three facts which would necessarily help to work out the original stock of strictly pictorial words imitating mere sounds, to a large class of words, including all the most important verbs which language in its early stages required. The first of these facts is, that most actions which attract the notice of men are, in the first place, accompanied by certain sounds or noises, which serve to indicate the approach, and to express the manner and intensity of their energy. The second fact is, that between sounds and certain feelings and ideas, not accompanied by any sound, there are certain strong analogies, such as that which the blind man indicated, when he said that he thought scarlet colour was like the sound of a trumpet; and these analogies, taken advantage of by the dexterous and economic framers of language, would necessarily lead to the designation of a number of ideas expressive of noiseless vision or touch, by words possessing some vocal and audible analogy. The third fact is, that all external impressions made upon our senses, which, if not the cause, are certainly one of the necessary factors of all human knowledge, are never expressed without the production of certain pleasant or unpleasant feelings, and certain affection of the nervous system, on which the utterance of articulate sound depends; and as effects always correspond to causes, it cannot but be that the vocal utterance from within educed by any strong impression from without, shall in some way or other represent the character of the source from which it sprang. Let us examine these three facts separately, and see to what classes of results in the formation of language they unavoidably lead. Take the word *Kill* to begin with. You ask what connection is there between the sound of this word and the action signified? I reply, that I do not know, because there are many words in all languages, derivative both in meaning and form, whose original type is not now recoverable; but there is another *English* word, *slay*, signifying the same thing, the original form of which is the German word, *schlagen*, to *strike*, and here I distinctly see a phonic congruity between the rough action signified, and the rough word *Schlag*, by which it is expressed. The act of striking is generally accompanied by a hard, sharp noise; and so, hard, sharp syllables, as in the English words, *knock*, *rap*, are used to express that act. Or take the Sanscrit root MAR, of which MÜLLER has made so much, and who does not see that it expresses something rude and harsh, as much as the English word *crush*, and the French word *ecraser?* In the same way the root AR, signifying to plough, and which appears in the Hebrew ארץ the earth, as well as in the Greek adverb ἔραζε, is evidently a phonetic expression of the rough sound of earth or gravel when stirred, containing a combination of letters which, when inverted, appears in *gravel*, *grain*, γράφω, *scratch*, χαράσσω.

In the same way, actions accompanied by slender soft sounds are expressed by weak vowels, as to *creep*, to *sneak*, and to *slink*. Is it not also plain, that whether we take the Greek κλέπτω or the English *steal*, we find that these words are so formed as to present a dramatic contrast to ἁρπάζω and *rob*, which signify the same kind of abstraction, accompanied with violence and noise? And if you say that the Latin *fūr* does not express anything of this kind, I thank you for the observation, and reply that the Greek verb φωράω, from which *fūr* is derived, does not originally imply the silent stealthiness of felonious appropriation, but rather the sudden, rude act, by which a thief is apprehended. Contrast with these words the English word *tumble*, and you will observe that the awkward, clumsy, hollow roll with which the act of accidental falling is generally accompanied, finds expression here to such a degree that the words to *tumble* and to *stand* seem as much opposed to one another as a round rowley-powley pudding is to the sharp, thin, clear knife which cuts it. And this brings me to my second great fact,—Why has the word *knife* a *k* in it? Why the Gaelic *sgian*, why the Latin *culter?* Is this altogether accidental? Certainly not. K is a sharp letter, perhaps the sharpest in the alphabet, and therefore in all languages appears in words which signify sharpness, as in the Latin word *acies*, Greek ἄκρος, Sanscrit *krit, to cut,* with the Latin *cædo,* and probably the Gaelic *cath*, a *battle*. The Greek κόπτω contains the same initial letter, although from the intrusion of the labial π it is a less perfect word to express a clean, sharp stroke than the simple dental which appears in the other roots. For the labials, being uttered by rounded, unpointed organs, are naturally used to express bluntness, as the very word *blunt*, Greek ἀμϐλύς, plainly proves. Hollow vowels and hard consonants will in all cases be applied to express the reverse of what is sharp and thin. So *tundo* in Latin is to beat, not sharply, like our word *rap*, but broadly and bluntly, as with a mallet. Hence *obtundere aures*, to bore a person with talking, to be constantly beating, and thumping, and drumming your crotchets upon the tympanum of his ear. So, when a man's intellect is not very sharp, he is said to be *muddled* or *fuddled;* and if muddled is only a verbal form of *mud*, you will easily understand that something soft, broad, round, not at all clear, and not very stable, is understood by the verb as well as by the noun. We thus see how not only sound, but everything perceptible to vision or to touch—that is to say, the whole range of phenomenal knowledge—comes under the derided principle of ὀνοματοποιία; and if there can be any stronger proof given of the unlimited range of articulate sound, in mimetically expressing things which have nothing to do with sound, the English word *mum*, for silence, contains that proof. M is the labial which most completely closes the lips, and sends the breath up through the nose; hence it appears in the Latin *mutus*, the Greek μύω for closing or shutting, not the mouth, but the eyes, and in the English *dumb*, which in German is *dumm*, stupid, because stupid people have often the sense to sit silent in company, and thus not betray their stupidity. I conclude these illustrations of the second of the three great facts by a remark on the word *stand*, previously used. This word, which is a bastard present, formed from the old past tense, like the Alexandrian Greek στήκω, has for its root the Sanscrit *sthâ*, in Latin *stare*. Now, any one may see that this word stands more firmly on its legs than the word *tumble*, with which we contrasted it. Why is this? There is no firmness or decision in any part of this word, just as in the cognate word *mumble* there is a plain want of determinative emphasis in the conglomeration of the letters. But when I say *sta*, I bring my teeth together with a decision which shows that I am suiting the word to the action, and that the firmness which I exhibit in the muscles of my legs is not to be accompanied with any looseness in the action of my jaws. And that this is not a mere fancy will be obvious to any one who considers the wide application which this combination of letters enjoys in words expressive of strength and decision in all languages. Thus in English, *stop*, *strength*, *strike*, *stride*, *sturdy*, *start*; in Greek, στράγγω, στρέφω, στρήνης, στρυφνός, στείβω, most of which have their Latin representatives, as *stringo*, *strenuus*, *stipo*. So in German, *starr*, *streng*, *stössig*; and many others. There remains now, to complete the pictorial process by which language is formed, the third fact mentioned above—according to which all external expressions necessarily affect in a certain way the whole nervous system and mental economy, and through the motion in the vital spirits thereby produced, modify in a corresponding way the articulation of human speech. Here we have a different principle altogether, as it would appear at first sight, from mere ὀνοματοποιία; for to imitate an internal sound, and to express an internal feeling, seem not only different, but quite contrary actions. Nevertheless, they are in their effects, as in their origin, substantially one; and Professor MÜLLER has accordingly put what he calls the POOH! POOH! theory as much under his ban as the BOW-WOW! For the fact of the matter is, that an interjection, such as *ah!* or οἴμοι, or *eheu*, and all such vocal expressions of pleasure or pain, must, by the laws of vitality, exhibit a certain correspondence with the sensations of which they are the expression. Thus any oppressive, heavy feeling in the chest will naturally cause a slow, protracted, dull flow of breath to proceed from the throat. The vowels α and ω, the diphthongs *ai* and *oi*, are exactly such a flow of breath. Hence the interjections ω, αι, οι, amplified into the verbs ὤζω, αἰάζω, οἰμώζω.

There is here, therefore, a sort of natural drama enacted—a correspondence of the *within* and *without*—which springs fundamentally out of the same root as the ὀνοματοποιία proper. When Aristotle called all poetry mimetic, he probably meant something of this kind; for while dramatic poetry only is strictly imitative of outward objects, lyric poetry is dramatically expressive of inward feelings; and to this the *Bow-wow* and the *Pooh! pooh!* departments of early word-making plainly correspond.

If we now inquire what the objections are that are brought against these facts, indicative of the operation of the pictorial principle in the world of vocal utterance, we find that they require no very laboured refutation, but resolve themselves into a few misunderstandings and prejudices, which a single touch can brush aside. In the first place, if it ever was asserted by any writer that all the presently existing roots in any language are onomatopœtic, and that all current words are to be explained on this principle alone, with such assertion I have nothing to do. I only maintain that the original stock of which language was made up consisted of such roots, and that a great proportion of them, after the changes of thousands of years, bear their origin distinctly on their face. I do not say, however, that all the words now existing in a language are to be dealt with on the supposition that they contain some pictorial element of the original phonic drama of human speech. Syllables are like sixpences, and are apt to be rubbed down in the course of time, till their original image and superscription can no more be traced.

Besides, as in the Greek language the word ἀδελφός, signifying *uterinus*, or born of the same womb, took the place of φράτωρ, which no doubt originally was used as *frater* in Latin, *bhṛātri* in Sanscrit, and *brother* in English, so many of the oldest dramatically significant roots of language may have been replaced by secondary roots, in which the real character that belonged to the first pictorial roots is lost. I do not therefore deny that *equus* may come from the root *ā́'su*, *swift*, and a horse signify the swift animal. Though I have no doubt that *bo*, an ox, is merely a human imitation of the bovine sound, I by no means insist that all animals should have received their names from the cries which they make. I only say that, in the original formation of language, this was one of the simplest and most obvious methods of designation, and a method that extended a great deal further than superficial observation might lead the modern speculator to believe.

As little can I see why Professor MÜLLER should feel it his duty to declare war wholesale against onomatopœia in language, because on this or the other occasion some men have handled it wildly, and ridden rough-shod with it over GRIMM'S law, and the whole body of ascertained facts with regard to phonic transmigrations and transmutations. A man may talk ingenious nonsense on any branch of philological science with the utmost ease, in the teeth of GRIMM'S law, or even with the help of it; but that great principle of interlingual change has nothing to do with the question how roots, variable according to certain laws of phonic change, were originally formed. The Sanscrit *pitṛi* may become the English *father*, and the Scotch *fader*, without touching the question whether *PA* and *MA* have anything to do with imitation by parents of the first untutored labial utterances of a child. Finally, I must be allowed to express my conviction that the opposition to onomatopœia seems to arise in the minds of some speculators partly from a certain horror of a sort of merely animal element in the creation of language, which in ancient times had found acceptance with the low sensuous philosophy of EPICURUS,[*] and partly, so far as the Germans are concerned, from a certain instinct in them which leads them to prefer what is remote to what is obvious, what is conceptional, to what is sensational, what is fanciful to what is real, what is mystical to what is plain. If they blame us, not unjustly altogether, for having no ideas in our scholarship, we may with equal reason retort that they have too many, and use them often with a wild ingenuity, rather than with a sober discretion. If we do not make such brilliant discoveries as they do beyond the flaming walls of the universe, we do not, on the other hand, so often fail to see what directly lies before us. The same national habit of thought which led FORCHHAMMER to find in the Iliad a geological account of the struggle betwixt land and water in the Troad, and leads Professor MÜLLER to discover in the same great historic tradition a mythological fight between light and darkness, seems to determine the position of this distinguished philologer in reference to the original formation and growth of roots in language. How they were formed he nowhere tells us; he does not pretend to know; but of one thing he feels assured, that there is more of mystery in the matter than the easy mimicry of natural sounds can explain. "Are not Abana and Pharpar rivers of Damascus? may I not wash in them and be clean?" He will have nothing to do with word-painting, because it is too simple a process, seems to deal with facts rather than with ideas, and is not at all mysterious. For my own part, I think all is mysterious with language in one sense, nothing in another. It is as natural for men to speak, as for birds to sing, and fountains to flow; and that when they did speak, they spoke originally from imitation of natural sounds, and a cunning adaptation of the expressive power of the audible element, not only to things audible, but also to things visible and tangible, I shall continue to believe till some principle shall be propounded that may explain all known facts in a manner equally obvious and satisfactory.

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[*] MÜLLER, vol. ii. p. 87, quotes a passage of PROCLUS from EPICURUS as having suggested his soubriquet of the *Bow-wow* theory.

I have only to say in conclusion, that my faith in imitation as the great principle in the formation of the original stock of human speech, is not in any degree affected by the vexed question whether man was originally created full-grown or a baby, whether he made language for himself, or got it, as some think there is a peculiar piety in imagining, ready-made from the Deity. I do not believe that Adam got language ready-made from his Creator, for the very plain reason that we get nothing ready-made from the Creator, but we make it ourselves after a fashion, by the indwelling power of His infinite virtue and grace, who is never far from the meanest of His creatures. But even if the Supreme Being did make a present to our primal sire of a ready-made language (though I think this contrary to the words of Moses in Genesis ii. 19), still the fact remains that the grand vocal organism so presented, bears on its front the most evident marks of an onomatopœtic or imitative construction. Those, therefore, who hold that God made human language must maintain that He made it on the same principle on which I maintain that man made it; for the facts are undeniable; and surely it cannot be more pious to suppose that the Father of all men coined words for the use of His reasonable children in a manner altogether arbitrary, rather than on the principle of a reasonable congruity, and a beautiful adaptation.

II.—*On the Cause and Cure of Cataract.* By Sir **David Brewster**, K.H., F.R.S.

(Read 16th January 1865.)

My attention was called to the subject of Cataract, in consequence of having, about forty years ago, experienced an incipient attack of that complaint, and studied its progress and cure.

While engaged in a game at chess with Sir **James Hall**, who was a very slow player, I amused myself in the intervals with looking at the streams of light which radiated from the flame of a candle in certain positions of the eyelids. In one of these observations I was surprised by a new phenomenon, of which I did not immediately see the cause. The flame of the candle was surrounded with lines of light, of an imperfectly triangular form, some parts of which were deeply tinged with the prismatic colours. Upon going home from the chess club, this optical figure was seen more distinctly round the moon, and of course it appeared, with more or less brightness, round every source of light.

Having been engaged in examining the structure of the crystalline lens in animals of all kinds, I soon discovered the cause of the phenomenon which I have described. The laminæ of the crystalline lens had separated near its centre, and the separation had extended considerably towards its margin. The albuminous fluid, the *liquor Morgagni*, which so wonderfully unites into one transparent body, as pure as a drop of water, the mass of toothed fibres which compose the crystalline lens, had not been sufficiently supplied, and if this process of desiccation had continued, the whole laminæ of the lens would have separated, and that state of white opacity induced, which no attempt has ever been made to remove.

The continuance of this affection of the lens was naturally a subject of much anxiety, and I never entertained the slightest hope of a cure. My medical friends recommended the use of what were then called Eye Pills, but having received no benefit from them, and having learned from experience the sympathy between the eye and the stomach, I used every day, and copiously, the *Pulvis salinus compositus*, and at the end of about eight months, when playing at chess in the same apartment, I had the happiness of seeing the laminæ of the lens suddenly brought into optical contact, and the entire disappearance of the luminous and coloured apparition with which I had been so long haunted.

In speculating on the process by which the crystalline lens is supplied with the necessary quantity of fluid, it occurred to me that it might be derived from the aqueous humour, and that cataract might be produced when there was too little water and too much albumen in the fluid which filled the aqueous chamber.

Upon this hypothesis, incipient cataract might be cured in two ways:—

1st, By discharging a portion of the aqueous humour, in the hope that the fresh secretion, by which the loss is repaired, may contain less albumen, and counteract the desiccation of the lens.

2d, By injecting distilled water into the aqueous chamber, to supply the quantity of humour discharged from it.

The first of these methods I knew to be practicable and safe, from the fact that a surgeon in the Manchester Infirmary, many years ago, tapped the aqueous chamber of a female patient forty times, in the vain hope of curing a case of conical cornea, which he attributed to an excess of aqueous humour. The frequent repetition of this operation shows how rapidly the humour is secreted, and how reasonable it is to suppose that, in the case of cataract, a healthier secretion might be produced under medical treatment.

Although the second method of injecting distilled water into the aqueous chamber presents greater difficulties, yet they do not appear to be insuperable. In 1827, when I happened to be in Dublin, I mentioned this method to the celebrated comparative anatomist, Dr Macartney, who considered it quite practicable. He mentioned to me that a foreign oculist, whose name I forget, had actually injected distilled water into the eye of a patient with the view of supplying the aqueous humour that was lost during the extraction of the lens.

My attention was recalled to these suggestions for treating incipient cataract, by the results of a series of experiments on the changes which take place in the crystalline lenses of the sheep, the cow, and the horse, after death. In these experiments, which were published in the Philosophical Transactions for 1837, the lenses were placed in a glass trough of distilled water, and exposed to polarised light; and the changes thus produced were indicated by variations in the number and character of the polarised rings, and more palpably by the gradual enlargement of the lens. The distilled water passed through the elastic capsule of the lens. The lens increased in size daily, and at the end of several days the capsule burst, leaving the lens in a disorganised state, the outer laminæ being reduced to an albuminous pulp by the water admitted through the capsule.

These experiments have an obvious importance in reference to the cause and cure of the two kinds of cataract to which the human eye is subject. The aqueous humour is in immediate contact with the capsule of the crystalline lens. When the humour, therefore, contains too little water, the lens has not a sufficient supply of the fluid which keeps its fibres and laminæ in optical contact, and hence the laminæ separate, and the lens becomes opaque and hard. When, on the contrary, the aqueous humour contains too much water, the capsule introduces the excess into the lens, and produces the more dangerous affection of soft cataract, in which it is difficult either to depress or extract the lens.

In order to cure the first of these kinds of cataract, we must discharge a portion of the aqueous humour, and either supply its place by injecting distilled water, or leave it to nature to supply a more healthy secretion. In order to cure the second kind, we must supply the place of the discharged humour with a solution of albumen; or, as in the first case, leave to nature the production of a more albuminous secretion.

These views have received a remarkable confirmation from recent experiments on the artificial production and removal of cataract in the eyes of animals. Dr KIND, a German physiologist, whom I met at Nice in 1857, informed me that he had produced cataract in guinea-pigs, by feeding them with much salt, and that the cataract disappeared when there was no salt in their food. More recently, Dr MITCHELL,* an American physician, produced cataract by injecting syrup into the subcutilar sacs of frogs; and Dr RICHARDSON† did the same by injecting syrup into the aqueous chamber of the recently dead eye of a sheep. In the experiment of Dr MITCHELL, the cataract was removed from the living eye of the frog by surrounding the animal with water; and in that of Dr RICHARDSON, the cataract was removed from the dead eye of the sheep by replacing the syrup with distilled water.

Neither Dr MITCHELL nor Dr RICHARDSON seem to have been acquainted with my experiments on the changes in the lens after death, published in 1837, and with the theory of the cause and cure of cataract there referred to; nor with the distinct statement of it published in 1836,‡ and twenty years later, in 1856.§ Dr RICHARDSON, however, has borne ample testimony to its practicability and safety, when he suggests, almost in my own words, "that it would be worth while, in the earliest stage of cataract in the human subject, to let out the aqueous humour, and to refill the chamber with simple water." And he has borne a still stronger testimony to its value by congratulating "Dr MITCHELL in having been the earliest experimentalist to elucidate the synthesis of cataract, and to take the first steps towards a rational interpretation of the disease."

The tendency of the human crystalline lens to indurate or soften, by a defect or excess of water in the aqueous humour, may occur at any period of life, and may arise from the general state of health of the patient; but it is most likely to occur at that age, between 40 and 60, and often much earlier, when the lens experiences that change in its condition which requires the aid of spectacles. This change commences at one part of the margin of the lens, where its density is either increased or diminished. Its action in forming a picture on the retina thus becomes unsymmetrical, and vision is sensibly impaired. But when the change has taken place round the margin of the lens, its symmetrical action is restored, and by the use of spectacles the vision becomes as perfect as it was before the change. If glasses are not used when the change is completed, the eye must either strain itself, or use a strong light, to produce distinct vision in reading the small type and the imperfect printing which characterises the daily press; and by both these processes it will, in a greater or less degree, be injured.

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* American Journal of the Medical Sciences. January 1860.

† Medical Times and Gazette. March 31, 1860.

‡ Report of the British Association, 1836, p. 111; and 1837, p. 12.

§ North British Review, vol. xx. p. 167. November 1856.

It is a strange delusion, arising either from ignorance or vanity, which induces most people to put off the use of spectacles as long as possible. From the instant they are required, spectacles of different focal lengths ought to be used for the different purposes for which distinct vision is required, and the eyes should never do any work, unless they can do it with perfect distinctness and satisfaction. There is no branch of the healing art where science comes so directly and immediately to the relief of impaired functions as that which relates to vision, and none where science has been so imperfectly applied. When the change in question takes place, the eye requires to be carefully watched, and used with the greatest caution; and if there is any appearance of a separation of the fibres or laminæ, those means should be adopted which, by improving the general health, are most likely to restore the aqueous humour to its usual state. Nothing is more easy than to determine the condition of the crystalline lens; and by the examination of a small luminous object placed at a distance, and the interposition of small apertures, and small opaque bodies of a spherical form, we can ascertain the exact point in the lens where the fibres and laminæ have begun to separate, and may observe from day to day whether the disease is gaining ground or disappearing.

[Since the preceding paper was read I have seen a remarkable work, entitled "*Etudes Cliniques sur l'evacuation de l'Humeur Aqueuse dans les Maladies de l'Œil*," par CASIMIR SPIRINO, Turin, 1862. Pp. 500. M. SPIRINO had, in the course of little more than a year, operated upon forty-five cases of cataract. In many of these the cataract was perfectly cured, and in others the sight was improved. The first case was that of a lady of eighty-one, who had cataract in both eyes. After thirty-two evacuations of the aqueous humour by the same aperture, and almost always two or three times at the same sitting, both cataracts disappeared, the lady was able to read, without glasses, Nos. 3 and 4 of JAEGER's scale, at the distance of 4 or 5 inches, and even to thread a small needle.]

III.—*On Hemiopsy, or Half-Vision.* By Sir **David Brewster**, K.H., F.R.S.

(Read 20th February 1865.)

The affection of Half-vision, or Half-blindness as it has been called, was first distinctly described by Dr **Wollaston**, in a paper " On Semidecussation of the Optic Nerves," published in the Philosophical Transactions for 1824. " It is now more than twenty years," he says, " since I was first affected with this peculiar state of vision, in consequence of violent exercise I had taken for two or three hours before. I suddenly found that I could see but half the face of a man whom I met, and it was the same with every object I looked at. In attempting to read the name **Johnson** over a door, I saw only **son**, the commencement of the name being wholly obliterated from my view. In this instance, the loss of sight was towards my left, and was the same, whether I looked with my right eye or my left. This blindness was not so complete as to amount to absolute blackness, but was a shaded darkness, without definite outline. The complaint lasted only about a quarter of an hour." In 1822, Dr **Wollaston** had another attack of hemiopsy, with this difference, that the blindness was to the right of the centre of vision, and he has referred to three other cases among his friends; but in these, the affection was accompanied with headache and indigestion.

In republishing Dr **Wollaston**'s paper in the "*Annales de Chimie et Physique*,"* M. **Arago** says, that he knows *four* cases of hemiopsy, and that he himself had experienced three attacks of it, followed by headache above the right eye.

In the " Cyclopædia of Practical Surgery," published in 1841, Mr **Tyrrell** describes Hemiopsy as " Functional amaurosis from general disturbance." He informs us that " he has experienced this form of amaurosis several times," and that he has been consulted by several fellow-sufferers of both sexes. In all these cases the affection was attended with severe headache, giddiness, and gastric irritation, sometimes preceding, and sometimes following, the attack.

In the accounts which have been given of these different cases of hemiopsy, no attempt has been made to ascertain the optical condition of the eye when it is said to be half-blind, or to determine the locality and immediate cause of the complaint. Dr **Wollaston** describes the blindness as a shaded darkness without definite outline. M. **Arago** says nothing about darkness; and the insensibility of the retina, of which he speaks, must mean its insensibility to visual and not to luminous impressions. Mr **Tyrrell**, on the other hand, simply states, that the obscurity takes place in different portions of the retina, and varies in its extent at different times.

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* 1824, vol. xxvii. p. 109.

Having myself experienced several attacks of hemiopsy, I have been enabled to ascertain the optical condition of the retina when under its influence, and to determine the extent of the affection, and its immediate cause.

In reading the different cases of hemiopsy, we are led to infer that there is vision in one-half of the retina, and blindness in the other. But this is not the case. The blindness, or insensibility to distinct impressions, exists chiefly in a small portion of the retina to the right or left hand of the *foramen centrale*, and extends itself irregularly to other parts of the retina on the same side, in the neighbourhood of which the vision is uninjured. In some cases the upper half of the object is invisible, the part of the retina paralysed being a little below the *foramen centrale*. On some occasions, in absolute darkness, when a faint glow of light was produced by some uniform pressure upon the whole of the retina, I have observed a great number of black spots, corresponding to parts of the retina upon which no pressure was exerted.

In the case of ordinary hemiopsy, as observed by myself, there is neither darkness nor obscurity, the portion of the paper from which the letters disappear being as bright as those upon which they are seen. Now, this is a remarkable condition of the retina. While it is sensible to luminous impressions, it is insensible to the lines and shades of the pictures which it receives of external objects; or, in other words, the retina is in certain parts of it in such a state that the light which falls upon it is irradiated, or passes into the dark lines or shades of the pictures upon it, and obliterates them. This irradiation exists to a small degree, even when the vision is perfect at the *foramen centrale*, and it may be produced artificially in a sound eye, on parts of the retina remote from the foramen, and as completely, though temporarily, as in hemiopsy. In order to prove this, we have only to look obliquely at a narrow strip of paper placed upon a green cloth, that is, to fix the eye upon a point a little distant from the strip of paper. After a short time the strip of paper will disappear partially or wholly, and the space which it occupied will be green, or the colour of the ground upon which it is laid.*

This temporary insensibility of the retina in the part of it covered by the picture of the strip of paper, or its inability to maintain constant vision of it, can arise only from its being paralysed by the continued action of light, an effect not likely to be produced, and never observed, in the ordinary use of the eye.

The insensibility of the retina, in cases of hemiopsy, and the consequent irradiation of the light into the space occupied with the letters, or the objects which disappear, though a phenomenon of the same kind as that which takes place in oblique vision, has yet a very different origin. The parts which are in these cases affected extend irregularly from the *foramen centrale* to the margin of the retina, as if they were related to the distribution of its blood-vessels, and hence it was probable that the paralysis of the corresponding parts of the retina was produced by their pressure. This opinion might have long remained a reasonable explanation of hemiopsy, had not a phenomenon presented itself to me, which places it beyond a doubt. When I had a rather severe attack, which never took place unless I had been reading for a long time the small print of the *Times* newspaper. and which was never accompanied either with headache or gastric irritation, I went accidentally into a dark room, when I was surprised to observe that all the parts of the retina which were affected were slightly luminous, an effect invariably produced by pressure upon that membrane. If these views be correct, hemiopsy cannot be regarded as a case of amaurosis, or in any way connected, as has been supposed, with cerebral disturbance.

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* Letters on Natural Magic. Let. II. p. 13.

Dr WOLLASTON endeavoured to explain the phenomena of hemiopsy, and the fact of single vision with two eyes, by what he calls the semidecussation of the optic nerves, a doctrine which Sir ISAAC NEWTON had suggested, and employed to account for single vision.* A fibre of the right-hand side of the optic nerve is supposed to decussate or divide itself into two fibres, sending *one* to the right side of the right eye, and *another* to the right side of the left eye, while a fibre on the left-hand side of the optic nerve also decussates, sending *one* fibre to the left side of the left eye, and *another* to the left side of the right eye. Hence, Sir ISAAC NEWTON drew the conclusion, that an impression on each of the two half fibres would convey a single sensation to the brain; and hence, Dr WOLLASTON concluded that hemiopsy in one eye must be accompanied with hemiopsy in the other.

Ingenious as these explanations are, the anatomical facts by which alone they could be supported have not been established. Dr ALISON,† who has adopted the opinion of NEWTON, and reasoned upon it, admits that the anatomical evidence is still defective; and the late Mr TWINING‡ has adduced nine cases of disease in the optic nerves and thalami, which stand in direct opposition to the hypothesis of semidecussation. Dr MACKENZIE, too, adopting the same view of the subject as Mr TWINING, distinctly asserts that "the great mass of facts in Pathology and Experimental Anatomy, touching this question, go to prove that injuries and diseases affecting one side of the brain, instead of *hemiopsia* in both eyes, produce amaurosis only in the opposite eye."

The two great facts of hemiopsy in both eyes, and of what is called single vision with two eyes, do not require the hypothesis of semidecussation to explain them. If hemiopsy is produced by the distended blood-vessels of the retina, these vessels must be similarly distributed in each eye, and similarly affected by any change in the system; and, consequently, must produce the same effect upon each retina, and upon the same part of it.

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* Optics, p. 320.

† Edinburgh Transactions, vol. xiii. p. 479.

‡ Trans. Med. Soc., Calcutta, vol. ii. p. 151; or, Edin. Journal of Science, July 1828, vol. ix. p. 143.

In explaining single vision with two eyes, we have no occasion to appeal to double fibres in the optic nerves, or to corresponding points on the retina. There is, in reality, no such thing as single vision, that is a single image seen by both eyes. With two sound eyes every object is seen double, and it appears single only when, by the law of visible position, the one image is placed above the other. But even in this case the object is seen double, by means of two dissimilar images of it which are not coincident. By shutting the right eye, we lose sight of a part on the right side of the double image, which is seen only by the right eye; and by shutting the left eye, we lose sight of a part on the left side of the double image, which is seen only by the left eye. If one eye gives a better picture than the other, the duplicity of the apparently single image is more easily seen. By shutting the good eye the imperfect picture is seen, and by shutting the bad eye we insulate the perfect picture. It is difficult to understand how optical writers and physiologists should have so long demanded a single sensation for the production of a single picture from the two pictures imprinted on the two retinas. If we had the hundred eyes of Argus, the production of an apparently single picture would have been the necessary result of the Law of Visible Position.

IV.—*Miscellaneous Observations on the Blood.* By John Davy, M.D., F.R.S., Lond. & Ed., &c.

(Read 6th March 1865.)

On a fluid of so much importance as the blood, observations with any pretension to accuracy can hardly be too often made and repeated, more especially when we consider its great instability, its little uniformity, and the differences of opinion entertained by physiologists respecting some of its most remarkable properties.

Such is the persuasion which has influenced me in engaging in the present inquiry, and in submitting its results to the Society.

I. *On the Action of Water on the Red Corpuscles of the Blood.*

As is well known, the red corpuscles are altered in form and appearance on admixture with water, the most obvious change being, that from discs they expand into globules.

In some trials made with the view to ascertain something more precise, I have selected the blood of birds, that chiefly of the common fowl and duck, the corpuscles of their blood, from their elliptical shape, being peculiarly fit, as it seemed, for the inquiry.

The first trials made were to ascertain the proportion of water that was required to effect any material change. The results obtained were the following:—

When one measure of water was added to one of serum holding red corpuscles in suspension, but few of them experienced an immediate change of form and became globular.

On the addition of two of water, the majority of the corpuscles underwent this change, a few only retaining their normal form.

On the addition of three of water, none of a normal form could any longer be seen; all that were visible were rounded, much reduced in apparent size, and were much less distinct; indeed, a nice adjustment was required to detect them. Many of them had a jagged outline; and from some there was a slight projection, suggestive of a rupture of their capsule.

Dried by evaporation at about 100° Fahr., very many of them were found to have recovered their original form and size. Some of them, however, appeared to be ruptured, the excluded nuclei adhering to their surface; others retained their nuclei, of irregular appearance; all appeared to be wasted.

On the addition of four of water, the corpuscles were seen less distinctly, yet they were to be seen, the adjustment being as accurate as possible, and using a warm object-glass, a precaution needed to prevent the dimming of the glass ($\frac{1}{8}$th inch power) from the vapour rising from the fluid in such close proximity.

When more water was added, the only material difference that I am aware of was not in the effect on the corpuscles, but in their wider diffusion, thus increasing the difficulty of observing them. To counteract this, a portion of cruor was mixed with water in a tall vessel, stirred occasionally, and after having been some hours left at rest, the greater part of the coloured fluid—coloured by the solution of the colouring matter of the corpuscles—was drawn off. What remained afforded an interesting result. A drop of this fluid under the microscope exhibited much the same appearance as that from the addition of four parts of water; and on drying at the same temperature, the appearances were also similar but more strongly marked, suggestive of ruptured capsules and the loss of their contents, with the exception, as in that instance, of some of them retaining their nuclei, these more or less altered. Most of the corpuscles, if that term be applicable to their remains, were circular or portions of circles, portions of them having been broken of. Some showed a rent, a few were elliptical, and with the exception of being wasted, but little altered in appearance.

The agency of water on the red corpuscles has commonly been attributed to imbibition or endosmosis, to solution of the soluble matter which these cells contain, and to exosmosis. The appearances which I have described seem to harmonise well with this view, with the addition of rupture of the cell-wall or capsule, and the occasional exclusion of the neuclei. They accord, too, tolerably with those noticed by Professor LEHMANN, in his Physiological Chemistry,* with the exception of two particulars. He states, that when largely diluted, the corpuscles become invisible under the microscope, which he attributes to their refractive power, after the action of water, differing but little from that of water itself. As already mentioned, when using certain precautions, I have found them, only much less distinct. The other particular relates to their remains. According to him, these are mere shreds, and not empty and more or less broken capsules, as I have found them to be. The subject, it must be admitted, is one in the investigation of which it is not easy to obtain uniform and satisfactory results, there are so many interfering and disturbing circumstances concerned, not omitting the influence of the serum, and especially keeping in mind the powerful attraction the corpuscles have for water, and their hygroscopic properties; and further, the changes to which they are liable as dead matter, from the influences to which they are exposed.

As to the last mentioned, I have found that the longer the blood is kept, the smaller is the quantity of water that is required to alter their form. As to their hygroscopic property, this is shown by the simple experiment of breathing on them, or by exposing them over water for a few hours, keeping them, of course, out of contact with the water. In the instance of the warm vapour of the breath, one expiration is sufficient to deprive them, previously dried, of their elliptical form.

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* Vol. ii. p. 184.

It is worthy of remark, that when the corpuscles are coloured by the addition of a weak solution of iodine, not only the action of the warm vapour of the breath is in a great degree arrested, but even the action of water, and this after immersion in water on a glass support for twelve hours, when they were found to retain their normal form, only slightly contracted, with their nuclei distinct. May it not be conjectured from this, that iodine medicinally used may operate in a degree similarly, and thus may arrest undue metamorphic disintegration?

II. *On the Changes which take place in the Blood when excluded from the Air.*

The changes to which the blood is subject when exposed to the air, at ordinary atmospheric temperatures, are pretty well known. To endeavour to ascertain what would happen were air as much as possible excluded, the following experiments were made:—

A bottle full of water, deprived of air by the air-pump, was emptied the instant before receiving blood from the divided cervical vessels of a barn-door fowl; so soon as full to overflowing, it was closed with a glass stopper lubricated with oil, and inverted in water.

During the first hour the blood retained its original hue, bright vermilion, and this throughout. After two hours the colour had lost something of its brightness. The following day the colour had become uniformly chocolate brown. The day after there was no appreciable change. The serum which had separated was of a wine yellow, and the crassamentum had contracted considerably. On the third day the serum was beginning to show a reddish tinge. From this day, viz., the 9th of November, to the 4th of December, the serum became of a darker red, and, like the crassamentum, was almost black, as seen by reflected light. During the time mentioned, the temperature of the room in which the blood was kept varied from about 55° to 58°. The bottle was now taken out of the water, which was as clear as at first, showing that the closure was complete. The stopper was drawn out with ease, proving—as it was introduced when the blood was warm—that there was pressure from within rather than from without, though there was no appearance of any gas evolved. The serum decanted was of a dark purplish-red, as seen in thin layers by transmitted light, but black by the same light, and opaque, in a tube of one-half inch diameter. The crassamentum, of the same colour, was soft and easily broken up, and had, as well as the serum, an offensive putrid smell, but less so than if air had been allowed access

With hydrate of lime, both yielded a strong ammoniacal odour. The blood corpuscles, as seen under the microscope, were found to have become rounded and globular. The fibrin seemed to be permanently dyed red; it retained this colour after having been well washed, and after maceration in water for many hours. Its structure was finely granular; it showed no appearance of fibres under gentle pressure, and viewed with a high power.

The experiment was repeated, using the blood of a turkey and also of a bullock. The results were similar. That on the blood of the turkey was of the same duration as the preceding. That on the bullock's blood was begun on the 14th of December, and ended on the 7th of January. In the latter instance, though no gas was evolved, the putrid blood, when subjected to the air-pump, entered into violent ebullition from the copious disengagement of air, and this even before the vacuum was nearly complete. It may also be mentioned, that a silver probe plunged into the clot became, after a few minutes, strongly discoloured, indicative thus of the presence of sulphuretted hydrogen.

A fourth experiment was made with the blood of a duck. In this instance, instead of emptying the bottle of water before receiving the blood, the water, deprived of air, was to a certain amount expelled by the blood as it flowed from the divided vessels. The specific gravity of the mixture was 1033. After having been kept from the 23d of November to the 12th of January, at a temperature varying from about 40° to 50° and 55°, the changes observed on examination were so similar to those already specified that they need not be described.

A fifth experiment was made with the blood of a fowl. As in the last the blood was mixed with water; but it differed from the last in being subjected to the air-pump as soon as it had become sufficiently cool. No air was thus extricated. The bottle was again closed and inverted in water. This was on the 16th of February; it was examined on the 17th of March. Some difficulty was experienced in withdrawing the stopper. The blood bore marks of an incipient putrefaction; its smell was offensive, and some muriate of ammonia was formed on a plate of glass moistened with hydrochloric acid put over it during a few hours.

These results, all so well marked, seem to be nearly identical with those which occur when blood of the same temperature is exposed to the air, almost the only difference that I am aware of being in degree. The change of colour is the same, with the exception, that when exposed to the atmosphere, the blood at the surface, especially if it be venous, becomes florid before it darkens; the change of form of the corpuscles is the same, and the solution in the serum of their colouring matter. The same gases likewise are formed, and the same alkali is generated, accompanied by the characteristic putrid odour.

That blood should thus undergo change when air is excluded, is no more, perhaps, than might be expected when we reflect on its composition, and that oxygen is contained in it in a state, it is presumed, free to act and give rise to putrefactive fermentation.

What is more remarkable is the fact, that blood may be retained in the living body, stagnant, at rest, without undergoing similar changes, at a temperature so favourable to these changes. I may refer to HEWSON's collected works, edited by Mr GULLIVER, for instances of the kind. In a note, page 17, to mention one, the editor remarks, "occasionally blood is extravasated and stagnant in the living body for an indefinite time, and yet retains fluidity, as Mr HUNTER and Mr CÆSAR HAWKINS have noticed." He adds, "I saw a case in a soldier, who had received a bruise in his loins, from his horse bolting with him over a bridge in Hyde Park; the injured part quickly swelled, evidently from effused fluid, which was let out twenty-eight days afterwards. It measured five ounces, was as liquid as blood just drawn from a vein, and coagulated in a cup in less than three minutes. The corpuscles were observed to be unchanged, and readily collected together in the usual way by their broad surfaces. Next day the clot was moderately firm, scarlet at the top, somewhat contracted, and surrounded by a little serum." What a contrast this presents to the blood from which atmospheric air was excluded in the experiments detailed! Can the difference have been owing to the stagnant blood in the living body having been exposed to the action of the surrounding tissues, by which it is possible that, though a change may have been going on slowly in the blood, the degraded or altered particles may have been carried away as they were produced, leaving the residue in its normal state? I have witnessed something analogous when a mass of fibrin, enclosed in a muslin bag, has been immersed in water under a cock, from which there was a constant small stream keeping the water round the included fibrin in motion. During about a month that the fibrin was thus exposed at a temperature of about 40°, it had undergone little change; it was firm and only slightly tainted. In instances of aneurism, it is well known that not only the fibrin, but also the crassamentum enclosed in the sac resist for a long time putrid decomposition. May not this resistance be referred to the same cause? This explanation is submitted conjecturally. The fact that extravasated blood, from contusion and vascular rupture, is commonly absorbed with discoloration of the bruised part, may be adduced as somewhat in its favour. The physiologists of the School of Hunter would doubtless refer the liquidity of the blood, in the case in question, to the vitality of the blood; but that is a doctrine which at present is hardly tenable.

III. *On the Action of the Air-Pump on the Blood.*

The air-pump I have used in the trials I am about to describe is the same as that with which I made some former experiments on the blood,* and, as then, it was in excellent order.

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* Anatom. and Physiolog. Res. vol. ii. p. 214.

The chief precautions taken were to receive the blood as it flowed from the divided vessels of the animal killed into phials, immediately after they had been emptied of water from which the air had been expelled by the action of the air-pump, and after closing with a glass stopper, cooling the blood rapidly by immersion in water.

Though these precautions were taken, I believe they were not absolutely necessary for good results, as I find that when water exhausted of air is poured into a carefully washed phial from which water containing air has been poured out, on submitting it to the air-pump, no air is extricated either from the water or from the side of the phial.

The experiments on exhaustion have been made on the blood of the common fowl, of the duck, of the sheep, bullock, and pig; they have most of them been several times repeated.

The results have varied more than I could have expected, tending to show that the quantity of air extricable from the blood by the air-pump is far from constant, and depends on circumstances, some of which are appreciable, others obscure.

1. From the blood of the common fowl, the quantity of air disengaged has commonly been less than from that of the duck, sheep, bullock, and pig.

2. The blood of all the animals, when taken from them shortly after feeding, has commonly afforded more air than from animals of the like kind when fasting.

3. Florid blood, which it may be inferred is chiefly arterial, has yielded less air than dark blood, which probably is chiefly venous, and, accordingly, that which flows first, when an animal has been blooded to death, less than that which flows last.

4. In a small number of instances, those of animals killed after a fast of many hours, the fresh blood yielded no air. In some of the trials which gave this result, the blood was mixed as it flowed with an equal quantity of water deprived of air.

5. In no instance have I witnessed the disengagement of air from fresh serum, proving that the air, when extricated from the blood, is derived from the clot, and it may be presumed, from the red corpuscles which are entangled in it.

6. As might be expected, I have found the disengagement of air from the action of the pump more copious in summer than in winter; and also more copious from blood, the fibrin of which has been broken up by having been agitated with shot previously freed from adhering air, than from the clot left entire. In the instance of the blood of the common fowl, which coagulates rapidly, affording a firm coagulum, even the puncturing of it makes a difference; air then escapes, which before was retained.

7. In many instances, blood which had yielded air on exhaustion, has, after exposure for a few hours to the atmosphere, on repetition of the exhaustion,

ceased to yield air, and this when the first trial was stopped before the exhaustion of the air was nearly complete. This result, seemingly paradoxical, may have been owing to ammonia formed, which may have fixed carbonic acid; and that ammonia was formed, was proved by the hydrochloric test and the production of muriate of ammonia. It has been witnessed in the instance of both venous and arterial blood, but most remarkably in the latter, and in warm weather oftener than in cold. In support of the explanation offered, I may mention an experiment on the blood of a calf, which had no food for about twenty hours before it was killed. This blood,—it was arterial,—even at first gave off no air on careful exhaustion. It was kept under an exhausted receiver from the 23d of April to the 13th of May, during the whole of which time it gave off no air, though the vacuum was as perfect as it could be made, and the pump was worked daily. At the end of this time the serum had become dark red, and on examination the blood was found in a state of incipient putrefaction and giving off ammonia.

What struck me as most remarkable in these experiments with the air-pump, was the comparatively small quantity of air, in most instances, disengaged from the blood, and its total absence in others, taking into account the quantity of carbonic acid liberated in the lungs during life in normal respiration, and also the quantity of air, both oxygen and carbonic acid, found in the blood by the German physiologists. Difference of temperature, comparing that of the hot blood circulating in the lungs in birds as high as 106°-108°, and in the sheep, ox, and pig, as high as 104°-106°,* with that of the blood of the same animals cooled to 50°-55°, may partly account for the result first referred to, but the second adverted to I cannot attempt to explain.

Besides the foregoing trials with the air-pump, I have made some on the blood-corpuscles, using very small quantities suspended in serum on a glass support. The corpuscles were from the blood of the animals already mentioned, and also of the frog and common trout. The results were all nearly similar: so long as the corpuscles were floating in serum there was no appreciable change of form, but if they were kept some hours under the exhausted receiver until they were left apparently dry on the object-glass, then a change was perceptible in them. Under the microscope they were found to have become greatly reduced in size, so as to be seen with difficulty, and not without the nicest adjustment, and also altered in form—the elliptical, as those of the bird, the fish, and batrachian, having become rounded. These changes were very similar to those produced by the action of water, and they may be accounted for, perhaps, on the idea that they were owing to the hygroscopic quality of the corpuscles. I may further remark that the effects on the corpuscles of the blood of the several animals tried somewhat varied; it seemed greatest in those of the common fowl, least in those of the ox.

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* I have found the temperature of the blood of a pig, flowing in a full stream, 106°. The pig was in high condition; the blood used was from it.

IV. *On the Effect of a Low Temperature on the Blood.*

It was ascertained by HEWSON that the blood, by rapid freezing, is not deprived of its property of coagulating when thawed;[*] besides this and the change of form of the corpuscles from refrigeration which I have observed,[†] I am not aware that any thing has hitherto been published respecting the agency of a low temperature on this fluid.

During the frost which prevailed in the Lake District the winter before last, from the 2d to the 10th of January, I had an opportunity of renewing the inquiry. The blood used was that of the turkey, of the common fowl, and of the sheep. The most remarkable result obtained was that a low temperature, like a high temperature, appears to promote not only a change of form of the red corpuscles, but also a change of composition, as indicated by the production of ammonia and the solution in the serum of the colouring matter of the blood. As the changes were the same whichever blood was the subject of experiment, I shall restrict myself to what was observed in the trials on that of the common fowl. On the 4th of January a wine-glass was nearly filled with the blood of a full-grown fowl as it flowed from the divided great cervical vessels; it coagulated in less than two minutes. A plate of glass, moistened with a drop of dilute hydrochloric acid, was placed over it. After ten minutes there was a copious deposition of dew on its inner surface, vapour from the warm blood beneath, and there condensed. On examination with microscope, after evaporation, not a trace of muriate of ammonia could be detected. The trial was repeated, and for six hours in the open air, at the temperature of 28° Fahr.; now a trace barely of the salt was found. The blood was moderately florid, preserving its original appearance, and was not yet frozen. It was left out during the night. The temperature during the time, as shown by a register thermometer, was as low as 12°; on the following morning, at 9 A.M., it had risen to 18°. The blood was found to be frozen hard and thoroughly; it was greatly darkened in colour, and had lost entirely its florid hue. Distinct crystals of muriate of ammonia, and these not a few, were detected on the covering glass after the evaporation of the acid, and the red corpuscles from elliptical had become circular and globular.

The observations were continued until the morning of the 10th, when a thaw set in. The blood was examined twice daily, viz., at 9 A.M. and at 3 P.M. During the period the temperature was always below 20°, but not lower than 15°, excepting once, as already mentioned. The day temperature ranged as high as 27°, it was never lower than 22°. At the former temperature, a softening of the blood was observed from incipient thaw. During the whole time, as denoted by the test employed, ammonia was evolved, and, as well as I could judge, the lower the temperature the larger was the quantity. I need hardly remark that there was no appreciable contraction of the crassamentum, no further separation of serum after congelation had taken place. The serum which first exuded before congelation—a very small quantity—after having been frozen, became coloured, and, finally, of a red nearly as dark as the general mass, and this owing in part to blood corpuscles suspended in it of altered form, and in part to solution of their colouring matter. After thawing, the blood had no smell indicating putridity, nor did it discolour silver; yet it continued, at a temperature of 50°, to evolve ammonia, and much in the same proportion as when frozen. Now, however, the contraction of the crassamentum, *i.e.*, of its fibrin, before arrested, took place, and to an extent seemingly differing but little from what would have occurred had the blood not been frozen. The blood corpuscles now were so reduced in size, and had become so transparent, that unless dried, they were seen with difficulty, and not without the most accurate adjustment.

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[*] HEWSON'S Works, p. 25.

[†] Physiological Researches, p. 369.

These results, viz., the disengagement of ammonia, and, we must infer, its formation, when blood is frozen, are hardly such as could be expected; and they are the more remarkable, as seeming to be independent of putrefaction and the action of oxygen, and owing to a new arrangement of elementary parts produced by a low temperature, one ranging from about 50° to many degrees below the freezing point. That congelation was not essential to the formation of the ammonia was shown in other experiments, in which, when blood was exposed to a temperature ranging in one trial from 32° to 34°, the volatile alkali was produced, and in others at a temperature varying from 40° to 50°; and, in the latter, even when continued several days, without any indications of putridity, judging from the absence of the smell such as denotes putrefaction, and from silver immersed remaining untarnished.

At first view what has been described may seem anomalous, yet the results are not without analogies. The potato, as is well known, becomes sweet from the conversion of starch into sugar by "frosting;" and the ripening of the grape, the sweetening of its juice, it is also well known, is hastened by the setting in of frost at the time of the vintage in Switzerland, and in other countries with a similar climate. The formation of peat is another example of the efficiency of a comparatively low temperature in producing new compounds. Familiar with the effects of heat—*i.e.*, of a high temperature—as an active agent, it is not perhaps surprising that cold—*i.e.*, a low temperature—should be little thought of except as the opposite and the antagonist of heat, disregarding the fact that they differ merely in degree; and how inconsiderable that is, whether measured by our sensations or by the thermometer.

After witnessing the effects of congelation on blood, the question occurred, Is meat liable like it to change from freezing—that change which the evolution of ammonia indicates? As it is well known that meat may be kept for weeks frozen without being spoiled as an article of diet, the obvious answer was in the negative. The only experiments I have made have afforded results leading to the same conclusion. A portion of fresh mutton, cut into small pieces, was exposed on the 6th of January to the open air. During the following night the register thermometer was as low as 16°; the next morning it was 22°. The meat was not frozen; its fibre was soft and flexible; a bare trace of muriate of ammonia was found on the glass above it prepared as a test of the volatile alkali. Before night it became frozen and rigid, and it continued so until the thaw began on the 10th. Examined twice daily, no traces could be detected of the production of ammonia. The fibre, indeed, was evidently softened, and its striated structure, as seen under the microscope, was less conspicuous, so much so, that without a good light and a careful adjustment it could not be seen.

A like question occurred respecting manures—Does frost arrest their decomposition? I have made trial of stable-dung, and have found it when frozen to exhale ammonia in an unmistakable manner, proving that a low temperature, as in the instance of blood, promotes its decomposition, or that change on which the evolution of ammonia depends. A similar result has been obtained from the exposure of impure lithate of ammonia (the urinary excrements of the pelican), of the mixed excrements, partly urinary, partly alvine, of the barn-door fowl, and of guano. From all of them, using the same test, the production of ammonia was conspicuous. Should not these results suggest the propriety of reconsidering the treatment of manures, and if not the time of their application to the land, at least whether an addition should not be made to them to fix the ammonia?

V. *On the Action of Ammonia on the Blood.*

Since the hypothesis has been advanced, that the escape of ammonia from the blood is the cause of its coagulation, additional interest is attached to the action of the volatile alkali on this fluid.

The following experiments have been made with a view not so much to test the correctness of that hypothesis, as to show what are the effects of ammonia on the blood as a whole, and on its several parts:—

1. *On the Entire Blood.*—On the 8th of December 241 grs. of the blood of a duck were received, as it flowed from the divided cervical vessels, into a bottle containing 72·5 grs. of aqua ammoniæ of sp. gr. ·95. The bottle was immediately closed with a glass stopper. This was at 10.37 A.M. At 12.15 P.M. a semi-fluid viscid coagulum had formed, of a rich Turkey-red colour. A glass rod applied to it, it yielded to gentle pressure, without adhering to, or in the slightest degree soiling the rod. At 2.30 P.M. it was somewhat firmer. At 10 P.M. it was more

so; now, when the bottle was turned on its side, it ceased to flow. On the following day it was so firm that it bore inversion without flowing; no serum had separated. Examined on the 13th of December, the only change perceptible was that it was rather firmer. On the 1st of January it was more carefully examined. On withdrawing the stopper, as might have been expected, the smell of the ammonia was very powerful, indeed unendurable. The coagulum was found of the consistence of a pretty firm jelly, readily yielding to pressure, but not adhering to the glass rod impressing it. The whole mass was easily removed, retaining its form unbroken; and such was the adhesiveness of its substance—*i.e.*, of its particles to one another—that the mass admitted of being divided with a scissors without its soiling the instrument. A portion of it put into water did not immediately colour the water; from black it became dull brown. Examined with the microscope under compression, it exhibited a finely granular surface, through which were scattered globules of a less diameter than the blood corpuscles—these, it may be inferred, contracted.

In other experiments, made within a few days of each other, with smaller proportions of the volatile alkali, the effect has been found to vary.

When 12 grs. of aqua ammoniæ, of the same strength as that last mentioned, were mixed with 465 grs. of the blood of a turkey, the instant it was shed, the coagulation was retarded about 20 minutes. On withdrawing the stopper the following morning there was a strong smell of ammonia; the crassamentum was found of tolerable consistence, and was surrounded and covered with red serum, which owed its colour chiefly to red corpuscles suspended in it of a globular form, a change from their normal form evidently owing to the action of the alkali. Examined again after twenty days the coagulum was found firmer; it admitted of being taken out as an entire mass.

In a third experiment 2·5 grs. of aqua ammoniæ were mixed, as in the former instances, with 572 grs. of the blood of a fowl. After two hours the blood was found feebly coagulated and viscid, in a semifluid state, sluggishly flowing like tar. When a portion of it was poured into water, it did not mix with the water, but kept entire, retaining its viscidity. What remained, examined the following morning, was found divided into a somewhat denser crassamentum, still semifluid and viscid, and a red somewhat viscid serum, abounding in red corpuscles, more or less altered in form, many of them diminished in volume, and almost all of them rounded. Both the soft coagulum and the serum smelt of ammonia.

A fourth experiment was made on the blood of a sheep, after the same manner as the preceding. The quantity of the aqua ammoniæ was 1 gr., of the blood 587 grs. Examined after about half-an-hour, the blood, still warm, was found pretty firmly coagulated, and already some serum had separated. The glass stopper was withdrawn, and instantly after a plate of glass, moistened with dilute hydrochloric acid, was placed over the mouth of the bottle, and left for two minutes.

Now, after evaporation, crystals of muriate of ammonia, of a large size and in abundance, were found on it.

A fifth experiment, similar in manner to the last, with the exception that the aqua ammoniæ (1 gr.) was spread as much as possible over the inside of the phial, was made on the blood of a fowl (590 grs.) The blood coagulated in eight minutes, and pretty firmly. Another portion, caught in a wine-glass, coagulated in about a minute. Each was tested for ammonia, as in the preceding trial. The blood in the wine-glass, after five minutes—the time that the acid was kept over it—afforded no distinct trace of ammonia. The blood in the bottle, after one minute, afforded ample proof of the evolution of ammonia in the large crystals of the muriate which were formed on evaporation on the incumbent glass. The contrast, indeed, was very striking, comparing the blood with and without the addition of ammonia, as thus tested, and also by test-paper; the one, the former, having no effect during a minute that moistened test-paper was held over it; the other, in the same time, producing a decided alkaline reaction. On the following morning the crassamentum in the bottle was found slightly contracted, though less than that in the wine-glass; some reddish serum had separated; the blood corpuscles, whether suspended in the serum or retained in the clot, were little if at all altered.

From these experiments it would appear that the effect of aqua ammoniæ varies as to the quantity used, and this in a manner that could hardly be expected; 31 per cent. occasioning a thick adhesive coagulum, with a change of form of the red corpuscles, without the separation of any serum; 2·5 per cent. retarding the coagulation many minutes, but not preventing the separation of serum and a certain contraction of the crassamentum; 0·44 per cent. retarding the coagulation and rendering the coagulum soft and viscid, barely semifluid, with little separation of serum, and that viscid; lastly, 0·17 per cent. had little effect, except that of retarding for a few minutes the coagulation,—the coagulum, when formed, having very much its normal appearance.

2. *On the Fibrin of the Blood.*—The fibrin used was obtained by washing the clot which had formed in the first experiment. In its moist state it was slightly viscid. By drying it lost 93 per cent.; 2·5 grs. thus dried were put into a phial with 273 grs. of aqua ammoniæ of sp. grav. ·89, and secured by a glass stopper. After eleven days it was not apparently diminished in volume: 41 grs. of the clear fluid decanted and evaporated yielded only ·1 gr. As the fluid became concentrated during the process, which was conducted at a low temperature, its fluidity diminished, and when reduced to a drop it was still transparent. In its dry state it appeared as a transparent film, and as seen under the microscope with a high power it had a finely granular appearance.

A second experiment was made on the same fibrin in its moist state, using in place of aqua ammoniæ alone a dilute solution, consisting of 65·4 grs. of the alkali, and of 305 grs. of water. The fibrin was equal to 31 grs. After ten days its volume seemed little diminished—58·4 grs. of the clear fluid evaporated left ·1 gr. The undissolved residuary portion, constituting so large a proportion of the whole, was soft, glutinous, and adhesive; it might be called ropy, as it allowed of being drawn out, and when agitated by a circular motion, it rose spirally in the liquid. It thus differed from the dried fibrin, which was softened in a slight degree, but not rendered glutinous. Examined after thirty-four days, it seemed little altered in bulk, and nowise in its properties. The fluid was slightly viscid: a portion of it, 41·4 grs., evaporated to dryness, yielded only ·05 gr. It became slightly turbid during evaporation at a temperature of about 180°, and when the ammonia was driven off, it lost the little viscidity it before had. The smaller proportion of residue in this instance might have been owing to the circumstance that the phial holding the fibrin and the dilute aqua ammoniæ not being firmly corked, some of the ammonia might have escaped.

These results demonstrate how feeble is the solvent power of ammonia on fibrin. Many other experiments which I have made, of which an account is hardly needed, have been amply confirmatory of the fact, and also of the well-known effect of ammonia in rendering fibrin viscid and glutinous, and of increasing its transparency. This last effect should be kept in mind, otherwise, as the refractive power of fibrin differs but little from that of water, it may in some instances be imagined to be dissolved, when it is only diffused.*

3. *On the Serum of the Blood.*—On this fluid the effect of ammonia is less distinct. It appears to diminish rather than to increase the viscidity of the serum, as is shown by the following experiment: a portion of the serum of the blood of a pig, equal to 314 grs., was mixed with 274·4 grs. of aqua ammoniæ of sp. gr. ·89, in a glass-stoppered phial; and about an equal quantity of the serum of the same blood was poured into a similar phial. This was on the 30th March. Each was shaken daily: froth was produced in each instance, but that from the ammoniacal mixture subsided more rapidly than that from the serum alone; and the longer the trial was continued—it was continued more than a month—the more marked was the difference.

At the end of this time the ammoniacal mixture had deposited a white matter, which was readily diffused on gentle agitation, rendering the fluid, which was before transparent, turbid.

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* When well-washed fibrin, still slightly coloured by the colouring matter of the blood, is placed under the microscope, it appears to consist of translucent granules forming under gentle pressure a connected tissue. On the addition of aqua ammoniæ it becomes clear and transparent, like jelly, with a brightening of its colour. Compressed, it shows elasticity, and when extended by continued pressure, so as to be very thin, its appearance is hyoloid; no granules are to be seen in it except a few scattered ones, which, it may be, were derived from blood corpuscles.

Fibrin which has been rendered viscid by ammonia, after the removal of the ammonia by repeated washing, gradually contracts, and from being transparent becomes, in consequence of condensation, opaque, or nearly so. Thus contracted it often exhibits an imitative form, like that of hydatids. Its retention of the colouring matter of the blood is remarkable; it is greater even than that of the capsule or walls of the corpuscles.

Under the microscope the deposit exhibited thin crystalline plates, their length exceeding their width about a third, some minute spicula, somewhat like raphides, and some granules. As the matter was not viscid, it may be inferred that fibrin did not form a part of it.

The transparent fluid separated by decantation from this sediment yielded a coagulum with the sulphuric, muriatic, nitric, and acetic acids, added each in slight excess, that is, in a quantity a little more than was sufficient to neutralise the ammonia. The precipitate was redissolved by the sulphuric and muriatic acids, and in great part by the nitric—these acids concentrated—but not by the acetic.

When the clear ammoniacal fluid was boiled until the whole of the volatile alkali was expelled, it was rendered gelatinous, that is, the coagulum formed was soft and transparent, like the albumen of the eggs of some birds similarly treated.

The same fluid, evaporated at a low temperature, left a brownish transparent matter, which was soluble almost entirely in water. The solution frothed when boiled and gelatinised. It had a slight alkaline reaction, like serum, and had no unpleasant smell. Evaporated again, little of it was redissolved on the addition of water, and still less on repeating the operation—thus resembling ordinary serum.*

The serum without the addition of ammonia, kept during the same time, had also yielded a deposit, which was of a greyish hue, and under the microscope exhibited only amorphous particles. The fluid had acquired a reddish hue, and had an offensive putrid smell, and it afforded when boiled a firm coagulum.

Comparing, then, the two, it appears that ammonia renders serum less viscid, prevents its putrefaction,† and modifies in some degree its coagulable property. Whether the serum of the blood of other animals under the influence of ammonia would show the same properties, I have not ascertained with sufficient accuracy. From the few comparative trials I have made, I am disposed to infer that there would be no material difference.

4. *On the Red Corpuscles of the Blood.*—On these the effect of the volatile alkali is more decided, as is shown by the following experiment,—one of the

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* Serum of blood, such as I have tried, and I have made many trials, on first evaporation affords a residue which is almost entirely soluble in water, but on repetition again and again, it is so altered as to become insoluble.

† Ammonia does not appear to arrest entirely the putrefactive decomposition of the blood: thus a mixture of 257 grs. of blood, and of 2·5 grs. of aqua ammoniæ, the subject of the third experiment, after having been kept twenty days, had, besides an ammoniacal odour, an offensive smell, indicative of incipient putrefaction. In great excess, it certainly retards the change in the instance of the entire blood, and in a great degree in the instance of the fibrin, and in that of the serum. A portion of the coagulum left from the first experiment and kept three months, had, after the ammonia had been rapidly expelled, an offensive odour, only in a slight degree.

many which I have made. The cruor used was from fresh bullock's blood, its fibrin separated in the usual way; 45 grs. of it were mixed with 48 grs. of aqua ammoniæ. The colour was immediately darkened, so much so, that by reflected light it appeared almost black; by transmitted, of a garnet-red, similar to the change of colour observed when the entire blood was used, and it was accompanied by the same alteration in the corpuscles, these being reduced in size and rendered globular. Another obvious effect was an increase of viscidity. Examined after eight days, and again after twenty-four, the only further change noticeable was the disappearance of the corpuscles, as if they had in great part been dissolved; they were not to be seen under the microscope; minute granules only were visible, and these were seen only after evaporation.

It is worthy of remark, that when the whole of the volatile alkali was expelled by heat at a temperature below 160°, and the residuary fluid was tried by test-paper, only the feeblest alkaline reaction was observable; in this respect, differing from the serum, which under the same circumstances showed a distinct alkaline reaction. The solution was coagulated at a temperature of about 160°. Another portion evaporated at a low temperature was resoluble, *i.e.*, the colouring matter; seeming to show that this matter had suffered no change from the action of the volatile alkali.

The bearings of the results of these several experiments on the hypothesis adverted to, hardly need be dwelt on, they are so obvious. Seeing that ammonia, in so large a quantity as that used in the first experiment, did not prevent the coagulation of the blood, or, in other words, of its fibrin—its coagulable part—it would be strange, indeed, if the escape of a very minute quantity of the volatile alkali, hardly an appreciable one at most, should be the cause of the phenomenon.

Considering that ammonia renders the fibrin viscid and alters the shape of the red corpuscles, is there not ground for caution as regards its medicinal use, and of more than doubt of its efficacy when administered with the intention of dissolving a coagulum in cases of thrombosis? The marked difference as to alkaline reaction of the serum and cruor, as already mentioned, was suggestive of analogy between the blood and the contents of the egg. It is stated that an aqueous solution of the colouring matter of the former is neutral.* Whatever care I have taken in preparing it, draining off the serum as much as possible from the clot before the action of water, I have always found it feebly alkaline.† Nor is this surprising, considering the impossibility of getting rid of all the serum by drain-

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* BRANDE and TAYLOR'S "Chemistry," 1863, p. 833.

† In one experiment, the clot, from six ounces of bullock's blood, after draining off as much as possible of the serum, was cut into small pieces and macerated in water, using the ordinary means to separate the fibrin. The solution formed, loaded with colouring matter, was evaporated at a temperature below 160°, until reduced to the sp. gr. 1033; its alkaline reaction then was very slight, so as to be hardly discernible when the delicate test-paper used was dried.

After evaporation of the solution to dryness, the residue was exposed to the fire in a platina capsule. In its charred state, after it had ceased to burn with flame, its particles were slightly ing. Further, I have found the ash of hæmatine prepared by a more elaborate process also feebly alkaline; and the latest analyses of the several ingredients of the blood, those most to be depended on, indicate the same.* May not this difference, slight though it appears, warrant the conjecture, that as in the egg, so in the blood, there may be an action of a galvanic kind between its several proximate parts? And may not the differences which are known to exist between the serum and the red corpuscles be adduced in favour of the conjecture?

VI. *On the Coagulation of the Blood.*

Of the many hypotheses which have been advanced at different times to account for the coagulation of the blood, each has been supported, as hypotheses usually are, by some facts, but few of them have for any length of time maintained their ground, facts having been adduced hostile to them.

Of the latest hypotheses brought forward, one is that of Dr Richardson, briefly designated the ammonia-theory, to which I have already adverted; another is that of Professor Lister, in which he considers the phenomenon as mainly depending, out of the body, on a kind of catalytic action produced by the contact of any foreign substance, and, within the body, as owing to an analogous cause, contact with a part, either dead or *quasi* dead,—as he supposes a tissue to be under the influence of inflammation.†

This hypothesis, as it appears to me, is open to certain objections. I shall now notice merely a few of the facts which seem to me most opposed to it.

1. Were it true, ought not the phenomenon of coagulation to take place in every instance in which dead matter comes in contact in the living body with the blood? Instances of ossification, in which concretions of phosphate of lime are formed in the arterial coats, and often project into the vessels themselves,—concretions differing but little from the "tartar," deposited so often on the teeth, and inorganic,—are familiar to every one acquainted with pathological anatomy, and yet in the majority of these cases the coagulation of the blood has not taken place.

2. In instances of aneurism, with a rupture of the vessel, the seat of it, a coagulum of blood is invariably formed, though in contact with parts which, it may be presumed, until the contrary is proved, still retain their vitality.

3. Examples of the coagulation of the blood in the veins, in the arteries, and in the ventricles of the heart, during life, in persons reduced to a feeble state by

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attracted by the magnet, the coal after cooling having been reduced to powder. The particles of its ash, after the charcoal had been burnt off, were also similarly attracted. The magnet used, it may be mentioned, was a needle that had been magnetised by a fœtal torpedo, and which (as the result showed) still retained its power, after the elapse of 32 years. The residuary ash, on the addition of a little water, showed so feeble an alkaline reaction, that it was hardly as well marked as that of the saliva.

* Lehmann's "Physiological Chemistry," ii. pp. 160, 212.

† Proceedings Roy. Soc. Vol. xii. p. 580.

disease, are not of unfrequent occurrence, and this often without any apparent lesion in the coats of the vessels themselves, or in the lining membrane of the ventricles.

4. Confirmatory of the last, many examples are on record of the blood, in its coagulated state after death, having been found broken up in the left ventricle of the heart, proving that its coagulation must have taken place whilst the heart was still forcibly acting, and this in cases in which the organ appeared to be sound.*

5. Certain poisons influence the coagulation, some accelerating it, some retarding it. As an example of both, may be mentioned the poison of a snake, the tic-polonga of Ceylon (*Daboia Russellii*, Gray), which on fowls acts with extreme rapidity, so much so, that simultaneously with their death, it occasions the coagulation of the blood in the heart and great vessels, and this even before the former has ceased to act; whilst, in larger animals, such as the dog, in which it takes effect less rapidly, causing death in an hour instead of about a minute, it has a contrary influence, that of preventing the coagulation of the blood.† There are other considerations which seem to me to cast a doubt on the accuracy of this hypothesis. To reconcile it with certain facts, its author is under the necessity of assuming that a clot is a "living tissue in relation to the blood;" if so, then does it not follow, in strictness of reasoning, that such must be its state under all conditions, whether formed within the body during life, or in blood abstracted by the ordinary operation of blood-letting; and he is further under the necessity of assuming that inflamed parts are *quasi* dead parts, or, in other words,—and they are his—"have lost for a time their vital properties, and comport themselves like ordinary solids."

The vagueness, moreover, of the hypothesis renders it open to objection. The referring the phenomenon to a catalytic action, seems to be little more than the accounting for what is obscure by that which is equally or hardly less obscure.

To conclude, I fear it must be confessed that, strictly speaking, the theory of the coagulation of the blood, its *vera causa*, is still an unsolved problem, there being, to all the hypotheses which have hitherto been propounded, opposing facts logically in strictness prohibiting the establishment of any one of them.

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* See the author's Anatom. and Physiolog. Research., ii. 196.

† Idem, vol. i. p. 123.

V.—*A Study of Trilinear Co-ordinates: being a Consecutive Series of Seventy-two Propositions in Transversals.* By the Rev. Hugh Martin, M.A., Free Greyfriars', Edinburgh. Communicated by Professor Kelland.

(Read 20th March 1865.)

Introductory Remarks.

The following series of theorems is given as an illustration of the modern method of trilinear co-ordinates, having been wrought out after perusal of Mr Ferrar's very lucid and elegant treatise on that subject. The demonstrations present no difficulty, requiring nothing more complicated than the formation of determinants of two and three places. Accordingly, after exhibiting the method of proof in a few instances, I have merely given the enunciations of the remaining propositions. As the series of theorems advances the manipulation becomes, of course, a little more complicated; but the co-ordinates and co-efficients always appear in such symmetry as very greatly abbreviates the task, and guarantees its accuracy. Two, or perhaps three, of these seventy-two theorems are known mathematical truths; but that so many new consecutive propositions should be so easily found, and so easily proved, is a convincing evidence of the simplicity, fertility, and power of this new and beautiful method.

Treated according to the ancient geometry, the contents of the following pages would constitute a volume of no mean dimensions; and some of the propositions, such as those which affirm that the six points P_1, P_2, P_3, P_4, P_5, P_6 range in a straight line, and that the seven straight lines U_1U_2; R_1R_2, R_3R_4; S_1S_2, S_3S_4; Q_1Q_3, Q_4Q_5, Q_2Q_6 all meet in a point, would probably have been undiscoverable.

In the admirable treatises of Mulcahy and Townsend a few analogous propositions are demonstrated geometrically. Mr Townsend, in particular, has a chapter in his *first* volume, on concurrent lines and co-linear points, which falls in very closely with the kind of propositions which the following series embraces. His *second* volume I have not been fortunate enough to see; but the subject is only ripening for a systematic gathering-up of the propositions that have been discovered in this line of investigation, and the following pages are presented as a humble contribution towards that desirable result.

A word or two may be permitted in reference to the additions to the terminology which must be made, and generally sanctioned by mathematicians, ere such systematic digest can be successfully accomplished. I have ventured—of course only provisionally—on one or two such additions. When the co-ordinates of two points are respectively the algebraical inverses of each other, I have called these points, in reference to each other, "inverse points;" and it is evident that a very fine vein of mathematical truth opens up in reference to them, which it needs only a little ingenuity to work advantageously. Thus, at a glance, it is evident that if a point moves in the straight line $l\alpha+m\beta+n\gamma=0$, its inverse moves in the locus, $\frac{l}{\alpha}+\frac{m}{\beta}+\frac{n}{\gamma}=0$; which is a conic passing through the angular points of the triangle of reference. Since writing the following pages I find Mr TOWNSEND has a chapter entitled "Theory of inverse points with respect to a circle;" and although not treated according to the trilinear method, these points, so called, will be found, I rather think, *if* so treated, to present only a case of what I have called "inverse points" in general. In the same manner I have called two lines "inverse" with respect to each other when the co-efficients of the co-ordinates are respectively the algebraical inverses of each other.

There is another relation between a special point and line which I have not ventured to designate, but to which I would respectfully call attention as requiring designation. When lines from the angular points of a triangle are drawn through any point to intersect the opposite sides, the intersections constitute the angular points of an inscribed triangle, whose sides are known to meet the corresponding sides of the original triangle in points which range in a straight line. Instead of giving a particular designation to this line, I have used the general functional symbol; and, as its position depends exclusively on the point —say P, I have called the line ϕ (P), in a few theorems in reference to it (Theorems XXXI.—XXXV.). Of course the inverse functional symbol ϕ^{-1} indicates the point in reference to the line, as the direct symbol indicates the line with reference to the point. This *point* and *line* are, indeed, with respect to each other, a species of pole and polar,—the line being the ordinary polar, not of the point but of its inverse,—to the imaginary conic $\alpha^2+\beta^2+\gamma^2=0$. Manifestly a special designation is necessary in a case like this, in order to secure that ease of reference and that brevity of treatment without which the pioneering work of farther investigation is brought to a stand.

Theorem LXVI. is the prize question of the "Gentleman's Diary" for 1841; and some long but good geometrical demonstrations of it are given. The proof is perfectly simple according to the trilinear method, and the co-ordinates of the point appear in a form so elegant that one could not help seeing that it must have some singular relations and be worthy of a name. I have accordingly ventured to call it the Anapole of the two given points; and, connecting it with some of the preceding results, I find a few propositions easily deducible, such as that the anapole of two inverse points and the line joining them are pole and polar, to the imaginary conic, $\alpha^2+\beta^2+\gamma^2=0$. For the three concluding theorems I am indebted to a young mathematical friend—destined, I believe and trust, to scientific eminence—Mr GEORGE M. SMITH, student in the Aberdeen University. On proposing to him the problems of finding the locus of the anapole of a central body and its planet, and the locus of the anapole of two points which should move away from each other in a straight line with uniform velocities, I was delighted to receive demonstrations, perfectly simple and elegant, to the effect that, in the former case, the anapole moves in a straight line, let the planet move as it may; and that, in the latter case, the locus of the anapole is a conic section, and becomes a straight line if the uniform velocities are equal; and farther, that the anapole of any two points in an ellipse circumscribing the triangle of reference is invariable. Geometers, I am sure, will admire these theorems of a rising young mathematician, and will recognise the vein thus struck as promising to be a fertile one. Mr Smith added another very beautiful property of the anapole, which turned out, on investigation, to be identical with Theorem LXV. of the following series.

Instead of defining a point by the equations $\frac{\alpha}{l}=\frac{\beta}{m}=\frac{\gamma}{n}$, we shall say the point is—(l, m, n). Instead of defining a straight line by the equation $l\alpha+m\beta+n\gamma=0$, we shall say the line is—(l, m, n).

The straight line joining the points (l_1, m_1, n_1), (l_2, m_2, n_2) is;—

$$\begin{vmatrix} m_1, & n_1 \\ m_2, & n_2 \end{vmatrix}, \quad \begin{vmatrix} n_1, & l_1 \\ n_2, & l_2 \end{vmatrix}, \quad \begin{vmatrix} l_1, & m_1 \\ l_2, & m_2 \end{vmatrix}.$$

The intersection of the two straight lines (l_1, m_1, n_1), (l_2, m_2, n_2) is defined by the same expression. This identity of form is, *in reality*, the earliest germ of the doctrine of pole and polar; and gives rise to what is *usually* regarded as the first promise of that doctrine, namely, the identity of the condition that the three points (l_1, m_1, n_1), (l_2, m_2, n_2), (l_3, m_3, n_3) shall range in a straight line, with the condition that the three straight lines (l_1, m_1, n_1), (l_2, m_2, n_2), (l_3, m_3, n_3) shall intersect in a point; which is, in both cases,

$$\begin{vmatrix} l_1, & m_1, & n_1 \\ l_2, & m_2, & n_2 \\ l_3, & m_3, & n_3 \end{vmatrix} \equiv 0.$$

THEOREMS.

Theorem 1.

On the sides of the triangle ABC, as bases, are constructed three triangles, A_1BC, AB_1C, ABC_1, similar to each other, and so placed that the angles $A_1BC=AB_1C=ABC_1$; $B_1CA=BC_1A=BCA_1$; and $C_1AB=CA_1B=CAB_1$. Then AA_1, BB_1, CC_1 meet in a point.

The sides of ABC, taken as the triangle of reference, being a, b, c, the perpendiculars from A_1 on a, b, c are, respectively,

$$\frac{a.\sin B_1.\sin C_1}{\sin A_1}, \quad \frac{a.\sin B_1.\sin(C+C_1)}{\sin A_1}, \quad \frac{a.\sin C_1.\sin(B+B_1)}{\sin A_1}.$$

$$\begin{array}{llccc} \text{Hence the trilinear co-ordinates of } A_1 \text{ are} & & -1, & \dfrac{\sin\,(C+C_1)}{\sin C_1}, & \dfrac{\sin\,(B+B_1)}{\sin B_1}. \\ \text{,,} \quad \text{,,} & \text{of } B_1 \text{ are} & \dfrac{\sin\,(C+C_1)}{\sin C_1}, & -1, & \dfrac{\sin\,(A+A_1)}{\sin A_1}. \\ \text{,,} \quad \text{,,} & \text{of } C_1 \text{ are} & \dfrac{\sin\,(B+B_1)}{\sin B_1}, & \dfrac{\sin\,(C+C_1)}{\sin C_1}, & -1. \end{array}$$

For $\dfrac{\sin(A+A_1)}{\sin A_1}$, $\dfrac{\sin(B+B_1)}{\sin B_1}$, $\dfrac{\sin(C+C_1)}{\sin C_1}$, put f, g, h. Then ;—

$$\left.\begin{array}{lrrr} A_1 \text{ is} & -1, & h, & g. \\ B_1 \text{ is} & h, & -1, & f. \\ C_1 \text{ is} & g, & f, & -1. \end{array}\right\}. \quad \text{Also, } \left\{\begin{array}{lccc} A \text{ is} & 1, & 0, & 0. \\ B \text{ is} & 0, & 1, & 0. \\ C \text{ is} & 0, & 0, & 1. \end{array}\right\} \text{ Hence ;—}$$

$$\begin{array}{r} A\,A_1 \text{ is} \\ B\,B_1 \text{ is} \\ C\,C_1 \text{ is} \end{array} \left|\begin{array}{rrr} 0, & g, & -h. \\ -f, & 0, & h. \\ f, & -g, & 0. \end{array}\right| \equiv 0. \quad \text{Therefore } A\,A_1,\ B\,B_1,\ C\,C_1 \text{ intersect in a point—say } P_1.$$

P_1 is gh, hf, fg, or f^{-1}, g^{-1}, h^{-1}.

THEOREM 2.

$$\left.\begin{array}{llll} B_1C_1 & \text{is } 1-f^2, & h+fg, & g+hf \\ C_1A_1 & \text{is } h+fg, & 1-g^2, & f+gh \\ A_1B_1 & \text{is } g+hf, & f+gh, & 1-h^2 \end{array}\right\}. \quad \text{Also, } \left\{\begin{array}{lccc} BC \text{ is} & 1, & 0, & 0. \\ CA \text{ is} & 0, & 1, & 0. \\ AB \text{ is} & 0, & 0, & 1. \end{array}\right\} \text{ Hence ;—}$$

$$\begin{array}{llll} BC,\ B_1C_1 & \text{meet in} & \mathfrak{A}_1 & \text{which is} \\ CA,\ C_1A_1 & \text{,,} & \mathfrak{B}_1 & \text{,,} \\ AB,\ A_1B_1 & \text{,,} & \mathfrak{C}_1 & \text{,,} \end{array} \left|\begin{array}{ccr} 0, & g+hf, & -(h+fg). \\ -(f+gh), & 0, & h+fg. \\ f+gh, & -(g+hf), & 0. \end{array}\right| \equiv 0. \quad \text{Therefore ;—}$$

$\mathfrak{A}_1\mathfrak{B}_1\mathfrak{C}_1$ is a straight line, viz., $(f+gh)^{-1}$, $(g+hf)^{-1}$, $(h+fg)^{-1}$.

THEOREM 3.

$$\left.\begin{array}{lrrr} B\,C_1 \text{ is} & 1, & 0, & g \\ C\,A_1 \text{ is} & h, & 1, & 0 \\ A\,B_1 \text{ is} & 0, & f, & 1 \end{array}\right\}. \text{ Also} \left\{\begin{array}{lrrr} B_1C \text{ is} & 1, & h, & 0 \\ C_1A \text{ is} & 0, & 1, & f \\ A_1B \text{ is} & g, & 0, & 1 \end{array}\right\}. \text{ Hence} \left\{\begin{array}{llll} B\,C_1,\ B_1C & \text{meet in} & A_2, & \text{viz.} \\ C\,A_1,\ C_1A & \text{,,} & B_2, & \text{,,} \\ A\,B_1,\ A_1B & \text{,,} & C_2, & \text{,,} \end{array}\right. \left|\begin{array}{rrr} -hg, & g, & h \\ f, & -hf, & h \\ f, & g, & -fg \end{array}\right|$$

$$\begin{array}{r} A\,A_2 \text{ is} \\ B\,B_2 \text{ is} \\ C\,C_2 \text{ is} \end{array} \left|\begin{array}{rrr} 0, & -h, & g \\ h, & 0, & -f \\ -g, & f, & 0 \end{array}\right| \equiv 0. \quad \text{Therefore } A\,A_2,\ B\,B_2,\ C\,C_2 \text{ intersect in a point—say } P_2.$$

P_2 is f, g, h.

P_1 and P_2 may be called inverse or reciprocal points.

THEOREM 4.

$$\begin{array}{r} BC \text{ is} \\ B_1C_1 \text{ is} \\ B_2C_2 \text{ is} \end{array} \left|\begin{array}{rcc} 1, & 0, & 0. \\ 1-f^2, & h+fg, & g+hf. \\ -\,gh(1-f^2), & f(h+fg), & f(g+hf). \end{array}\right| \equiv 0. \quad \text{Therefore ;—}$$

$$\begin{array}{lcl} BC,\ B_1C_1,\ B_2C_2 & \text{intersect in} & \mathfrak{A}_1. \\ {}^{*}\ CA,\ C_1A_1,\ C_2A_2 & \text{,,} & \mathfrak{B}_1. \\ AB,\ A_1B_1,\ A_2B_2 & \text{,,} & \mathfrak{C}_1. \end{array}$$

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[*] By symmetry.

THEOREM 5.

$$\begin{array}{l} \mathrm{A_1A_2} \text{ is} \\ \mathrm{B_1\,B_2} \text{ is} \\ \mathrm{C_1\,C_2} \text{ is} \end{array} \left| \begin{array}{rrr} g^2-h^2, & -h\,(1-g^2), & g\,(1-h^2) \\ h\,(1-f^2), & h^2-f^2, & -f\,(1-h^2) \\ -g\,(1-f^2), & f\,(1-g^2), & f^2-g^2 \end{array} \right| \equiv 0. \text{ Therefore ;—}$$

$\mathrm{A_1A_2}$, $\mathrm{B_1B_2}$, $\mathrm{C_1C_2}$ intersect in a point,—say $\mathrm{P_3}$, viz., $f-gh$, $g-hf$, $h-fg$.

THEOREM 6.

$$\begin{array}{l} \mathrm{P_1} \text{ is} \\ \mathrm{P_2} \text{ is} \\ \mathrm{P_3} \text{ is} \end{array} \left| \begin{array}{ccc} gh, & hf, & fg. \\ f, & g, & h. \\ f-gh, & g-hf, & h-fg. \end{array} \right| \equiv 0. \text{ Therefore } \mathrm{P_1,\,P_2,\,P_3} \text{ range in a straight line.}$$

$\mathrm{P_1P_2P_3}$ is $f\,(g^2-h^2)$, $g\,(h^2-f^2)$, $h\,(f^2-g^2)$.

THEOREM 7.

$$\begin{array}{llll} \mathrm{A_1B,\,B\,C} & \text{meet in} & \mathfrak{A}_2, & \text{viz.,} \\ \mathrm{B_1\,C,\,C\,A} & \text{,,} & \mathfrak{B}_2, & \text{,,} \\ \mathrm{C_1A,\,A\,B} & \text{,,} & \mathfrak{C}_2, & \text{,,} \end{array} \left\{ \begin{array}{rrr} 0, & 1, & -f \\ -g, & 0, & 1 \\ 1, & -h, & 0 \end{array} \right\}. \text{ Also} \left\{ \begin{array}{llll} \mathrm{C_1A,\;B\,C} & \text{meet in} & \mathfrak{A}_3 & \text{viz.,} \\ \mathrm{A_1B,\,C\,A} & \text{,,} & \mathfrak{B}_3 & \text{,,} \\ \mathrm{B_1C,\,A\,B} & \text{,,} & \mathfrak{C}_3 & \text{,,} \end{array} \right. \left\{ \begin{array}{rrr} 0, & f, & -1 \\ -1, & 0, & g \\ h, & -1, & 0 \end{array} \right\}.$$

Hence ;—

$$\begin{array}{ll} \mathfrak{B}_2\mathfrak{C}_2 & \text{is} \\ \mathfrak{C}_2\mathfrak{A}_2 & \text{is} \\ \mathfrak{A}_2\mathfrak{B}_2 & \text{is} \end{array} \left\{ \begin{array}{ccc} h, & 1, & gh \\ hf, & f, & 1 \\ 1, & fg, & g \end{array} \right\} ; \quad \begin{array}{ll} \mathfrak{B}_3\mathfrak{C}_3 & \text{is} \\ \mathfrak{C}_3\mathfrak{A}_3 & \text{is} \\ \mathfrak{A}_3\mathfrak{B}_3 & \text{is} \end{array} \left\{ \begin{array}{ccc} g, & gh, & 1, \\ 1, & h, & hf, \\ fg, & 1, & f, \end{array} \right\} ;$$

Also ;—

$$\begin{array}{llll} \mathfrak{B}_2\mathfrak{C}_2,\ \mathfrak{B}_3\mathfrak{C}_3 & \text{meet in} & \mathrm{A_3}, & \text{viz.,} \\ \mathfrak{C}_2\mathfrak{A}_2,\ \mathfrak{C}_3\mathfrak{A}_3 & \text{,,} & \mathrm{B_3}, & \text{,,} \\ \mathfrak{A}_2\mathfrak{B}_2,\ \mathfrak{A}_3\mathfrak{B}_3 & \text{,,} & \mathrm{C_3}, & \text{,,} \end{array} \left\{ \begin{array}{ccc} 1-g^2h^2, & h\,(g^2-1), & g\,(h^2-1) \\ h\,(f^2-1), & 1-h^2f^2, & f\,(h^2-1) \\ g\,(f^2-1), & f\,(g^2-1), & 1-f^2g^2 \end{array} \right\}.$$

Farther ;—

$$\begin{array}{ll} \mathfrak{B}_2\mathfrak{C}_3 & \text{is} \\ \mathfrak{C}_2\mathfrak{A}_3 & \text{is} \\ \mathfrak{A}_2\mathfrak{B}_3 & \text{is} \end{array} \left\{ \begin{array}{ccc} 1, & h, & g \\ h, & 1, & f \\ g, & f, & 1 \end{array} \right\} ; \quad \begin{array}{ll} \mathfrak{B}_3\mathfrak{C}_2 & \text{is} \\ \mathfrak{C}_3\mathfrak{A}_2 & \text{is} \\ \mathfrak{A}_3\mathfrak{B}_2 & \text{is} \end{array} \left\{ \begin{array}{ccc} gh, & g, & h \\ f, & hf, & h \\ f, & g, & fg \end{array} \right\} ;$$

Hence ;—

$$\begin{array}{llll} \mathfrak{B}_2\mathfrak{C}_3,\ \mathfrak{B}_3\mathfrak{C}_2 & \text{meet in} & \mathrm{A_4}, & \text{viz.,} \\ \mathfrak{C}_2\mathfrak{A}_3,\ \mathfrak{C}_3\mathfrak{A}_2 & \text{,,} & \mathrm{B_4}, & \text{,,} \\ \mathfrak{A}_2\mathfrak{B}_3,\ \mathfrak{A}_3\mathfrak{B}_2 & \text{,,} & \mathrm{C_4}, & \text{,,} \end{array} \left\{ \begin{array}{ccc} h^2-g^2, & h\,(g^2-1), & g\,(h^2-1) \\ h\,(f^2-1), & f^2-h^2, & f\,(h^2-1) \\ g\,(f^2-1), & f\,(g^2-1), & g^2-f^2 \end{array} \right\}.$$

Now ;—

$$\begin{array}{ll} \mathrm{A} & \text{being} \\ \mathrm{A_3} & \text{,,} \\ \mathrm{A_4} & \text{,,} \end{array} \left| \begin{array}{ccc} 1, & 0, & 0 \\ 1-g^2h^2, & h\,(g^2-1), & g\,(h^2-1) \\ h^2-g^2, & h\,(g^2-1), & g\,(h^2-1) \end{array} \right| \equiv 0. \text{ Therefore ;—}$$

$\mathrm{A\,A_3A_4}$ is a straight line.
$\mathrm{B\,B_3\,B_4}$,, ,,
$\mathrm{C\,C_3\,C_4}$,, ,,

THEOREM 8.

$$\begin{array}{l} \mathrm{A\,A_3A_4} \text{ is} \\ \mathrm{B\,B_3B_4} \text{ is} \\ \mathrm{C\,C_3C_4} \text{ is} \end{array} \left| \begin{array}{ccc} 0, & g\,(1-h^2), & -h\,(1-g^2) \\ -f\,(1-h^2), & 0, & h\,(1-f^2) \\ f\,(1-g^2), & -g\,(1-f^2), & 0 \end{array} \right| \equiv 0. \text{ Therefore ;—}$$

$\mathrm{A\,A_3A_4}$, $\mathrm{B\,B_3B_4}$, $\mathrm{C\,C_3C_4}$ meet in a point $\mathrm{P_4}$, viz., $\dfrac{1-f^2}{f}$, $\dfrac{1-g^2}{g}$, $\dfrac{1-h^2}{h}$.

THEOREM 9.

$$\begin{array}{l} A_1 \text{ is} \\ A_2 \text{ is} \\ A_3 \text{ is} \end{array} \left| \begin{array}{ccc} -1, & h, & g \\ -gh, & g, & h \\ 1-g^2h^2, & h\,(g^2-1), & g\,(h^2-1) \end{array} \right| \equiv 0. \quad \text{Therefore ;—}$$

$$\begin{array}{lll} A_1A_2A_3 & \text{is a straight line.} & \\ B_1B_2B_3 & ,, & ,, \\ C_1C_2C_3 & ,, & ,, \end{array}$$

THEOREM 10.

$$\left. \begin{array}{llllll} B_3C_3 \text{ is} & 1+fgh, & h+fg, & g+hf; & \text{meeting } BC & \text{in } \mathfrak{A}_1 \\ C_3A_3 \text{ is} & h+fg, & 1+fgh, & f+gh; & ,, \quad CA & ,, \ \mathfrak{B}_1 \\ A_3B_3 \text{ is} & g+hf, & f+gh, & 1+fgh; & ,, \quad AB & ,, \ \mathfrak{C}_1 \end{array} \right\}. \quad \text{Therefore ;—}$$

$$\begin{array}{lll} BC,\ B_1C_1,\ B_2C_2,\ B_3C_3 & \text{meet in} & \mathfrak{A}_1. \\ CA,\ C_1A_1,\ C_2A_2,\ C_3A_3 & ,, & \mathfrak{B}_1. \\ AB,\ A_1B_1,\ A_2B_2,\ A_3B_3 & ,, & \mathfrak{C}_1. \end{array}$$

THEOREM 11.

$$\begin{array}{lll} BC,\ B_4C_4 \text{ meet in} & \mathfrak{A}_4, & \text{viz.,} \\ CA,\ C_4A_4 \quad ,, & \mathfrak{B}_4, & ,, \\ AB,\ A_4B_4 \quad ,, & \mathfrak{C}_4, & ,, \end{array} \left| \begin{array}{ccc} 0, & (f-gh).(h+fg), & -(f+gh).(g-hf) \\ -(g+hf).(h-fg), & 0, & (g-hf).(f+gh) \\ (h-fg).(g+hf), & -(h+fg).(f-gh), & 0 \end{array} \right| \equiv 0.$$

Therefore $\mathfrak{A}_4\mathfrak{B}_4\mathfrak{C}_4$ is a straight line, viz., $\dfrac{1}{(g+hf).(h-fg)}, \ \dfrac{1}{(h+fg).(f-gh)}, \ \dfrac{1}{(f+gh).(g-hf)}.$

THEOREM 12.

$$\begin{array}{lll} B_1C,\ A_1B \text{ meet in} & A_5, & \text{viz.,} \\ C_1A,\ B_1C \quad ,, & B_5, & ,, \\ A_1B,\ C_1A \quad ,, & C_5, & ,, \end{array} \left\{ \begin{array}{ccc} -h, & 1, & gh \\ hf, & -f, & 1 \\ 1, & fg, & -g \end{array} \right\}. \quad \begin{array}{lll} BC_1,\ CA_1 \text{ meet in} & A_6, & \text{viz.,} \\ CA_1,\ AB_1 \quad ,, & B_6, & ,, \\ AB_1,\ BC_1 \quad ,, & C_6, & ,, \end{array} \left\{ \begin{array}{ccc} -g, & gh, & 1 \\ 1, & -h, & hf \\ fg, & 1, & -f \end{array} \right\}.$$

$$\begin{array}{lll} A_5A_6,\ BC \text{ meet in} & \mathfrak{A}_5, & \text{viz.,} \\ B_5B_6,\ CA \quad ,, & \mathfrak{B}_5, & ,, \\ C_5C_6,\ AB \quad ,, & \mathfrak{C}_5, & ,, \end{array} \left| \begin{array}{ccc} 0, & g\,(1-h^2), & -h\,(1-g^2) \\ -f\,(1-h^2), & 0, & h\,(1-f^2) \\ f\,(1-g^2), & -g\,(1-f^2), & 0 \end{array} \right| \equiv 0. \quad \text{Therefore ;—}$$

$\mathfrak{A}_5\mathfrak{B}_5\mathfrak{C}_5$ is a straight line, viz., $\dfrac{1-f^2}{f}, \ \dfrac{1-g^2}{g}, \ \dfrac{1-h^2}{h}.$

THEOREM 13.

$$\begin{array}{l} BC \text{ is} \\ B_5C_6 \text{ is} \\ B_6C_5 \text{ is} \end{array} \left| \begin{array}{ccc} 1, & 0, & 0 \\ f^2-1, & f\,(g+hf), & f\,(h+fg) \\ hg\,(1-f^2), & g+hf, & h+fg \end{array} \right| \equiv 0. \quad \text{Therefore ;—}$$

$$\begin{array}{lll} BC,\ B_5C_6,\ B_6C_5 & \text{meet in a point:} & \mathfrak{A}_6 \\ CA,\ C_5A_6,\ C_6A_5 & ,, \quad ,, & \mathfrak{B}_6 \\ AB,\ A_5B_6,\ A_6B_5 & ,, \quad ,, & \mathfrak{C}_6 \end{array}$$

THEOREM 14.

$$\begin{array}{l} \mathfrak{A}_6 \text{ is} \\ \mathfrak{B}_6 \text{ is} \\ \mathfrak{C}_6 \text{ is} \end{array} \left| \begin{array}{ccc} 0, & h+fg, & -(g+hf) \\ -(h+fg), & 0, & f+gh \\ g+hf, & -(f+gh), & 0 \end{array} \right| \equiv 0. \quad \text{Therefore ;—}$$

$\mathfrak{A}_6\mathfrak{B}_6\mathfrak{C}_6$ is a straight line, viz., $f+gh,\ g+hf,\ h+fg$.

$\mathfrak{A}_1\mathfrak{B}_1\mathfrak{C}_1$ and $\mathfrak{A}_6\mathfrak{B}_6\mathfrak{C}_6$ are inverse or reciprocal lines.

THEOREM 15.

$$\begin{array}{l} \mathfrak{B}_1\mathfrak{C}_6 \text{ is} \\ \mathfrak{C}_1\mathfrak{A}_6 \text{ is} \\ \mathfrak{A}_1\mathfrak{B}_6 \text{ is} \end{array} \left| \begin{array}{ccc} (h+fg)\,.\,(f+gh), & (g+hf)\,.\,(h+fg), & (f+gh)^2 \\ (g+hf)^2, & (f+gh)\,.\,(g+hf), & (h+fg)\,.\,(f+gh) \\ (f+gh)\,.\,(g+hf), & (h+fg)^2, & (g+hf)\,.\,(h+fg) \end{array} \right| \begin{array}{l} \text{and meets BC in } \mathfrak{A}_7 \\ \quad ,, \quad \text{CA in } \mathfrak{B}_7 \\ \quad ,, \quad \text{AB in } \mathfrak{C}_7 \end{array}$$

$$\begin{array}{l} \mathfrak{A}_7 \text{ is} \\ \mathfrak{B}_7 \text{ is} \\ \mathfrak{C}_7 \text{ is} \end{array} \left| \begin{array}{ccc} 0, & (f+gh)^2, & -(g+hf)\,.\,(h+fg) \\ -(h+fg)\,.\,(f+gh), & 0, & (g+hf)^2 \\ (h+fg)^2, & -(f+gh)\,.\,(g+hf), & 0 \end{array} \right| \equiv 0. \quad \text{Therefore ;—}$$

$\mathfrak{A}_7\mathfrak{B}_7\mathfrak{C}_7$ is a straight line, viz., $\dfrac{g+hf}{h+fg},\ \dfrac{h+fg}{f+gh},\ \dfrac{f+gh}{g+hf}$.

THEOREM 16.

$$\text{Similarly,} \left\{ \begin{array}{lr} \mathfrak{B}_6\mathfrak{C}_1 \text{ meeting} & \text{BC in } \mathfrak{A}_8 \\ \mathfrak{C}_6\mathfrak{A}_1 \quad ,, & \text{CA in } \mathfrak{B}_8 \\ \mathfrak{A}_6\mathfrak{B}_1 \quad ,, & \text{AB in } \mathfrak{C}_8 \end{array} \right\}, \ \mathfrak{A}_8\mathfrak{B}_8\mathfrak{C}_8 \text{ is a straight line, viz., } \frac{h+fg}{g+hf},\ \frac{f+gh}{h+fg},\ \frac{g+hf}{f+gh}.$$

$\mathfrak{A}_7\mathfrak{B}_7\mathfrak{C}_7$ and $\mathfrak{A}_8\mathfrak{B}_8\mathfrak{C}_8$ are reciprocal lines.

THEOREM 17.

$$\begin{array}{l} \text{A A}_1 \text{ is} \\ \text{B B}_5 \text{ is} \\ \text{C C}_6 \text{ is} \end{array} \left| \begin{array}{rrr} 0, & g, & -h \\ -1, & 0, & hf \\ 1, & -fg, & 0 \end{array} \right| \equiv 0. \quad \text{Also} \left\{ \begin{array}{l} \text{A A}_2 \text{ is} \\ \text{B B}_6 \text{ is} \\ \text{C C}_5 \text{ is} \end{array} \right. \left| \begin{array}{rrr} 0, & h, & -g \\ -hf, & 0, & 1 \\ fg, & -1, & 0 \end{array} \right| \equiv 0. \quad \text{Therefore ;—}$$

A A_1, B B_5, C C_6 meet in a point A_7 viz., $fgh,\ h,\ g$.
B B_1, C C_5, A A_6 „ „ B_7 „ $h,\ fgh,\ f$.
C C_1, A A_5, B B_6 „ „ C_7 „ $g,\ f,\ fgh$.

THEOREM 18.

A A_2, B B_6, C C_5 meet in a point A_8 viz., $1,\ fg,\ hf$.
B B_2, C C_6, A A_5 „ „ B_8 „ $fg,\ 1,\ gh$.
C C_2, A A_6, B B_5 „ „ C_8 „ $hf,\ gh,\ 1$.

But instead of continuing the manipulation, we shall gather up these results, and continue the series of propositions.

THEOREM I.

AA_1, BB_1, CC_1 intersect, in a point, P_1.

THEOREM II.

$$\left.\begin{array}{lcc} \text{Let } BC,\ B_1C_1 & \text{meet in} & \mathfrak{A}_1 \\ CA,\ C_1A_1 & ,, & \mathfrak{B}_1 \\ AB,\ A_1B_1 & ,, & \mathfrak{C}_1 \end{array}\right\}.\quad \text{Then;—}\mathfrak{A}_1\mathfrak{B}_1\mathfrak{C}_1 \text{ is a straight line.}$$

THEOREM III.

$$\left.\begin{array}{lcc} \text{Let } BC_1,\ B_1C & \text{meet in} & A_2 \\ CA_1,\ C_1A & ,, & B_2 \\ AB_1,\ A_1B & ,, & C_2 \end{array}\right\}.\quad \text{Then;—}AA_2,\ BB_2,\ CC_2 \text{ meet in a point, } P_2.$$

P_1 and P_2 are reciprocal points.

THEOREM IV.

$$\begin{array}{lcccc} BC,\ B_1C_1,\ B_2C_2 & \text{intersect in a point, and that point is} & & & \mathfrak{A}_1 \\ CA,\ C_1A_1,\ C_2A_2 & ,, & ,, & ,, & \mathfrak{B}_1 \\ AB,\ A_1B_1,\ A_2B_2 & ,, & ,, & ,, & \mathfrak{C}_1 \end{array}$$

THEOREM V.

A_1A_2, B_1B_2, C_1C_2 intersect in a point, P_3.

THEOREM VI.

$P_1P_2P_3$ is a straight line.

THEOREM VII.

$$\left.\begin{array}{lcc} \text{Let } A_1B,\ BC & \text{meet in} & \mathfrak{A}_2 \\ B_1C,\ CA & ,, & \mathfrak{B}_2 \\ C\ A,\ AB & ,, & \mathfrak{C}_2 \end{array}\right\};\qquad \left.\begin{array}{lcc} \text{And let } C_1A,\ BC & \text{meet in} & \mathfrak{A}_3 \\ A_1B,\ CA & ,, & \mathfrak{B}_3 \\ B_1C,\ AB & ,, & \mathfrak{C}_3 \end{array}\right\}.$$

$$\left.\begin{array}{lcc} \text{Also let } \mathfrak{B}_2\mathfrak{C}_2,\ \mathfrak{B}_3\mathfrak{C}_3 & \text{meet in} & A_3 \\ \mathfrak{C}_2\mathfrak{A}_2,\ \mathfrak{C}_3\mathfrak{A}_3 & ,, & B_3 \\ \mathfrak{A}_2\mathfrak{B}_2,\ \mathfrak{A}_3\mathfrak{B}_3 & ,, & C_3 \end{array}\right\};\qquad \left.\begin{array}{lcc} \text{And let } \mathfrak{B}_2\mathfrak{C}_3,\ \mathfrak{B}_3\mathfrak{C}_2 & \text{meet in} & A_4 \\ \mathfrak{C}_2\mathfrak{A}_3,\ \mathfrak{C}_3\mathfrak{A}_2 & ,, & B_4 \\ \mathfrak{A}_2\mathfrak{B}_3,\ \mathfrak{A}_3\mathfrak{B}_2 & ,, & C_4 \end{array}\right\}.$$

$$\begin{array}{lcc} \text{Then,—}AA_3A_4 & \text{is a} & \text{straight line.} \\ BB_3B_4 & ,, & ,, \\ CC_3C_4 & ,, & ,, \end{array}$$

THEOREM VIII.

AA_3A_4, BB_3B_4, CC_3C_4 meet in a point, P_4.

THEOREM IX.

$$\begin{array}{lcc} A_1A_2A_3 & \text{is a} & \text{straight line.} \\ B_1B_2B_3 & ,, & ,, \\ C_1C_2C_3 & ,, & ,, \end{array}$$

THEOREM X.

$$\begin{array}{llllll} BC, & B_1C_1, & B_2C_2, & B_3C_3 & \text{all meet in} & \mathfrak{A}_1. \\ CA, & C_1A_1, & C_2A_2, & C_3A_3 & ,, & \mathfrak{B}_1. \\ AB, & A_1B_1, & A_2B_2, & A_3B_3 & ,, & \mathfrak{C}_1. \end{array}$$

THEOREM XI.

$$\left.\begin{array}{llll} \text{Let } BC, & B_4C_4 & \text{meet in} & \mathfrak{A}_4 \\ CA, & C_4A_4 & ,, & \mathfrak{B}_4 \\ AB, & A_4B_4 & ,, & \mathfrak{C}_4 \end{array}\right\}. \quad \text{Then ;—}\mathfrak{A}_4\mathfrak{B}_4\mathfrak{C}_4 \text{ is a straight line.}$$

THEOREM XII.

$$\left.\begin{array}{llll} \text{Let } B_1C, & A_1B & \text{meet in} & A_5 \\ C_1A, & B_1C & ,, & B_5 \\ A_1B, & C_1A & ,, & C_5 \end{array}\right\};\text{—} \qquad \left.\begin{array}{llll} \text{And let } BC_1, & CA_1 & \text{meet in} & A_6 \\ CA_1, & AB_1 & ,, & B_6 \\ AB_1, & BC_1 & ,, & C_6 \end{array}\right\}.$$

$$\left.\begin{array}{llll} \text{Also let } BC, & A_5A_6 & \text{meet in} & \mathfrak{A}_5 \\ CA, & B_5B_6 & ,, & \mathfrak{B}_5 \\ AB, & C_5C_6 & ,, & \mathfrak{C}_5 \end{array}\right\}. \quad \text{Then ;—}\mathfrak{A}_5\mathfrak{B}_5\mathfrak{C}_5 \text{ is a straight line.}$$

THEOREM XIII.

$$\begin{array}{llllll} BC, & B_5C_6, & B_6C_5 & \text{meet in} & \text{a point,} & \mathfrak{A}_6. \\ CA, & C_5A_6, & C_6A_5 & ,, & ,, & \mathfrak{B}_6. \\ AB, & A_5B_6, & A_6B_5 & ,, & ,, & \mathfrak{C}_6. \end{array}$$

THEOREM XIV.

$\mathfrak{A}_6\mathfrak{B}_6\mathfrak{C}_6$ is a straight line; reciprocal to $\mathfrak{A}_1\mathfrak{B}_1\mathfrak{C}_1$.

THEOREM XV.

$$\left.\begin{array}{llll} \text{Let } BC, & \mathfrak{B}_1\mathfrak{C}_6 & \text{meet in} & \mathfrak{A}_7 \\ CA, & \mathfrak{C}_1\mathfrak{A}_6 & ,, & \mathfrak{B}_7 \\ AB, & \mathfrak{A}_1\mathfrak{A}_6 & ,, & \mathfrak{C}_7 \end{array}\right\}. \quad \text{Then ;—}\mathfrak{A}_7\mathfrak{B}_7\mathfrak{C}_7 \text{ is a straight line.}$$

THEOREM XVI.

$$\left.\begin{array}{llll} \text{Let } BC, & \mathfrak{B}_6\mathfrak{C}_1 & \text{meet in} & \mathfrak{A}_8 \\ CA, & \mathfrak{C}_6\mathfrak{A}_1 & ,, & \mathfrak{B}_8 \\ AB, & \mathfrak{A}_6\mathfrak{B}_1 & ,, & \mathfrak{C}_8 \end{array}\right\}. \quad \text{Then ;—}\mathfrak{A}_8\mathfrak{B}_8\mathfrak{C}_8 \text{ is a straight line; reciprocal to } \mathfrak{A}_7\mathfrak{B}_7\mathfrak{C}_7.$$

THEOREM XVII.

$$\begin{array}{llllll} AA_1, & BB_5, & CC_6 & \text{meet in} & \text{a point,} & A_7. \\ BB_1, & CC_5, & AA_6 & ,, & ,, & B_7. \\ CC_1, & AA_5, & BB_6 & ,, & ,, & C_7. \end{array}$$

THEOREM XVIII.

$$\begin{array}{llllll} AA_2, & BB_6, & CC_5 & \text{meet in} & \text{a point,} & A_8. \\ BB_2, & CC_6, & AA_5 & ,, & ,, & B_8. \\ CC_2, & AA_6, & BB_5 & ,, & ,, & C_8. \end{array}$$

THEOREM XIX.

A_7A_8, B_7B_8, C_7C_8 meet in a point P_5.

THEOREM XX.

$$\begin{array}{llll} BC, & B_7C_7, & B_8C_8 & \text{meet in a point, } \mathfrak{A}_9. \\ CA, & C_7A_7, & C_8A_8 & \quad ,, \quad ,, \quad \mathfrak{B}_9. \\ AB, & A_7B_7, & A_8B_8 & \quad ,, \quad ,, \quad \mathfrak{C}_9. \end{array}$$

THEOREM XXI.

$\mathfrak{A}_9\mathfrak{B}_9\mathfrak{C}_9$ is a straight line.

THEOREM XXII.

$$\begin{array}{lllll} BC, & AA_2, & A_8P_1, & A_7P_2 & \text{all meet in a point, } \mathfrak{A}_{10}. \\ CA, & BB_2, & B_8P_1, & B_7P_2 & \quad ,, \quad ,, \quad \mathfrak{B}_{10}. \\ AB, & CC_2, & C_8P_1, & C_7P_2 & \quad ,, \quad ,, \quad \mathfrak{C}_{10}. \end{array}$$

THEOREM XXIII.

$$\begin{array}{lllll} A\mathfrak{A}_9, & AC, & A\mathfrak{A}_{10}, & AB & \text{is a harmonic pencil.} \\ B\mathfrak{B}_9, & BA, & B\mathfrak{B}_{10}, & BC & \quad ,, \quad ,, \\ C\mathfrak{C}_9, & CB, & C\mathfrak{C}_{10}, & CA & \quad ,, \quad ,, \end{array}$$

THEOREM XXIV.

$$\begin{array}{llll} BC, & A_1P_2, & A_2P_1 & \text{meet in a point, } \mathfrak{A}_{11}. \\ CA, & B_1P_2, & B_2P_1 & \quad ,, \quad ,, \quad \mathfrak{B}_{11}. \\ AB, & C_1P_2, & C_2P_1 & \quad ,, \quad ,, \quad \mathfrak{C}_{11}. \end{array}$$

THEOREM XXV.

$A\mathfrak{A}_{11}$, $B\mathfrak{B}_{11}$, $C\mathfrak{C}_{11}$ intersect in a point, P_6.

THEOREM XXVI.

$$\begin{array}{llll} BC, & A_1A_8, & A_2A_7 & \text{meet in a point, } \mathfrak{A}_{12}. \\ CA, & B_1B_8, & B_2B_7 & \quad ,, \quad ,, \quad \mathfrak{B}_{12}. \\ AB, & C_1C_8, & C_2C_7 & \quad ,, \quad ,, \quad \mathfrak{C}_{12}. \end{array}$$

THEOREM XXVII.

$A\mathfrak{A}_{12}$, $B\mathfrak{B}_{12}$, $C\mathfrak{C}_{12}$ intersect in a point, Q_1.

THEOREM XXVIII.

$$\text{Let } \left.\begin{array}{llll} BC, & A_7A_8 & \text{meet in} & \mathfrak{A}_{13} \\ CA, & B_7B_8 & \quad ,, & \mathfrak{B}_{13} \\ AB, & C_7C_8 & \quad ,, & \mathfrak{C}_{13} \end{array}\right\} ;\text{—Then, } A\mathfrak{A}_{13},\ B\mathfrak{B}_{13},\ C\mathfrak{C}_{13} \text{ intersect in a point, } Q_2.$$

THEOREM XXIX.

$$\text{Let } \left.\begin{array}{llll} B_5B_6, & C_5C_6 & \text{meet in} & A_9 \\ C_5C_6, & A_5A_6 & \quad ,, & B_9 \\ A_5A_6, & B_5B_6 & \quad ,, & C_9 \end{array}\right\} ;\text{—Then, } AA_9,\ BB_9,\ CC_9 \text{ intersect in a point, } Q_3.$$

THEOREM XXX.

$$\text{Let } \left.\begin{array}{llll} A_1A_2, & A_5A_6 & \text{meet in} & A_{10} \\ B_1B_2, & B_5B_6 & \quad ,, & B_{10} \\ C_1C_2, & C_5C_6 & \quad ,, & C_{10} \end{array}\right\} ;\text{—Then, } AA_{10},\ BB_{10},\ CC_{10} \text{ intersect in a point, } Q_4.$$

THEOREM XXXI.

$$\text{Let}\ \left.\begin{array}{llr} BC,\ AA_7 & \text{meet in} & l \\ CA,\ BA_7 & \quad ,, & m \\ AB,\ CA_7 & \quad ,, & n \end{array}\right\};\qquad \text{And let}\ \left.\begin{array}{llr} BC,\ mn & \text{meet in} & l_1 \\ CA,\ nl & \quad ,, & m_1 \\ AB,\ lm & \quad ,, & n_1 \end{array}\right\}.$$

Then $l_1m_1n_1$ is a straight line, as is well known. As it depends entirely on the position of A_7 let it be called $\phi\ (A_7)$. Then ;—

$$\begin{array}{llllll} \phi\ (A_7) & \text{is the polar of} & A_8 & \text{to the imaginary} & \text{conic,} & \alpha^2+\beta^2+\gamma^2=0. \\ \phi\ (B_7) & \quad ,, & B_8 & \quad ,, & \quad ,, & \quad ,, \\ \phi\ (C_7) & \quad ,, & C_8 & \quad ,, & \quad ,, & \quad ,, \end{array}$$

THEOREM XXXII.

$$\begin{array}{llllll} \phi\ (A_8) & \text{is the polar of} & A_7 & \text{to the imaginary} & \text{conic,} & \alpha^2+\beta^2+\gamma^2=0. \\ \phi\ (B_8) & \quad ,, & B_7 & \quad ,, & \quad ,, & \quad ,, \\ \phi\ (C_8) & \quad ,, & C_7 & \quad ,, & \quad ,, & \quad ,, \end{array}$$

THEOREM XXXIII.

$$\begin{array}{llllll} \phi\ (P_1) & \text{is the polar of} & P_2 & \text{to the imaginary} & \text{conic,} & \alpha^2+\beta^2+\gamma^2=0. \\ \phi\ (P_2) & \quad ,, & P_1 & \quad ,, & \quad ,, & \quad ,, \end{array}$$

THEOREM XXXIV.

$$\begin{array}{lllll} \phi^{-1}\ (\mathfrak{A}_1\,\mathfrak{A}_1\,\mathfrak{C}_1) & \text{is the pole of} & \mathfrak{A}_6\,\mathfrak{B}_6\,\mathfrak{C}_6 & \text{to the conic,} & \alpha^2+\beta^2+\gamma^2=0. \\ \phi^{-1}\ (\mathfrak{A}_6\,\mathfrak{B}_6\,\mathfrak{C}_6) & \quad ,, & \mathfrak{A}_1\,\mathfrak{B}_1\,\mathfrak{C}_1 & \quad ,, & \quad ,, \end{array}$$

THEOREM XXXV.

$$\begin{array}{lllll} \phi^{-1}\ (\mathfrak{A}_7\,\mathfrak{B}_7\,\mathfrak{C}_7) & \text{is the pole of} & \mathfrak{A}_8\,\mathfrak{B}_8\,\mathfrak{C}_8 & \text{to the conic,} & \alpha^2+\beta^2+\gamma^2=0. \\ \phi^{-1}\ (\mathfrak{A}_8\,\mathfrak{B}_8\,\mathfrak{C}_8) & \quad ,, & \mathfrak{A}_7\,\mathfrak{B}_7\,\mathfrak{C}_7 & \quad ,, & \quad ,, \end{array}$$

The principle of reciprocation would introduce here a number of Propositions which it is unnecessary to enunciate.

THEOREM XXXVI.

$$\text{Let}\ \left.\begin{array}{llr} AA_1,\ B_7C_7 & \text{meet in} & l_2 \\ BB_1,\ C_7A_7 & \quad ,, & m_2 \\ CC_1,\ A_7B_7 & \quad ,, & n_2 \end{array}\right\};\qquad \text{And let}\ \left.\begin{array}{llr} BC,\ m_2n_2 & \text{meet in} & \alpha_1 \\ CA,\ n_2l_2 & \quad ,, & \beta_1 \\ AB,\ l_2m_2 & \quad ,, & \gamma_1 \end{array}\right\}.$$

Then ;—$\alpha_1\beta_1\gamma_1$ is a straight line.

THEOREM XXXVII.

$$\text{Let}\ \left.\begin{array}{llr} AA_2,\ B_8C_8 & \text{meet in} & l_3 \\ BB_2,\ C_8A_8 & \quad ,, & m_3 \\ CC_2,\ A_8B_8 & \quad ,, & n_3 \end{array}\right\};\qquad \text{And let}\ \left.\begin{array}{llr} BC,\ m_3n_3 & \text{meet in} & \alpha_2 \\ CA,\ n_3l_3 & \quad ,, & \beta_2 \\ AB,\ l_3m_3 & \quad ,, & \gamma_2 \end{array}\right\}.$$

Then ;—$\alpha_2\beta_2\gamma_2$ is a straight line.

THEOREM XXXVIII.

$$\text{Let}\ \left.\begin{array}{llr} \beta_1\gamma_2,\ \beta_2\gamma_1 & \text{meet in} & \alpha_3 \\ \gamma_1\alpha_2,\ \gamma_2\alpha_1 & \quad ,, & \beta_3 \\ \alpha_1\beta_2,\ \alpha_2\beta_1 & \quad ,, & \gamma_3 \end{array}\right\};\text{—Then, } \alpha_3\beta_3\gamma_3 \text{ is a straight line}$$

THEOREM XXXIX.

$$\text{Let}\ \left.\begin{array}{lll} \alpha_1\beta_2,\ \gamma_1\alpha_2 & \text{meet in} & \alpha_4 \\ \beta_1\gamma_2,\ \alpha_1\beta_2 & \text{,,} & \beta_4 \\ \gamma_1\alpha_2,\ \beta_1\gamma_2 & \text{,,} & \gamma_4 \end{array}\right\}; \qquad \text{And let}\ \left.\begin{array}{lll} \alpha_2\beta_1,\ \gamma_2\alpha_1 & \text{meet in} & \alpha_5 \\ \beta_2\gamma_1,\ \alpha_2\beta_1 & \text{,,} & \beta_5 \\ \gamma_2\alpha_1,\ \beta_2\gamma_1 & \text{,,} & \gamma_5 \end{array}\right\}.$$

$$\text{Then;—}\quad \begin{array}{lll} \text{A}\,\alpha_4\alpha_5 & \text{is a straight line.} & \\ \text{B}\,\beta_4\beta_5 & \text{,,} & \text{,,} \\ \text{C}\,\gamma_4\gamma_5 & \text{,,} & \text{,,} \end{array}$$

THEOREM XL.

A $\alpha_4\alpha_5$, B $\beta_4\beta_5$, C $\gamma_4\gamma_5$ meet in a point.

THEOREM XLI.

Postulating again, similarly as in Theorems XXXVI. and XXXVII.,—

$$\left.\begin{array}{l} \text{Let}\ \ \text{A}\,\text{A}_2,\ \text{B}_7\text{C}_7\ \text{meet in}\ \bar{l}_2;\ \text{and similarly} \\ \text{Also}\ \text{A}\,\text{A}_1,\ \text{B}_8\text{C}_8\ \ \text{,,}\ \ \bar{l}_3;\ \text{and similarly} \end{array}\right\}.\quad \text{Then;—}$$

$\bar{\alpha}_1\,\bar{\beta}_1\,\bar{\gamma}_1$ is a straight line.

THEOREM XLII.

$\bar{\alpha}_2\,\bar{\beta}_2\,\bar{\gamma}_2$ is a straight line.

THEOREM XLIII.

$\bar{\alpha}_3\,\bar{\beta}_3\,\bar{\gamma}_3$ is a straight line.

THEOREM XLIV.

$$\begin{array}{lll} \text{A}\,\bar{\alpha}_4\,\bar{\alpha}_5 & \text{is a straight line.} & \\ \text{B}\,\bar{\beta}_4\,\bar{\beta}_5 & \text{,,} & \text{,,} \\ \text{C}\,\bar{\gamma}_4\,\bar{\gamma}_5 & \text{,,} & \text{,,} \end{array}$$

THEOREM XLV.

A $\bar{\alpha}_4\,\bar{\alpha}_5$, B $\bar{\beta}_4\,\bar{\beta}_5$, C $\bar{\gamma}_4\,\bar{\gamma}_5$ meet in a point.

THEOREM XLVI.

$$\begin{array}{lllll} \text{Let}\ \alpha_1\alpha_2,\ \bar{\alpha}_1\bar{\alpha}_2 & \text{meet in} & \alpha_6 & & \\ \beta_1\beta_2,\ \bar{\beta}_1\bar{\beta}_2 & \text{,,} & \beta_6 & & \text{Then;—} \\ \gamma_1\gamma_2,\ \bar{\gamma}_1\bar{\gamma}_2 & \text{,,} & \gamma_6 & & \end{array}$$

A α_6, Bβ_6, Cγ_6; A_7A_8, B_7B_8, C_7C_8 all intersect in P_5.

Let P_3Q_3 and P_6Q_1 meet in Q_5; and let P_5Q_5 and P_4Q_4 meet in Q_6.

Then the points P_1, P_2, P_3, P_4, P_5, P_6; Q_1, Q_2, Q_3, Q_4, Q_5, Q_6 have very remarkable relations.

THEOREM XLVII.

P_1, P_2, P_3, P_4, P_5, P_6 all range in a straight line.

THEOREM XLVIII.

Q_1Q_4 and Q_3Q_5 intersect in P_3.

THEOREM XLIX.

Q_2Q_5 and Q_4Q_6 intersect in P_4.

THEOREM L.

Q_1Q_3 and P_1P_2 intersect in P_5.

THEOREM LI.

Q_1Q_5 and Q_3Q_4 intersect in P_6.

THEOREM LII.

P_4Q_5 and P_5Q_4 intersect in Q_2.

THEOREM LIII.

P_1P_2, Q_1Q_2, Q_3Q_6 intersect in a point.

THEOREM LIV.

Q_1Q_3, Q_4Q_5, Q_2Q_6 intersect in a point,—say $\mathfrak{P}$.

THEOREM LV.

Let $\left.\begin{array}{l} Q_3Q_5 \text{ and } Q_5P_3 \text{ meet in } S_1 \\ Q_1Q_3 \text{ and } Q_4P_5 \quad ,, \quad S_2 \end{array}\right\}$; And let $\left.\begin{array}{l} Q_3Q_5 \text{ and } Q_4Q_6 \text{ meet in } S_3 \\ Q_1Q_4 \text{ and } Q_2Q_5 \quad ,, \quad S_4 \end{array}\right\}$. Then ;—

$S_1S_2\mathfrak{P}$ is a straight line.

THEOREM LVI.

$S_3S_4\mathfrak{P}$ is a straight line.

THEOREM LVII.

Let $\left.\begin{array}{l} P_1Q_1 \text{ and } P_2Q_3 \text{ meet in } R_1 \\ P_1Q_3 \text{ and } P_2Q_1 \quad ,, \quad R_2 \end{array}\right\}$; And let $\left.\begin{array}{l} P_1Q_4 \text{ and } P_2Q_5 \text{ meet in } R_3 \\ P_1Q_5 \text{ and } P_2Q_4 \quad ,, \quad R_4 \end{array}\right\}$. Then ;—

$R_1R_2\mathfrak{P}$ is a straight line.

THEOREM LVIII.

$R_3R_4\mathfrak{P}$ is a straight line.

THEOREM LIX.

$$\text{Let} \left.\begin{array}{l} P_1Q_6 \text{ and } P_2Q_2 \text{ meet in } U_1 \\ P_1Q_2 \text{ and } P_2Q_6 \quad \text{,,} \quad U_2 \end{array}\right\} \text{Then ;—}$$

$U_1U_2\mathfrak{P}$ is a straight line.

THEOREM LX.

U_1U_2; R_1R_2, R_3R_4; S_1S_2, S_3S_4; Q_1Q_3, Q_4Q_5, Q_2Q_6; all intersect in $\mathfrak{P}$.

THEOREM LXI.

$\mathfrak{P}$ is the pole of $P_1P_2P_3P_4P_5P_6$ to the imaginary conic, $\alpha^2+\beta^2+\gamma^2=0$.

THEOREM LXII.

The lines AA_1, BB_1, CC_1; AA_2, BB_2, CC_2 cut the sides of the triangle ABC in six points which lie in the conic;—

$$\alpha^2+\beta^2+\gamma^2-\left(\frac{g}{h}+\frac{h}{g}\right)\beta\gamma-\left(\frac{h}{f}+\frac{f}{h}\right)\gamma\alpha-\left(\frac{f}{g}+\frac{g}{f}\right)\alpha\beta=0.$$

For the six points are inverse or reciprocal points. Substituting the co-ordinates of five of them (the third of the *second* set being omitted) in the general equation of the second degree, and eliminating the arbitrary constants, gives the conic as above. To substitute the co-ordinates of five of the points, omitting now the third of the *first* set, amounts evidently to inverting the separate terms in the constants of the above equation; and as this leaves it unaltered, the proof of the theorem is obvious.

THEOREM LXIII.

The lines AA_5, BB_5, CC_5; AA_6, BB_6, CC_6 cut the sides of the triangle ABC in six points which lie in the conic;—

$$\alpha^2+\beta^2+\gamma^2-\left(gh+\frac{1}{gh}\right)\beta\gamma-\left(hf+\frac{1}{hf}\right)\gamma\alpha-\left(fg+\frac{1}{fg}\right)\alpha\beta=0.$$

THEOREM LXIV.

The points $\mathfrak{A}_1\mathfrak{B}_1\mathfrak{C}_1$; $\mathfrak{A}_2\mathfrak{B}_2\mathfrak{C}_2$ lie in the conic;—

$$\alpha^2+\beta^2+\gamma^2+\left(f+\frac{1}{f}\right)\beta\gamma+\left(g+\frac{1}{g}\right)\gamma\alpha+\left(h+\frac{1}{h}\right)\alpha\beta=0.$$

THEOREM LXV.

If the point P_1 move in a straight line, the line $\phi\ (P_1)$ will always touch a conic which touches the three sides of the triangle ABC.

Let the equation of the line in which P_1 moves be $l\alpha + m\beta + n\gamma = 0$; or substituting the co-ordinates of P_1 now supposed variable (see Theorem 1),

$$\frac{l}{f} + \frac{m}{g} + \frac{n}{h} = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1).$$

$\phi\,(P_1)$ is
$$f\alpha + g\beta + h\gamma = 0 = u \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2).$$

Now h may be considered as constant, since it is the ratios only that are concerned. Take f as the independent variable, then g varies with f by (1); and by the theory of envelopes we have

$$\frac{du}{df} = 0 = \left(\frac{du}{df}\right) + \left(\frac{du}{dg}\right) \cdot \frac{dg}{df} \quad . \quad . \quad . \quad . \quad . \quad (3).$$

Elimination gives
$$\pm (l\alpha)^{\frac{1}{2}} \pm (m\beta)^{\frac{1}{2}} \pm (n\gamma)^{\frac{1}{2}} = 0 ;—$$

the equation of a conic touching the sides of the triangle of reference.—*Q. E. D.*

THEOREM LXVI.

Let lines be drawn from the angles of a triangle through any two points and terminating in the opposite sides; by joining the extremities of each set of lines so drawn, two other triangles will be formed. The three lines joining the intersections of corresponding sides of these two triangles with the corresponding angle of the original triangle meet in a point.

If the co-ordinates of the two assumed points be $a_1\, b_1\, c_1$ and $a_2\, b_2\, c_2$ those of the third point are

$$a_1 a_2 \begin{vmatrix} b_1, b_2 \\ c_1, c_2 \end{vmatrix}, \quad b_1 b_2 \begin{vmatrix} c_1, c_2 \\ a_1, a_2 \end{vmatrix}, \quad c_1 c_2 \begin{vmatrix} a_1, a_2 \\ b_1, b_2 \end{vmatrix}.$$

Let this point be called the anapole of the two assumed points.

THEOREM LXVII.

The anapoles of A_7, A_8; of B_7, B_8; and of C_7, C_8 are in a straight line,—say $\mathbb{A}_1\, \mathbb{B}_1\, \mathbb{C}_1$.

THEOREM LXVIII.

The straight line joining the anapoles of P_1, A_8 and P_2, A_7 passes through A; cutting BC (say) in $\mathbb{A}_2$.
" " " P_1, B_8 " P_2, B_7 " B; " CA " $\mathbb{B}_2$.
" " " P_1, C_8 " P_2, C_7 " C; " AB " $\mathbb{C}_2$.

$\mathbb{A}_2\, \mathbb{B}_2\, \mathbb{C}_2$ is a straight line, identical with $\mathfrak{A}_5 \mathfrak{B}_5 \mathfrak{C}_5$ of THEOREM XII.

THEOREM LXIX.

The lines $\mathbb{A}_1\, \mathbb{B}_1\, \mathbb{C}_1$, $\mathbb{A}_2\, \mathbb{B}_2\, \mathbb{C}_2$ intersect in the anapole of P_1, P_2, which is also the pole of the line $P_1 P_2$ to the imaginary conic, $\alpha^2 + \beta^2 + \gamma^2 = 0$. So that the anapole of inverse points is the pole of the line joining them—to the same imaginary conic.

THEOREM LXX.

If one of two points remain fixed and the other move in any manner whatever, the locus of the anapoles is a straight line.

THEOREM LXXI.

If two points move away from each other along a straight line with uniform velocities, their anapole will describe a conic section; and if the uniform velocities be equal, a straight line.

THEOREM LXXII.

The anapole of any two points in a conic section passing through the angles of the triangle of reference is invariable, and its co-ordinates are proportional to the sides of the triangle.

VI.—*Note on Confocal Conic Sections.* By H. F. Talbot, Esq.

(Read 17th April 1865.)

A short paper of mine on Fagnani's theorem, and on Confocal Conic Sections, was inserted in the twenty-third volume of the Transactions of the Royal Society. Some of the conclusions of that paper can, however, be obtained more simply, as I will now proceed to show.

I will, in the first place, resume the problem—

" To find the intersection of a confocal ellipse and hyperbola."

Since the curves have the same foci, and therefore the same centre, let the distance between the centre and focus be called *unity*, since it is the same for both curves. Let a, b, be the axes of the ellipse, A, B, those of the hyperbola Then we have $1 = a^2 - b^2 = A^2 + B^2$, which equation expresses the condition of confocality.

The equation of the ellipse will be $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, and that of the hyperbola $\frac{x^2}{A^2} - \frac{y^2}{B^2} = 1$. But at the point of intersection x and y are the same for both curves. We have therefore two equations from which to determine two unknown quantities. The result is one of unexpected simplicity. (See Vol. XXIII. p. 295.)

$$x = Aa\,, \qquad y = Bb\,.$$

The theory of the Conic Sections has been so much studied, that I can scarcely suppose that a result of such extreme simplicity, and so fruitful in remarkable results, should not have occurred to some previous mathematician. I have no had the opportunity of late of consulting many treatises on the Conic Sections, but in those which I have examined I have not found this theorem.

I will not here repeat the proof which I gave of it in my former paper, since it suffices to show that these values of x and y satisfy both the given equations. In fact, if we put $x = Aa$ and $y = Bb$, the equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ becomes $A^2 + B^2 = 1$, which is true, and the equation $\frac{x^2}{A^2} - \frac{y^2}{B^2} = 1$ becomes $a^2 - b^2 = 1$, which is also true.

This fundamental theorem being thus established, I shall proceed to show how easily the theorem of page 296 follows from it, viz., "If two ellipses and two hyperbolas have all of them the same centre and foci, and therefore intersect in four points, forming a curvilinear quadrilateral, the diagonals of this quadrilateral are equal."

There will, of course, be a similar quadrilateral on the other side of the axis major.

In proof of this theorem, it is sufficient to calculate the value of *one diagonal*, for since that is found to be a *symmetrical function* of the four greater axes of the given curves, the second diagonal has necessarily the same value.

This may be shown thus (Vol. XXIII., fig. 15, p. 296). Adopting the former notation, the square of one of the diagonals, or $D^2 = (x - x_{,})^2 + (y - y_{,})^2$

where

$$x = Aa \qquad y = Bb$$

$$x_{,} = A_{,}a_{,} \qquad y_{,} = B_{,}b_{,}$$

$$\therefore\ D^2 = (x^2 + y^2) + (x_{,}^2 + y_{,}^2) - 2xx_{,} - 2yy_{,}$$

But $xx_{,} = AA_{,}aa_{,}$, which is a symmetrical quantity, being the product of the four major axes:—and $yy_{,} = BB_{,}bb_{,}$ is a symmetrical also, being the product of the four minor axes.

Therefore it remains to show that $(x^2 + y^2) + (x_{,}^2 + y_{,}^2)$ is a symmetrical quantity.

Now

$$x^2 + y^2 = A^2a^2 + B^2b^2$$

but

$$B^2b^2 = (1 - A^2)(a^2 - 1) = -1 + (A^2 + a^2) - A^2a^2$$

$$\therefore\ x^2 + y^2 = (A^2 + a^2) - 1.$$

And similarly

$$x_{,}^2 + y_{,}^2 = (A_{,}^2 + a_{,}^2) - 1$$

$$\therefore\ (x^2 + y^2) + (x_{,}^2 + y_{,}^2) = (A^2 + A_{,}^2 + a^2 + a_{,}^2) - 2$$

which being a symmetrical quantity, the truth of the theorem in question is demonstrated.

From this theorem many others may be deduced; some of which I have given in my first memoir. The following elegant theorem was communicated to me by Charles H. Talbot, Esq.

"If the direction of one of the diagonals passes through the focus, that of the other diagonal passes through the other focus."

Demonstration.—First take the general case in which neither diagonal passes through a focus (see fig. 1). Let the diagonals be PP′, QQ′; join HP, HP′ and SQ, SQ′;—then I say that HP′−HP=SQ′−SQ.

For, by a theorem in my former paper (p. 295), if two confocals intersect, the *focal distance* of their intersection equals the distance between their vertices.

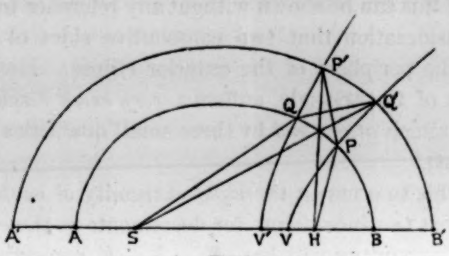

Fig. 1.

Thus, if AP be the ellipse, and VP the hyperbola (fig. 2), AB the major axis,

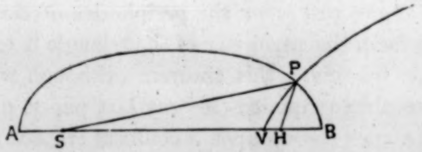

Fig. 2.

S, H, the foci; SP will be equal to AV, and HP to BV.

Therefore in fig. 1 we have

$$HP'=V'B', \quad HP=VB, \quad SQ'=A'V, \quad SQ=AV'$$

$$\therefore\ HP'-HP=V'B'-VB=V'V+BB'$$

$$\text{and}\ \ SQ'-SQ=A'V-AV'=V'V+AA'$$

$$\therefore\ (\text{since } AA'=BB')\ HP'-HP=SQ'-SQ.$$

This, of itself, is a curious theorem. The other follows immediately from it. For, in the particular case, where HPP' is a straight line, $HP'-HP$ is the diagonal PP', which is always equal to the diagonal QQ'.

Therefore, in this case, $SQ'-SQ=QQ'$, and therefore SQQ' is a straight line, which was to be proved.

Another theorem which I have found concerning these quadrilaterals is the following.

"If one of the diagonals is a tangent to the inner ellipse, the other diagonal is so likewise."

I omit, for the present, the demonstration of this, which is not difficult.

I deduced from Graves's theorem in my former paper the remarkable consequence, that if a triangle or other polygon is inscribed to the one, and circumscribed to the other, of two confocal ellipses, its perimeter is constant, at whatever point of the exterior ellipse it is supposed to commence.

But the truth of this can be shown without any reference to GRAVES's theorem, from the simple consideration that two consecutive sides of the triangle make equal angles with the periphery of the exterior ellipse. Hence if the point of departure, or vertex of the triangle, suffers a *very small* displacement, the three sides increase or diminish *at one end* by three small quantities δ, δ', δ'' (generally speaking all different).

Let us suppose this to occur at the *right* extremity of each of the three lines, then it is evident that the increments (or decrements as the case may be) which occur at their *left* extremities will be $-\delta''$, $-\delta$, $-\delta'$ respectively (because each side *gains* at one end what the *following* side *loses* there). Therefore the total increase of the perimeter $= (\delta-\delta'') + (\delta'-\delta) + (\delta''-\delta') = 0$. A much more general theorem can be proved in the same way. "If a triangle cirumscribes an ellipse, and its three angles rest upon the peripheries of three other ellipses (all four having the same foci), the perimeter of the triangle is constant."

I find that CHASLES has given this theorem (although without proof) in his memoir, which I have already quoted (see my last paper, p. 287). The same is true of polygons of n angles resting upon n confocal ellipses.

I will conclude this short note by giving a curious property of the circle, communicated to me by C. H. TALBOT, Esq.

"If three concentric circles (fig. 3) are described from any centre S, with

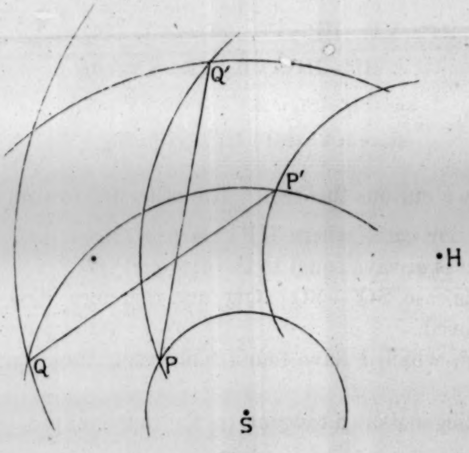

Fig. 3.

radii m, $m+h$, $m+2h$. And if three other concentric circles intersecting them are described from any other centre H, with radii n, $n+h$, $n+2h$ [m, n, h having any values]; then the chord of PQ′, the middle arc of one series, equals the chord of P′Q, the middle arc of the other series."[*]

Demonstration.—In each series of circles the radii have a common difference h, which may be called the *interval* between them. P and P′ are two points in the same ellipse of which S, H, are the foci, because in passing from P to P′, SP increases by one *interval* h, and HP diminishes by the same, therefore SP + HP remains constant.

By similar reasoning Q and Q′ are two points in a *second* ellipse having same foci. Moreover P and Q are two points in a hyperbola of which S, H, are foci; because in passing from P to Q, both HP and SP increase by one interval h, and therefore HP − SP remains constant, and equal to HQ − SQ.

By similar reasoning P′ and Q′ are two points in a *second* hyperbola having same foci. Therefore P, P′, Q, Q′ are the intersections of two ellipses and two hyperbolas, all confocal. Therefore the diagonals PQ′, P′Q are equal to each other.—*Q.E.D.*

This property of the circle should be readily demonstrable by EUCLID's Elements; a simple geometrical demonstration is, however, at present a desideratum.

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[*] The second or middle circle of one series must be understood to be limited by the first and third circles of the other.

VII.—*On the Motion of a Heavy Body along the Circumference of a Circle.* By EDWARD SANG, Esq.

(Read 20th March 1865.)

In the year 1861 I laid before the Royal Society of Edinburgh a theorem concerning the time of descent in a circular arc, by help of which that time can be computed with great ease and rapidity. A concise statement of it is printed in the fourth volume of the Society's Proceedings at page 419.

The theorem in question was arrived at by the comparison of two formulæ, the one being the common series and the other an expression given in the "Edinburgh Philosophical Magazine" for November 1828, by a writer under the signature J. W. L. Each of these series is reached by a long train of transformations, developments, and integrations, which require great familiarity with the most advanced branches of the infinitesimal calculus; yet the theorem which results from their comparison has an aspect of extreme simplicity, and seems as if surely it might be attained to by a much shorter and less rugged road. For that reason I did not, at the time, give an account of the manner in which it was arrived at, intending to seek out a better proof. On comparing it with what is known in the theory of elliptic functions, its resemblance to the beautiful theorem of HALLE became obvious; but then the coefficients in HALLE's formulæ are necessarily less than unit, whereas for this theorem they are required to be greater than unit.

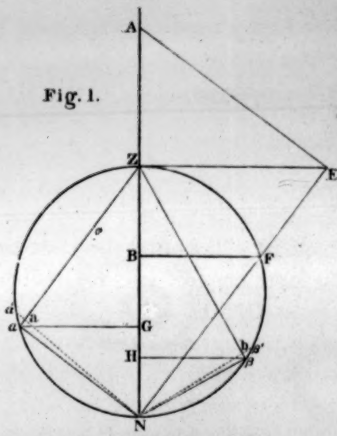
The search after the mutual relations of the two theorems has led me to the discovery of a few simple propositions which involve only the very first principles of the calculus, and the well-known law that the square of the velocity which a heavy body acquires in descending along a curve is proportional to the vertical distance, and to the intensity of gravitation jointly; and which, yet, contain the whole theory of motion in a circle whether that motion be oscillatory or continuous. I am thus enabled to present this hitherto intricate theory in a form which renders it intelligible to junior students of mechanical science.

By a well-known method of extension, the doctrine of the motion of a heavy physical point along the circumference of a circle can be made to include that of the rotation of any mass of matter on an axis not passing through its centre of gravity, whether that axis be horizontal or be inclined; hence, in the following investigation, I may confine my attention to the motion of a physical point in the circumference of a circle placed vertically.

2. Let N be the nadir and Z the zenith point of a circle, along the circumference of which a minute heavy body is free to move. If that body be projected

from N with some given velocity V, it will ascend along the circumference, losing velocity as it rises. When the initial velocity exceeds that which is due to a descent along the diameter ZN, the body will rise to the zenith-point Z, and will proceed onwards to descend along the other semicircumference; after that it will continue (all resistance being supposed away) to repeat revolution after revolution. But when the initial velocity is less than that which is due to a descent along ZN, the heavy body will have lost the whole of its velocity at some point below Z, from that point it will descend again to N, pass to the other side, on which it will reach to the same height, and thence descending it will continue to oscillate as in the familiar example of a pendulum. There are thus two distinct cases of circular motion, viz., the *continuous* and the *oscillatory*.

Fig. 1.



3. These two cases may be connected in the following manner :—

Let us suppose that a heavy body α, has been projected at N, with a velocity due to its descent from some point A beyond Z, and that it has now reached to the point marked α. Having drawn the horizontal line αG, we see that its velocity at the point α is that which is due to a fall through the distance AG; so that if we put V_A for the initial velocity, and v_α for the velocity at the point α, we must have the proportion

$$V_A^2 : v_\alpha^2 :: NA : AG :: NZ . NA : NZ . AG$$
$$:: NZ . NA : NZ . NA - NZ . NG .$$

Through Z draw the horizontal line ZE, and make it a mean proportional between NZ and ZA; join NE, EA, then the trigons NZE, NEA are similar, so that NE is a mean proportional between NZ and NA, wherefore the above analogy may be written,—

$$V_A^2 : v_\alpha^2 :: NE^2 : NE^2 - N\alpha^2 .$$

4. F being the intersection of NE with the circumference of the circle, draw FB horizontally, then the five lines NA, NE, NZ, NF, and NB, are in continued proportion; so that NA : NZ : : NZ : NB.

If a second body β be projected from N, with a velocity due to a descent from B, it will rise along the curve only to the point F, its velocity there being entirely exhausted. The greatest distance, then, which β can reach from N, viz., NF, is to the greatest distance to which α can attain, viz., NZ, in the ratio of NE to NA. Let us take an intermediate point β to correspond with α, by making $N\alpha : N\beta$

$:: \mathrm{NA} : \mathrm{NE} :: \mathrm{NE} : \mathrm{NZ} ::$ &c.; and seek the ratio of the two velocities, viz., of α at the point α, and of β at the point β.

5. Putting V_B for the initial velocity of β, and v_β for its velocity at the point β we have,—

$$\begin{aligned} V_B^2 : v_\beta^2 &:: \mathrm{BN} : \mathrm{BH} :: \mathrm{BN} : \mathrm{BN} - \mathrm{NH} \\ &:: \mathrm{NZ} . \mathrm{BN} : \mathrm{NZ} . \mathrm{BN} - \mathrm{NZ} . \mathrm{NH} \\ &:: \mathrm{NF}^2 \quad : \mathrm{NF}^2 - \mathrm{N}\beta^2 . \end{aligned}$$

But we have also

$$V_A^2 : V^2_B :: \mathrm{NE}^2 : \mathrm{NF}^2 ,$$

wherefore

$$v_\alpha^2 : v_\beta^2 :: \mathrm{NE}^2 - \mathrm{N}\alpha^2 : \mathrm{NF}^2 - \mathrm{NB}^2$$

Now, from our construction,

$$\mathrm{NE}^2 : \mathrm{NZ}^2 :: \mathrm{N}\alpha^2 : \mathrm{N}\beta^2$$

wherefore

$$\mathrm{NE}^2 : \mathrm{NZ}^2 :: \mathrm{NE}^2 - \mathrm{N}\alpha^2 : \mathrm{NZ}^2 - \mathrm{N}\beta^2$$

but

$$\mathrm{NZ}^2 - \mathrm{N}\beta^2 = \mathrm{Z}\beta^2 ,$$

wherefore

$$\mathrm{NE}^2 - \mathrm{N}\alpha^2 : \mathrm{NE}^2 :: \mathrm{Z}\beta^2 : \mathrm{NZ}^2$$

and similarly

$$\mathrm{NE}^2 : \mathrm{NA}^2 :: \mathrm{NF}^2 - \mathrm{N}\beta^2 : \mathrm{NZ}^2 - \mathrm{N}\alpha^2 ;$$

or,

$$\mathrm{NE}^2 : \mathrm{NF}^2 - \mathrm{N}\beta^2 :: \mathrm{NA}^2 : \mathrm{Z}\alpha^2 .$$

Compounding these ratios we obtain,—
whence

$$\begin{aligned} \mathrm{NE}^2 - \mathrm{N}\alpha^2 : \mathrm{NF}^2 - \mathrm{N}\beta^2 &:: \mathrm{NA}^2 . \mathrm{Z}\beta^2 : \mathrm{NZ}^2 . \mathrm{Z}\alpha^2 , \\ v_\alpha : v_\beta &:: \mathrm{NA} . \mathrm{Z}\beta : \mathrm{NZ} . \mathrm{Z}\alpha . \end{aligned}$$

6. Let us now suppose that the body α moves through an exceedingly minute distance, represented by $\alpha\alpha'$, and let us make the proximate chord $\mathrm{N}\beta'$, in the same ratio to $\mathrm{N}\alpha'$ as before; then, since $\mathrm{N}\alpha$ may be held equal to $\mathrm{N}\alpha$ and $\mathrm{N}b$ to $\mathrm{N}\beta$, we have $\alpha\alpha' : b\beta :: \mathrm{N}\alpha : \mathrm{N}\beta$.

The minute trigons $a\alpha\alpha'$ and $b\beta\beta'$ are similar, respectively, to $\alpha\mathrm{NZ}$ and $\beta\mathrm{NZ}$, wherefore

$$\begin{aligned} \alpha\alpha' : a\alpha' &:: \mathrm{NZ} : \mathrm{Z}\alpha \\ b\beta' : \beta\beta' &:: \mathrm{Z}\beta \ : \mathrm{NZ} . \end{aligned}$$

By compounding these three ratios we obtain

$$\begin{aligned} \alpha\alpha' : \beta\beta' &:: \mathrm{N}\alpha . \mathrm{Z}\beta : \mathrm{N}\beta . \mathrm{Z}\alpha \\ &:: \mathrm{NZ} . \mathrm{Z}\beta : \mathrm{NF} . \mathrm{Z}\alpha . \end{aligned}$$

7. On comparing the lengths of the arcs $\alpha\alpha'$ and $\beta\beta'$, and also the velocities with which they are passed over, we find that the minute intervals of time are in the ratio

$$\text{time in } \alpha\alpha' : \text{time in } \beta\beta' :: \frac{\mathrm{NZ}}{\mathrm{NA}} : \frac{\mathrm{NF}}{\mathrm{NZ}} :: \mathrm{NZ} : \mathrm{NE} .$$

Now, if we suppose that the semicircumference $N\alpha Z$ is divided into a multitude of minute portions, of which $\alpha\alpha$ may be taken as one; and if we divide the arc $N\beta F$ into as many corresponding portions by making the chords $N\beta$ always to the chords $N\alpha$ in the constant ratio NZ to NE; the time of describing each element $\alpha\alpha'$ of the semicircumference NZ is to that of describing the corresponding element $\beta\beta'$ of the arc NF in the constant ratio NZ to NE; and, consequently, the time of describing any portion as $N\alpha$ must be to that of the describing the corresponding portion $N\beta$ in the same ratio; and so also must be the periodic times of the two motions.

Hence, if we can discover the law of the motion in the arc NF, we shall be able thence to deduce the law of the continuous motion due to the velocity obtained by descent from the point A; and contrariwise.

For the sake of convenience, we shall call these two motions *conjugate* to each other.

8. It will conduce greatly to the clearness of our subsequent investigations to introduce here another consideration. The time of describing the arc NF is greater than that of describing the conjugate arc NZ in the ratio of NZ to NF; the oscillatory motion will thus fall behind the continuous motion. Now, if we were to suppose that the body β is acted on by gravitation of an intensity greater than that which acts on α in the ratio duplicate of the ratio of the actual periodic times, the two motions would be rendered alike.

We shall then suppose that the gravitation acting on α, which we may designate by G_α is proportional to NZ, while the intensity of the gravitation acting on β, appropriately denoted by G_β, is proportional to NA. And, as we have to do with the subduplicate of the ratio of these intensities, we shall, for the sake of additional convenience, put

$$G_\alpha = NZ\ .\ NZ; \qquad G_\beta = NZ\ .\ NA\ .$$

9. The height from which a body has fallen being denoted by h, and the intensity of gravitation being G, the velocity acquired is, according to the well-known law of motion, proportional to $\sqrt{(Gh)}$; we shall, therefore, put the general formula for that velocity thus:—

$$v = \sqrt{(G\ .\ NZ\ .\ h)}\ .$$

10. On inserting the above value of G_α in this general formula, and at the same time making $h = AG = NA - NG$ we have

$$v_\alpha = \sqrt{\{NZ\ .\ NZ\ .\ NZ\,(NA - NG)\}} = NZ\ .\ \sqrt{(NE^2 - N\alpha^2)}\ .$$

And similarly for the body β

$$v_\beta = \sqrt{\{NZ\ .\ NA\ .\ NZ\,(NB - NH)\}} = NE\ .\ \sqrt{(NF^2 - N\beta^2)}\ .$$

But since

$$\mathrm{NE} : \mathrm{NZ} :: \mathrm{NZ} : \mathrm{NF} :: \mathrm{N}\alpha : \mathrm{N}\beta$$

$$\mathrm{NZ} \,.\, \sqrt{(\mathrm{NE}^2 - \mathrm{N}\alpha^2)} = \mathrm{NE} \,.\, \sqrt{(\mathrm{NZ}^2 - \mathrm{N}\beta^2)} = \mathrm{NE} \,.\, \mathrm{Z}\beta\,,$$

and

$$\mathrm{NE} \,.\, \sqrt{(\mathrm{NF}^2 - \mathrm{N}\beta^2)} = \mathrm{NZ} \,.\, \sqrt{(\mathrm{NZ}^2 - \mathrm{N}\alpha^2)} = \mathrm{NZ} \,.\, \mathrm{Z}\alpha$$

wherefore

$$v_\alpha = \mathrm{NE} \,.\, \mathrm{Z}\beta\,; \qquad v_\beta = \mathrm{NZ} \,.\, \mathrm{Z}\alpha\,.$$

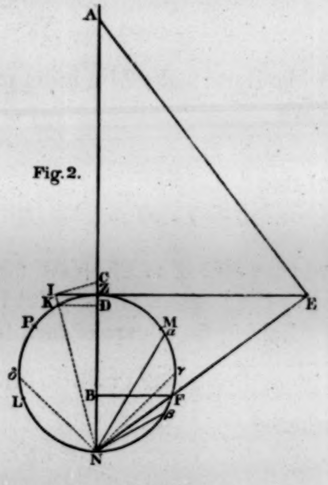
Hence, under this supposition of two distinct intensities of gravitation, we have

$$v_\alpha : v_\beta :: \mathrm{NE} \,.\, \mathrm{Z}\beta : \mathrm{NZ} \,.\, \mathrm{Z}\alpha\,,$$

but we have shown in Article 6 that the minute distance $\alpha\alpha'$ is to the corresponding distance $\beta\beta'$ in the very same ratio, wherefore the time in which the body α describes the distance $\alpha\alpha'$ is now equal to that in which β describes the distance $\beta\beta'$. And consequently if α and β start at the same instant from N, they will reach the points α and β simultaneously; and just when α has reached the highest point Z, β will have reached its highest point F; so that the periodic times of the two conjugate motions have been made alike.

11. In the figure 1 hitherto referred to, the points α and β have been placed on opposite sides of the diameter NZ for the sake of perspicuity. We shall now, in figure 2, suppose that they are both projected in the same direction from N and at the same instant, so that when α has reached the point α, β has reached β. Proceeding onwards, when α comes to Z, β arrives at F, the velocity of α being then that which is due to a fall from A to Z, and the velocity of β being zero. Subsequently, while α returns to N along the other semicircumference, β returns to N by retracing its previous path FN. In this way both bodies arrive at N at the same instant, but moving in opposite directions. While α, for the second time, describes the entire circumference of the circle, β ascends to L and thence returns to N at the same instant that α reaches that point. The two bodies are now moving in the same direction as at first, and these phases, all resistance being set aside, are again and again reproduced.

Fig. 2.



12. Let us now imagine that the arc $\alpha\beta$ is continually bisected in γ, and let us trace the motion of this middle point.

When α has reached Z and β has come to F, the point γ must be at M the middle of the arc FZ: when α has passed Z and β is descending from F towards

N, the point γ must be approaching to Z, and it must reach Z just when α and β have met at N. Thus the time in which γ describes the arc MZ is just equal to that in which it passes from N to M. By the time that α has reached Z again, β has reached L the extreme point of its motion on the other side, and therefore γ is at P the middle of ZL; lastly, when α has once more descended along ZPLN to N, β has descended along LN to the same point, and so γ also has come to N.

It thus appears that while the point α makes two complete revolutions, the point γ makes only one. In the progress of this revolution the velocity of γ varies; when at N it is half the sum of the initial velocities V_A and V_B, and when at Z it is reduced to be their difference; so that the motion of γ has the general characteristic of one due to the action of gravity upon a heavy body.

13. If the motion of γ can be truly represented by the action of gravitation upon a heavy body, we may determine the point from which γ may be supposed to have descended, and the intensity of the gravitation which must act upon it, by comparing the velocities at the lowest and highest points of its path. Let C be the point from which γ must have descended in order to acquire at N the velocity $\frac{1}{2}(V_A+V_B)$, or at Z the velocity $\frac{1}{2}(V_A-V_B)$, and put G_γ for the intensity of the gravitation to which it is subjected; then

$$\surd(G_\gamma \,.\, NZ \,.\, CN) = \tfrac{1}{2}(V_A+V_B); \quad \surd(G_\gamma \,.\, NZ \,.\, CZ) = \tfrac{1}{2}(V_A-V_B).$$

Now according to Article 10,

$$V_A = NE \,.\, NZ; \; V_B = NE \,.\, NF$$

while if we make ZI a mean proportional between CZ and ZN, and join NI, we have

$$NZ \,.\, CN = NI^2, \qquad NZ \,.\, CZ = IZ^2,$$

so that

$$\surd(G_\gamma) \,.\, NI = \tfrac{1}{2} NE\,(NZ+NF); \qquad \surd(G_\gamma) \,.\, IZ = \tfrac{1}{2} NE\,(NZ-NF),$$

whence $NI : IZ :: NZ + NF : NZ - NF$; a proportion which enables us to determine the position of the point I, and, consequently, that of C.

Taking the square of each term of that proportion we have

$$NI^2 : IZ^2 :: NZ^2 + 2NZ \,.\, NF + NF^2 : NZ^2 - 2NZ \,.\, NF + NF^2$$

whence

$$NI^2 : NZ^2 :: (NZ+NF)^2 : 4NZ \,.\, NF$$

and consequently

$$NI^2 = \frac{NZ^2\,(NZ+NF)^2}{4NZ \,.\, NF}; \; NI = NZ\,\frac{NZ+NF}{2\surd(NZ \,.\, NF)} = NZ\,\frac{NE+NZ}{2\surd(NE \,.\, NZ)},$$

whence also

$$\mathrm{NC} = \frac{(\mathrm{NZ} + \mathrm{NF})^2}{4\mathrm{NF}}; \; 4\mathrm{NF} \,.\, \mathrm{NC} = (\mathrm{NZ} + \mathrm{NF})^2 .$$

14. Having thus determined the position of the point C, we can determine also the intensity of the gravitation for, putting for NI the value just found,

$$\sqrt{G_\gamma} \,.\, \mathrm{NZ} \frac{\mathrm{NF} + \mathrm{NZ}}{2\sqrt{(\mathrm{NZ} \,.\, \mathrm{NF})}} = \tfrac{1}{2}\, \mathrm{NE}\, (\mathrm{NZ} + \mathrm{NF})$$

whence $G_\gamma = \mathrm{NZ} \,.\, \mathrm{NE}$; so that the intensity of the gravitation for γ must be a mean proportional between those for α and β.

15. A descent from C under the influence of gravitation of the intensity NZ . ZE would cause a heavy body to have at Z and at N the very velocities which the moveable point γ has at those places; and we have now to inquire whether the same influence would give to that body, when at any intermediate point, the corresponding velocity. Before treating of this matter generally, it may be instructive to inquire into the velocity of the point γ when it is at M the middle of FZ; the moveable point γ is at M when α is at Z and β at F, now at that instant the velocity of β is zero, while the velocity of α, proportional to the square root of NZ . ZA, is represented by NZ . ZE, so that the velocity of γ must then be $\tfrac{1}{2}$ NZ . ZE.

But the velocity of the heavy body when at M is given by the general formula,

$$\sqrt{(G_\gamma \,.\, \mathrm{NZ} \,.\, \mathrm{QC})} = \sqrt{\{G_\gamma \,.\, \mathrm{NZ}\, (\mathrm{NC} - \mathrm{NQ})\}} .$$

Substituting for G_γ the value above found we have

$$v_M^{\,2} = \mathrm{NZ} \,.\, \mathrm{NE}\, (\mathrm{NI}^2 - \mathrm{NM}^2)$$

but since M is the middle of the arc FZ

$$2\mathrm{NZ} : \mathrm{NZ} + \mathrm{NF} \text{ or } 2\mathrm{NE} : \mathrm{NE} + \mathrm{NZ} :: \mathrm{NZ}^2 : \mathrm{NM}^2$$

wherefore

$$\mathrm{NM}^2 = \mathrm{NZ}^2 \frac{\mathrm{NE} + \mathrm{NZ}}{2\mathrm{NE}}; \text{ but } \mathrm{NI}^2 = \mathrm{NZ}^2 \frac{(\mathrm{NE} + \mathrm{NZ})^2}{4\mathrm{NE} \,.\, \mathrm{NZ}}$$

wherefore

$$\mathrm{NI}^2 - \mathrm{NM}^2 = \mathrm{NZ}^2 \left\{ \frac{\mathrm{NE}^2 + 2\mathrm{NE} \,.\, \mathrm{NZ} + \mathrm{NZ}^2}{4\mathrm{NE} \,.\, \mathrm{NZ}} - \frac{2\mathrm{NE} \,.\, \mathrm{NZ} + 2\mathrm{NZ}^2}{4\mathrm{NE} \,.\, \mathrm{NZ}} \right\}$$

$$= \mathrm{NZ}^2 \frac{\mathrm{NE}^2 - \mathrm{NZ}^2}{4\mathrm{NE} \,.\, \mathrm{NZ}} = \mathrm{NZ}^2 \frac{\mathrm{ZE}^2}{4\mathrm{NE} \,.\, \mathrm{NZ}}$$

and consequently

$$v_M^{\,2} = \tfrac{1}{4} \,.\, \mathrm{NZ}^2 \,.\, \mathrm{ZE}$$

or

$$v_M = \tfrac{1}{2}\, \mathrm{NZ} \,.\, \mathrm{ZE}$$

and thus the velocity at M due to a descent from the level of C is exactly that which the moveable point γ has at the same place.

16. We may now examine the velocity which this same heavy body would have at any intermediate point as γ. The general expression for that velocity is

$$v_\gamma = \surd\{G_\gamma(NI^2 - N\gamma^2)\} = \surd\{NZ\ .\ NE\,(NI^2 - N\gamma^2)\}$$

but since γ is the middle of the arc $\alpha\,\beta$,

$$N\gamma^2 = \tfrac{1}{2}\{NZ^2 + N\alpha\,.\,N\beta - Z\alpha\,.\,Z\beta\}$$

subtracting this from the value of NI^2 and simplifying

$$NI^2 - N\gamma^2 = \frac{NE^2\,.\,NZ^2 + NZ^2\,.\,NZ^2 + 2NE\,.\,NZ\,.\,Z\alpha\,.\,Z\beta - 2NE\,.\,NZ\,.\,N\alpha\,.\,N\beta}{4NE\,.\,NZ}.$$

Now $NE\,.\,N\alpha = NZ\,.\,N\beta$ so that the continued product $NE\,.\,NZ\,.\,N\alpha\,.\,N\beta$ may be written either $NE^2.\,N\alpha^2$ or $NZ^2.\,N\beta^2$; writing it once each way we obtain

$$NI^2 - N\gamma^2 = \frac{NE^2\,.\,Z\alpha^2 + 2NE\,.\,NZ\,.\,Z\alpha\,.\,Z\beta + NZ^2\,.\,Z\beta^2}{4NE\,.\,NZ}$$

$$= \frac{(NE\,.\,Z\alpha + NZ\,.\,Z\beta)^2}{4NE\,.\,NZ}$$

wherefore

$$v_\gamma = \surd\left\{NE\,.\,NZ\,\frac{(NE\,.\,Z\alpha + NZ\,.\,Z\beta)^2}{4NE\,.\,NZ}\right\} = \tfrac{1}{2}(NE\,.\,Z\alpha + NZ\,.\,Z\beta);$$

but we have seen that $NE\,.\,Z\alpha$ is the velocity of β at the point β, $NZ\,.\,Z\beta$ that of α at the point α, so that

$$v_\gamma = \tfrac{1}{2}\{v_\alpha + v_\beta\}$$

and thus, at every point of the circumference, the velocity of a body projected from N with a velocity due to a descent from Z, and acted on by a gravitation having its intensity represented by NZ . NE, is equal to the velocity of the middle of the arc $\alpha\,\beta$.

17. The motion of the body γ round the circumference has for its *conjugate* that of a fourth, which we may name δ ascending from N to K, and thence returning to N, while γ rises from N to Z, and proceeding onwards, returns to N; the conjugation being analogous to that which connects the motions of α and β.

Hence, if we inflect the chord $N\delta$, a fourth proportional to NZ, NK, and $N\gamma$, we shall obtain the point at which the body δ is found when γ is at γ; and if we make G_δ a fourth proportional to NZ, NC, and G_γ, we shall obtain for the intensity of the gravitation to which δ must be subjected

$$NZ : NC :: NZ\,.\,NE : NC\,.\,NE = G_\delta.$$

18. In this way we have obtained two pair of conjugate motions, the periodic time of the second pair being double of that of the first, and the intensities of gravitation being

$$G_\alpha = NZ \,.\, NZ$$
$$G_\beta = NZ \,.\, NA$$
$$G_\gamma = NZ \,.\, NE$$
$$G_\delta = NC \,.\, NE$$

such that any one of the four motions being known the other three may be found.

19. Here it is to be observed, that the periodic time of γ and δ is double that of α and β; in order to bring it to be the same, we must quadruple the intensities of the gravitation acting on these bodies, so that for all the

Periodic Times alike

$$G_\alpha = NZ \,.\, NZ \,, \qquad G_\beta = NZ \,.\, NA$$
$$G_\gamma = 4NZ \,.\, NE \,, \qquad G_\delta = 4NC \,.\, NE \,.$$

20. And if the intensities of gravitation be supposed the same for all the four bodies, their periodic times will then be proportional to the square roots of the preceding intensities: so that if we put T_α, T_β, T_γ, T_δ for the periodic times on the supposition of one gravitation, we have

Gravitations alike

$$T_\alpha = NZ \;; \qquad T_\beta = NE$$
$$T_\gamma = 2\sqrt{(NZ \,.\, NE)} \;; \qquad T_\delta = 2\sqrt{(NC \,.\, NE)} \,.$$

21. These two pairs of conjugate motions are so connected, that from one of them the other can be found, the law of connection being contained in the proportion

$$NI : IZ :: NE + NZ :: NE - NZ$$

or in

$$NI + IZ : NI - IZ :: NE : NZ \,.$$

If then from the pair α, β, we deduce the pair γ, δ; we may again from this latter deduce another pair of conjugate motions which we may mark γ_1, δ_1; and from this again another pair γ_2, δ_2, and so on without end. Or if we regard γ, δ, as the original pair, and deduce α, β, from it, we may thence deduce a new pair α_1, β_1, and from that again, another α_2, β_2; and so on, so that we have a progression of conjugate motions extending indefinitely each way, and such that any one of the series being known, all the others can be thence deduced. The latter branch of the progression, viz., that from γ, δ, to α, β, and thence onwards is that which is available in our research.

22. The periodic time of a body descending from C is deducible from that of one which has the velocity belonging to a descent from A; that again is deducible from the motion of a body supposed to have fallen from A_1, and so on. Now, the distances NA, NA_1, NA_2, &c, increase with greater and greater rapidity as we proceed, so that after a few terms NA may become enormously great as compared with NZ. But EZ is proportional to the velocity at Z, while NE is proportional to that at N; and the ratio of NE to EZ must approach nearer and nearer to a ratio of equality as the point A rises; so that when NA is very great, the velocity becomes almost uniform, and the periodic time of the motion becomes the quotient of the circumference by that velocity. In this way, the study of the law of this progression may conduct us to a knowledge of the periodic time of the motion of γ.

23. Analogously the distance NB is deduced from NC, from NB again we may deduce NB_1, thence NB_2, and so on; and in this manner, we may reduce the question of the time of descent in the arc KN, to that of the time of ascent in an excessively minute arc.

24. Attending first to the case of oscillatory motion, let it be proposed to compute the periodic time of a body having its velocity due to a descent from the level of D.

For this purpose, let us put B_0 for the angle NZK measured by half of the extreme arc NK, and B_1 for the angle NZF measured by half of the arc NF; then

$$NK = NZ \,.\, \sin B_0, \qquad KZ = NZ \,.\, \cos B_0 \;;$$

but

$$NZ + ZK : NZ - ZK :: NZ : NF\,,$$

wherefore

$$1+\cos B_0 : 1-\cos B_0 :: (\cos \tfrac{1}{2} B_0)^2 : (\sin \tfrac{1}{2} B_0)^2 :: 1 : \sin B_1$$

and

$$\sin B_1 = (\tan \tfrac{1}{2} B_0)^2 \,.$$

But it has been shown in article 20 that the times of descent from K and from F, there marked by the symbols T_δ and T_β are in the ratio $2\sqrt{(NC \,.\, NE)} : NE$ or of $2\sqrt{NC} : \sqrt{NE}$. It is now convenient to indicate these times by the characters T sin. B_0 and time B_1, so that the above proportion may be written

$$\begin{array}{rlll}
\text{time } B_0 : \text{time } B_1 & :: & \sqrt{\overline{NC}} & : \sqrt{NE} \\
& :: & 2NI & : NI + IZ \\
& :: & 2NZ & : NZ + ZK \\
& :: & 2 & : 1 + \cos B_0 \\
& :: & 1 & : (\cos \tfrac{1}{2} B_0)^2 \\
& :: & (\sec \tfrac{1}{2} B_0)^2 & : 1
\end{array}$$

so that the time of descent from K is

$$\text{time } B_0 = \text{time } B_1 \,.\, (\sec \tfrac{1}{2} B_0)^2 \,.$$

Hence the following very simple construction:—Having made NS the fourth part of the arc NK and drawn a horizontal line through N, join OS and produce it to meet the horizontal tangent at T; draw TU perpendicular to OT, meeting ON produced in U; lastly inflect NF double of NU; then the time of descent along KN is to that of descent from F to N in the ratio of OU to ON.

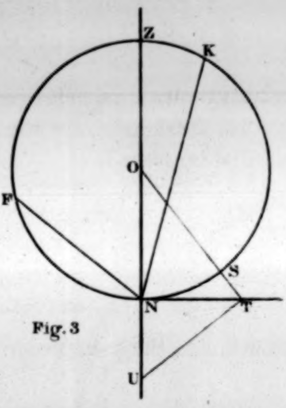


Fig. 3

By repeating this operation in regard to the arc NF, we should obtain a much smaller arc, and thence again one still smaller, and so on; the periodic times in these successive arcs bearing known ratios to each other. Now it is obvious, from a glance at the figure, that these arcs, which we may denote by $2B_0$, $2B_1$, $2B_2$, &c., form a very rapidly decreasing progression; and that the ratios of which ON : OU is the first, approach at the same time to a ratio of equality; hence the time of descent along KN may be deduced from the time of oscillation in an exceedingly minute arc. A very familiar investigation shows that the time of oscillation in a small arc is almost independent of the extent of the arc; but instead of founding on this well-known proposition, I prefer to deduce the truth of it from our present considerations.

25. If we suppose the arc NF of figure 2 to be very minute, the height NA, which is inversely proportional to NZ, must become very great in proportion to the diameter NZ, and hence the velocity at the point Z must be nearly equal to that at N, the two being in the ratio of ZE to EN; and the time in which a body descending from A describes the circumference must always be greater than that in which another moving with a uniform velocity equal to that at N would describe the same circumference. Putting g for the actual intensity of gravitation, the velocity acquired by falling from A to N is

$$V_A = \surd(2g\ .\ AN)$$

so that, as πNZ is the length of the circumference, the value of T_A must be greater than

$$\frac{\pi NZ}{\surd 2g\,.\,AN} = \pi\sqrt{\left(\frac{NZ}{2g}\right)}\ .\ \sqrt{\frac{NZ}{AN}} = \pi\sqrt{\left(\frac{NZ}{2g}\right)}\ .\ \frac{NF}{NZ}\quad .$$

while, since the velocity at Z is to that at N as ZE to NE, the same time must be less than

$$\pi\sqrt{\left(\frac{NZ}{2g}\right)}\ \frac{NF}{NZ}\ .\ \sec NZF.$$

26. Now we have seen that

$$T_\alpha\ :\ T_\beta\ ::\ NF\ :\ NZ$$

wherefore the time of descending from F to N, and thence rising to L on the other side, is between the limits

$$\pi\sqrt{\left(\frac{\mathrm{NZ}}{2g}\right)} \text{ and } \pi\sqrt{\left(\frac{\mathrm{NZ}}{2g}\right)} \cdot \sec \mathrm{NZF},$$

which limits approach closer to each other when the arc NF is made smaller.

27. Resuming now the investigation as in article 24, we find that the time of oscillation from K is

$$\text{time } \mathrm{B}_0 \succ (\sec \tfrac{1}{2}\mathrm{B}_0)^2\, \pi\sqrt{\left(\frac{\mathrm{NZ}}{2g}\right)}$$

$$\prec (\sec \tfrac{1}{2}\mathrm{B}_0)^2\, \pi\sqrt{\left(\frac{\mathrm{NZ}}{2g}\right)} \cdot \sec \mathrm{B}_1.$$

Continuing the same progression another step by making

$$\sin \mathrm{B}_2 = (\tan \tfrac{1}{2}\mathrm{B}_1)^2$$

we find

$$\text{time } \mathrm{B}_0 \succ (\sec \tfrac{1}{2}\mathrm{B}_0)^2\, (\sec \tfrac{1}{2}\mathrm{B}_1)^2\, \pi\sqrt{\left(\frac{\mathrm{NZ}}{2g}\right)}$$

$$\prec (\sec \tfrac{1}{2}\mathrm{B}_0)^2\, (\sec \tfrac{1}{2}\mathrm{B}_1)^2\, \pi\sqrt{\left(\frac{\mathrm{NZ}}{2g}\right)} \cdot \sec \mathrm{B}_2;$$

these limits being now closer to each other, since B_2 is a smaller angle than B_1.

If, then, we continue the progression indefinitely, by making

$$\sin \mathrm{B}_3 = (\tan \tfrac{1}{2}\mathrm{B}_2)^2;\ \sin \mathrm{B}_4 = (\tan \tfrac{1}{2}\mathrm{B}_3)^2;\ \&c.$$

we shall obtain for the entire time of an oscillation in the arc $4\mathrm{B}_0$,

$$\text{time } \mathrm{B}_0 = \pi\sqrt{\left(\frac{\mathrm{NZ}}{2g}\right)} \cdot (\sec \tfrac{1}{2}\mathrm{B}_0)^2 \cdot (\sec \tfrac{1}{2}\mathrm{B}_1)^2 \cdot (\sec \tfrac{1}{2}\mathrm{B}_2)^2 \cdot \&c.$$

28. As an example of the calculation we may require the time of oscillation over an arc of 180°, which is the extreme limit of a pendulum with a flexible thread. In this case $\mathrm{B}_0 = 45°$, whence $\log \tan \tfrac{1}{2}\mathrm{B}_0 = \log \tan 22° \ 30' = 9{\cdot}617\ 2243$; wherefore $\log \sin \mathrm{B}_1 = 9{\cdot}234\ 4486$; $\mathrm{B}_1 = 9°\ 52'\ 45''{\cdot}42$; from this again we have $\log \tan \tfrac{1}{2}\mathrm{B}_1 = \log \tan 4°\ 56'\ 22''{\cdot}71 = 8{\cdot}936\ 6506$; $\log \sin \mathrm{B}_2 = 7{\cdot}873\ 3012$, giving $\mathrm{B}_2 = 0°\ 25'\ 40''{\cdot}74$. And once again, $\log \tan \tfrac{1}{2}\mathrm{B}_2 = \log \tan 0°\ 12'\ 50''{\cdot}37 = 7{\cdot}572\ 2861$; $\log \sin \mathrm{B}_3 = 5{\cdot}144\ 5722$, $\mathrm{B}_3 = 0°\ 0'\ 02''{\cdot}88$. Here we observe that the log-secant of B_3 does not differ from zero by unit in the seventh decimal place; and that, therefore, we have brought our limits so close together that the difference cannot be appreciated by help of the ordinary logarithmic tables. The

logarithm of the ratio of the time of oscillation in a semicircumference to that in a small arc is thus

$$
\begin{array}{lrl}
 & \text{Log sec } \tfrac{1}{2}\,B_0 = & 0{\cdot}034\ 3847 \\
 & \text{Log sec } \tfrac{1}{2}\,B_1 = & 1\ 6160 \\
 & \text{Log sec } \tfrac{1}{2}\,B_2 = & 30 \\
\cline{3-3}
 & & 0{\cdot}036\ 0037 \quad \times\ 2 \\
\cline{3-3}
1{\cdot}180341 & & 0{\cdot}072\ 0074
\end{array}
$$

In other words, the time of oscillation in a semi-circumference is to that for a minute arc as 72 to 61 very nearly.

When the arc is nearly a whole circumference, the steps of the progression are more numerous, and the computation may, with advantage, be systematically arranged. I subjoin the work for an arc of 320°, with logarithms to ten places.

n	$\tfrac{1}{2}B_n$	2 log sec $\tfrac{1}{2}B_n$	2 log tan $\tfrac{1}{2}B_n$	B_{n+1}
0	40° 00′ 00″·000	·23149 20670	9·84762 70684	44° 45′ 21″·339
1	22 22 40·669	6800 53042	9·22920 47378	9 45 34·359
2	4 52 47·180	315 40158	7·86265 75152	25 03·441
3	12 31·721	57684	5·16071 41918	2·986
4	1·493	0		
		·30265 71554=log 2·00750740		

Thus, even in this extreme case, four terms of the progression are sufficient.

These examples show that for all cases in ordinary clock-work, or in experiments on the length of the pendulum, the time of oscillation is sufficiently well represented by the formula

$$\text{time } B = \pi \sqrt{\left(\frac{l}{g}\right)} \cdot (\sec \tfrac{1}{2} B)^2$$

B being one-fourth part of the entire arc of oscillation. Hence the number of beats per day is proportional to the square of the cosine of the eighth part of the arc, and the daily retardation to the square of the sine of the same eighth part.

29. From the periodic time of an oscillation that of the conjugate continuous motion may be readily deduced; and thus, so far as the entire motions are concerned, the theory may be said to be complete. The investigation of the time at which the moving body arrives at any proposed point in its path is carried on by the application of the principle just explained; but as it is of comparatively little interest, and, at the same time, more tedious, I shall not, for the present, go into its details.

VIII.—*Experimental Inquiry into the Laws of the Conduction of Heat in Bars.* Part II. *On the Conductivity of Wrought Iron, deduced from the Experiments of* 1851. By James D. Forbes, D.C.L., LL.D., F.R.S., V.P.R.S. Ed., Principal of St Salvator and St Leonard's College, St Andrews, and Corresp. Member of the Institute of France. (Plates I., II., III., IV., and V.)

(Read 20th February 1865.)

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INTRODUCTION.

The Articles are numbered in continuation of those in the First Part of this Paper.

39. In the first part of this paper, read to the Royal Society of Edinburgh in April 1862, and published in their Transactions,† I explained the principles of a method devised by me in 1850 for ascertaining the absolute conducting power of substances capable of being formed into long bars; and I also stated the general results of experiments made in 1851 on the Conductivity for heat of wrought Iron.

40. I explained in Art. 14 of that paper, that the publication of the results had been for ten years withheld, partly in consequence of the state of my health which completely interrupted the experiments, but still more from the defective graduation of some of the thermometers used, which made it necessary to submit the instruments to a careful scrutiny, and to repeat with the duly corrected numbers the whole of the elaborate projections of the curves and calculations from them, on which the accuracy of the final results of course depends.

41. I stated that the friendly aid and exemplary patience of the late Mr Welsh of the Kew Observatory had supplied me with data for correcting the readings of the most important, and at the same time the most inaccurately graduated of the series of French thermometers employed in these experiments.

* Sections III. and VII. have been added to this paper since it was read.

† Vol. XXIII. p. 133.

Without his help the present corrected reduction of the observations could never have been made; and even with the aid of the tables kindly prepared by him, it has been a work of no small labour and anxiety to bring to one strictly accordant scale the whole of the observations made with eight or ten thermometers, none of them deserving of being called standards, and in most of which the zero appears to have oscillated at different periods.

42. I have thought it unnecessary, as it would certainly have been most tedious, to print in this paper the crude observations and the numerous tables of reduction formed for the scales of the several thermometers. I have thought it sufficient to give the corrected results, which, in many cases, are the mean of independent readings of different thermometers.

43. Besides the correction of scale errors, an important correction required to be applied in order to reduce the readings to what they would have been had the column of the mercury in the thermometer partaken of the temperature of the bulb. Owing to the small transverse dimensions of the bars, whose temperatures were to be ascertained, the bulb of the thermometers was often little more than covered by the mercury with which the holes in the bars were filled (Art. 20). The stems were therefore necessarily exposed in their whole length to the temperature of the surrounding air. In the case of the higher temperatures to be measured, this correction was not only large (amounting sometimes to 3° Cent., always additive), but also in some degree uncertain, owing to the ascending currents of warm air in the neighbourhood of the heated bar, and enveloping the stem of the thermometer.[*] However, I believe that the formula in the note below leads to pretty accurate results, checked, as it has been, by occasional observations of a small auxiliary thermometer suspended in the air, touching the stem of the thermometer to be reduced, about its middle.

44. The hotter thermometers are probably slightly *over* corrected. I have stated that in extreme cases this correction amounts to about 3° Cent., a quantity which may possibly be erroneous in some cases to one-tenth of its amount, but

---

[*] The form of the correction is very simple, being

$$\frac{\text{Degrees exposed} \times \text{Excess of Temp. shewn over air.}}{\text{Dilatation of Merc. in Glass for } 1^\circ \text{ Cent.}}$$

always additive. If T be the temperature as read, t the temperature of the air, and a the scale reading of the commencement of the stem of the particular thermometer, the correction is very nearly

$$+\frac{(T-a)\ (T-t)}{6400}.$$

Since t and a are usually small numbers, the correction increases nearly as the square of the temperature to be measured.

Fortunately, the precision of this correction is not very important to the result. It chiefly affects the actual temperatures; for it will be more fully seen hereafter, that if the same instrument be used in the dynamical and statical experiments, being exposed in precisely the same way, the measures will be relatively correct, and the deduction of the conductivity will not thereby be sensibly affected.

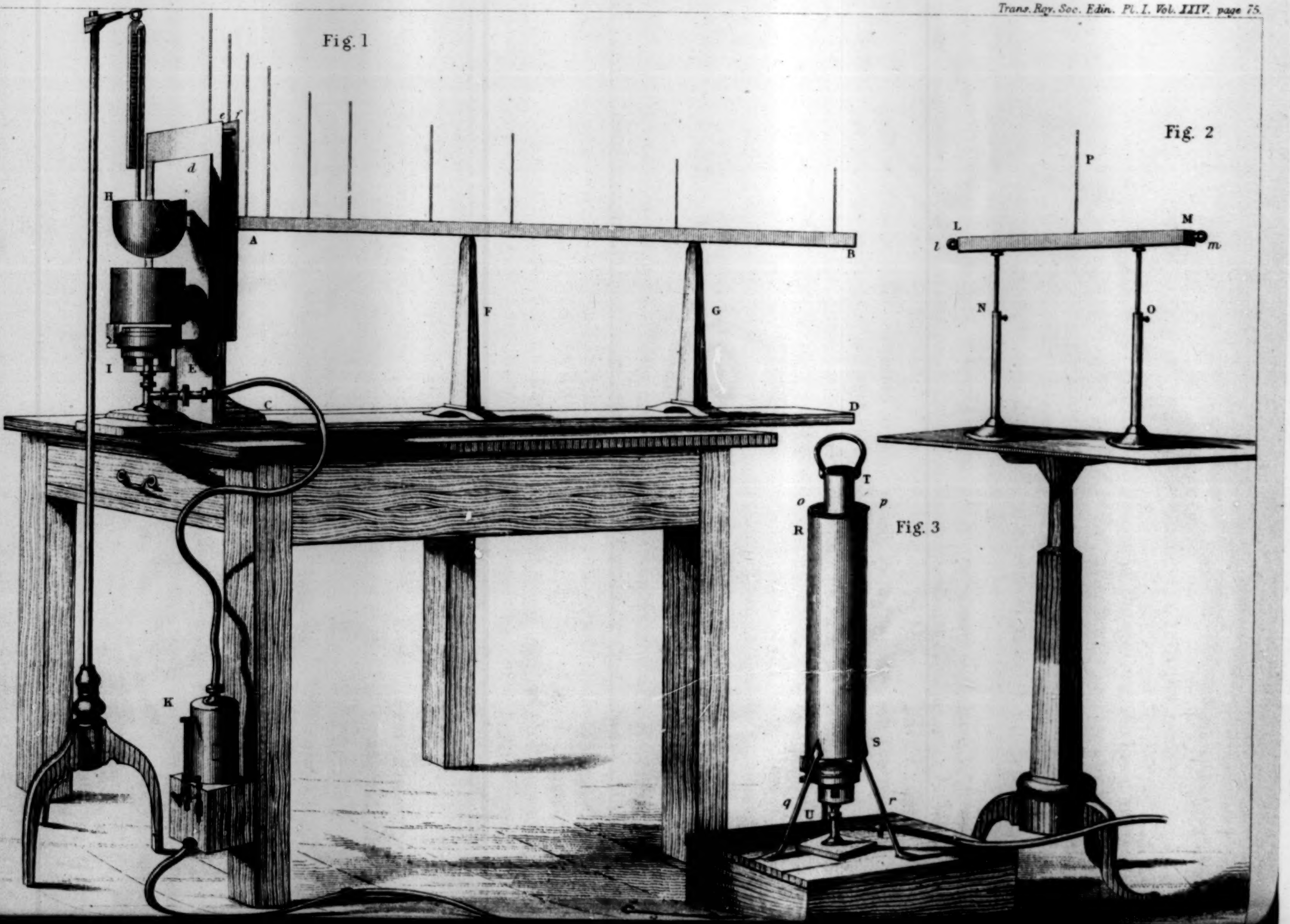


Fig. 1

Fig. 2

Fig. 3

Fig. 1

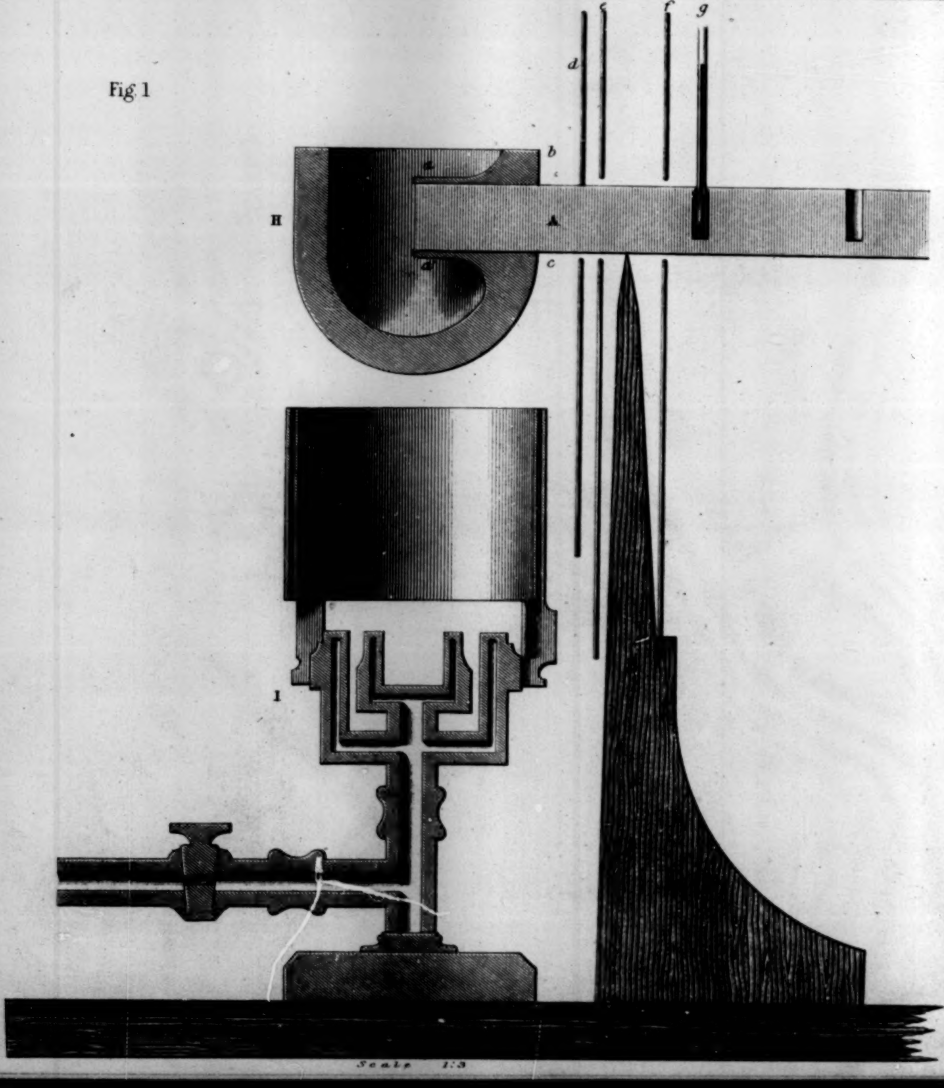
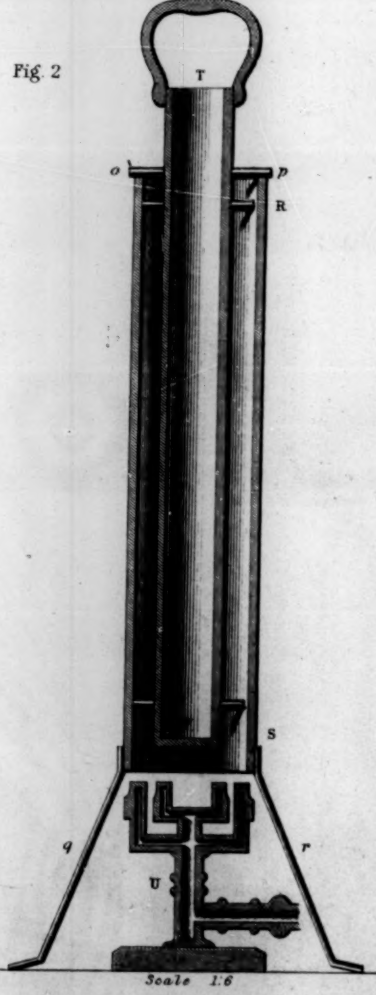


Fig. 2



I hope rarely so. Much larger corrections would have been inevitable at the highest temperatures (about 200° Cent.), had I not invariably employed for these a thermometer in which about 110° of the mercury was expelled from the bulb into the cavity at the top of the stem. The corrections for the reading of this thermometer were determined by Mr WELSH with extraordinary care. As its indications only commenced about the boiling-point of water, the length of the column exposed to the air was comparatively short.

45. For the principles on which the experimental investigations are founded, I refer to Art. 5, &c., of the former part of this paper. It will be recollected that there are two distinct classes of experiments, in one of which (the statical) the permanent temperatures at different points of a bar are to be observed; in the other (the dynamical) the velocity of cooling of a short bar of similar section, uniformly heated at first, is to be ascertained. I shall now proceed to describe these experiments severally more in detail than I have yet done, and to classify and discuss the results.

§ I. *Statical Experiments.*

46. *The Apparatus.*—A general account of this has been given in Arts. 17–20. It will, however, be rendered more intelligible by a reference to Plate I., fig. 1. The long wrought-iron bar AB was supported on a wooden frame CD by means of one fixed support E, and two moveable props F, G, which were all of wood, and were brought to a blunt edge at top, on which the bar rested, at about 15 inches above the top of a massive table, which stood in a spacious apartment attached to the Natural Philosophy Class-room (Edinburgh University). No fire was allowed during the experiments, and the south shutters being closed, the room was lighted from the north. At the end of the bar, towards the left side of the figure, was attached the heating apparatus, a cast-iron crucible H, usually filled with just-melting lead. It was kept hot by means of the powerful gas-furnace I, with a double metal chimney and two concentric rows of burners. The gas was derived from the main pipe by a flexible tube L, and passed through one of Milne's patent gas regulators, K, with a view to obtaining a uniform flame, which, however, remained subject to occasional fluctuation. The connection of the crucible with the conduction bar will be best understood from the sectional diagram in Plate II. fig. 1. An internal flange $a\,a'$ was cast on the crucible, leaving a square cavity 2·5 inches long, into which the extremity A of the conduction bar was thrust, and was retained there by friction only. The exterior face of the crucible $b\,c$ is almost vertical, and determines the position from which the distances of the thermometers along the bar are reckoned. Supposing the crucible itself to be maintained at the constant temperature of melting lead, it seems reasonable to assume that the bar A, so far as encased within it—that is, up to the zero line $b\,c$—may be regarded as having nearly

the same temperature.* The gas flame and the violently heated currents of air thence arising were prevented from playing against any part of the bar by a piece of metal plate fastened by wire to the crucible against the face *a b* (but to prevent confusion not shown in the figure), while the whole conduction bar was farther protected from heated currents, and from radiation from the crucible and gas chimney, by three polished metal screens *d, e, f,* placed parallel to one another, two to the left and one to the right of the wooden support E. The square apertures in the screens were 0·25 inch wider than the dimensions of the bar, and they were supported in such a manner as not usually to touch it in any part. These screens very effectually defend the thermometers, as well as the bar, from extraneous heat. The first thermometer only—that at 3 inches distance from the zero line *b c*—is seen at *g* in the section, fig. 1.

47. The conduction-bars have already been described in Arts. 17, 18. The more perfect one was 1·25 inch square, and fully 8 feet long, reckoning from the zero line above mentioned. It was used in two states, *first*, with a naked polished surface, and, *secondly*, when coated with thin paper. The other bar, also of wrought-iron, was 1 inch in the side and 7 feet long. In the present paper I shall discuss separately the results obtained with these two bars, presenting three quite independent cases, but which, as they ought to lead to an identical value of the conductivity of iron (assuming the quality of the bars to be alike), put the method here proved to a severe trial.

48. Throughout the remainder of this paper, when I speak of Case I., I mean the 1¼-inch bar with moderately *polished* surface; Case II. is the same bar with *paper* surface; Case III. is the 1-inch bar with naked, but less brightly polished surface.

49. The thermometers were inserted in holes in the bar 0·28 inch diameter and about ¾ inch deep. They were surrounded by mercury or (in the hotter holes) by fusible metal. (See Art. 20, and also Plate II. fig. 1.) Nine or ten thermometers were usually employed, and in the case of the principal bar (Cases I. and II.) they were usually (though not always) spaced as follows, reckoning from the zero line *a b* on the face of the crucible :—

0·25, 0·5, 0·75, 1, 1·5, 2 or 2·5, 4, 6, 8 feet.

50. The method of using a single standard thermometer for obtaining final results by the method of *stepping*, with its advantages, have been fully explained at Art. 22.

51. "The *free temperature*, or that to be deducted from the readings of the thermometers, in order to get the true excess of statical temperature along the bar, was obtained by inserting a well-compared thermometer into a hole containing mercury, drilled in a similar but short bar of iron, supported in the free air of the

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* See, however, note to Art. 70, below.

room in the neighbourhood of the long bar, and similarly exposed, but without artificial heat." (Art. 23.) The arrangement is shown in Plate I. fig. 2.

52. The gas furnace was usually lighted about 8 A.M., when the lead in the crucible was gradually melted. The readings of the thermometers were not recorded until about four hours had elapsed, and the experiment seldom lasted altogether less than eight hours, generally ten or eleven hours. It was difficult to keep the flame of the gas furnace steady, the "regulator" used for the purpose being apparently of little use. The lead in the crucible, after being quite fluid, sometimes solidified a little over the interior flange which grasps the bar. I at length found the best way of regulating the temperature to be, to keep the eye very constantly on the first thermometer in order, and whenever the slightest rise commenced, to dip a little cold lead into the crucible, and either let it melt or chill the mass by its contact, or the gas might be cautiously lowered. If the temperature was seen to be falling, the gas had to be raised. With experience I learned to keep the temperature of the three-inch hole within a range of 2° Cent. under favourable circumstances, the temperature being nearly 200° Cent. In some experiments in which solder was employed instead of lead, I used a thermometer whose bulb dipped into the crucible, where it stood about 460° Fahr.* My able assistant, Mr JAMES LINDSAY, learned to regulate this with great nicety.

53. When the temperature had been for a long time quite steady at the three-inch (or hottest) hole, the thermometers, disposed, as has been explained, along the bar, were successively read, and the readings recorded. This was done from left to right along the bar with all deliberation, without regard to any possible change during the process in the temperature at the hot end of the bar, since any such change is comparatively slowly propagated along the bar. In like manner, in "stepping" with one thermometer from point to point of the bar (Art. 2), a slight change in the temperature of the source is immaterial, since the "stepping" is performed faster than the wave of heat can follow.

54. A careful examination of the whole record of simultaneous readings was made, and those corresponding to the most stationary conditions of temperature were selected; and these being corrected for scale errors and temperature of column (Arts. 42, 43) were entered, after the free temperature indicated by the little bar (Art. 51) had been deducted, in the following tables as the Corrected Excesses of Statical Temperatures in centigrade degrees.

55. Looking cursorily over Table I., we may observe, *First*, that each day's observations are comparable only amongst one another, as no *exact* coincidence of the temperature of the crucible, or source of heat on different days, was attempted. *Secondly*, the bracketed observations are made with independent thermometers. *Thirdly*, comparing the first series for the bar covered with paper

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* By an oversight in the first part of this paper (Art. 19, *note*), it was stated that in this instance the thermometer was dipped in melted *lead*.

TABLE I.—CORRECTED EXCESSES OF STATICAL TEMPERATURE, ORIGIN AT OUTER SURFACE OF CRUCIBLE—DEGREES CENTIGRADE.

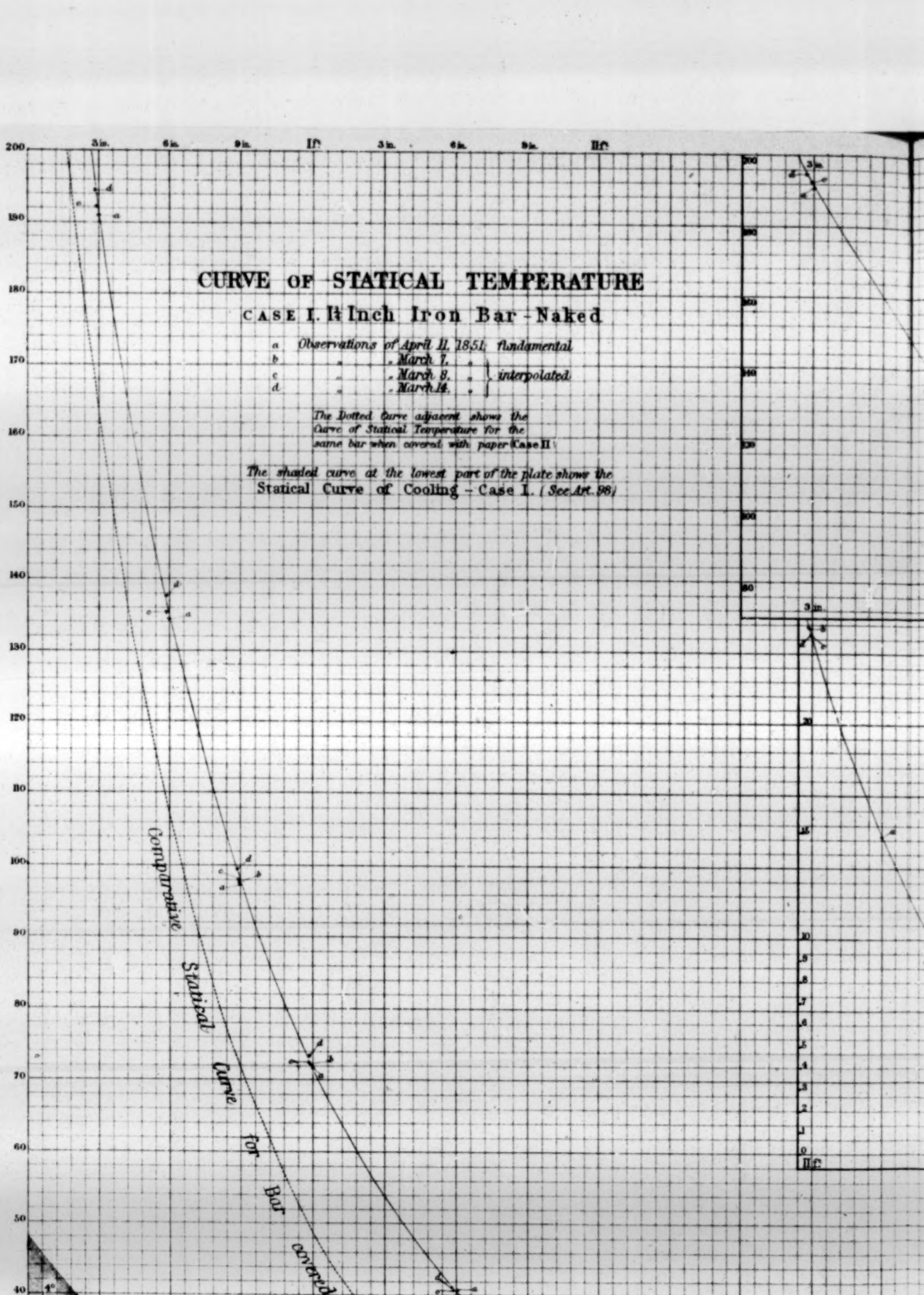
(The numbers bracketed together were obtained with different thermometers.)

	3 inch.	6 inch.	9 inch.	1 ft. 0 in.	1 ft. 6 in.	2 ft. 0 in.	2 ft. 6 in.	3 ft. 0 in.	4 ft. 0 in.	5 ft. 0 in.	6 ft. 0 in.	7 ft. 0 in.	8 ft. 0 in.
CASE I.—1¼ inch Iron Bar, naked. 1851, March 7 (Series A.)	191·1°	134·6°	97·7°	72·2°	March 7. Series B.	24·6°	…°	9·7°	…°	1·88°	…°	0·53°	0·34°
March 8, ...	192·3	135·3	97·95	72·4 72·4	40·9° 40·6*	24·1* 24·15	…	9·5	…	1·78	…	0·43	0·28
March 14, . . .	194·6	137·65	99·65 99·45	73·4	41·35	24·25 24·15	…	9·05 8·9*	…	1·70 1·61*	…	…	0·25* 0·35 0·45
(Standard) April 11, ...	191·0	134·7 134·3	97·15 97·4 97·25	71·85	40·9 40·65*	…	14·8 14·8	…	4·05 4·05	…	0·95 0·95	…	0·30 0·28
CASE II.—1¼ inch Iron Bar, covered with Paper.													
(Standard) 1851, April 15, . .	162·9	105·7 105·65	71·45 71·45	49·35	25·0 24·65*	[13·0?]	7·08 6·98	[3·85?]	1·28 1·28	0·50	0·18 0·18	…	0·0
April 16,† . .	120·55 120·3 120·5	80·0 80·15	54·95 54·65 54·8*	38·4 38·15*	19·45	10·3	5·65 5·65	…	1·10 1·10	…	0·15 0·15	…	0·0

	4 inch.	7 inch.	10 inch.	1 ft. 1 in.	1 ft. 7 in.	1 ft. 10 in.	2 ft. 4 in.	2 ft. 7 in.	4 ft. 4 in.	4 ft. 7 in.	4 ft. 10.	6 ft. 7 in.
CASE III.—1 inch Iron Bar, naked. 1850, Dec. 21, . . .	149·9	100·6	69·95	…	25·62*	…	10·2	7·65	1·25	1·0 1·0	0·78	0·07
1851, Jan. 11 (Series *a.*)	156·6	105·1	72·42	50·8	…	19·0	…	7·62	1·02	0·77 0·86	0·52	0·0
(Standard) … … (Series *b.*)	159·0	105·75 105·8	72·8	51·0	…	19·4	…	7·83	1·2	0·92	0·67	0·0

* Defective thermometer, or otherwise less reliable observations.

† The crucible was this day filled with a mixture of lead and tin (*solder*, which melts at a much lower temperature than pure lead), heated considerably above its melting-point.



CURVE OF STATICAL TEMPERATURE

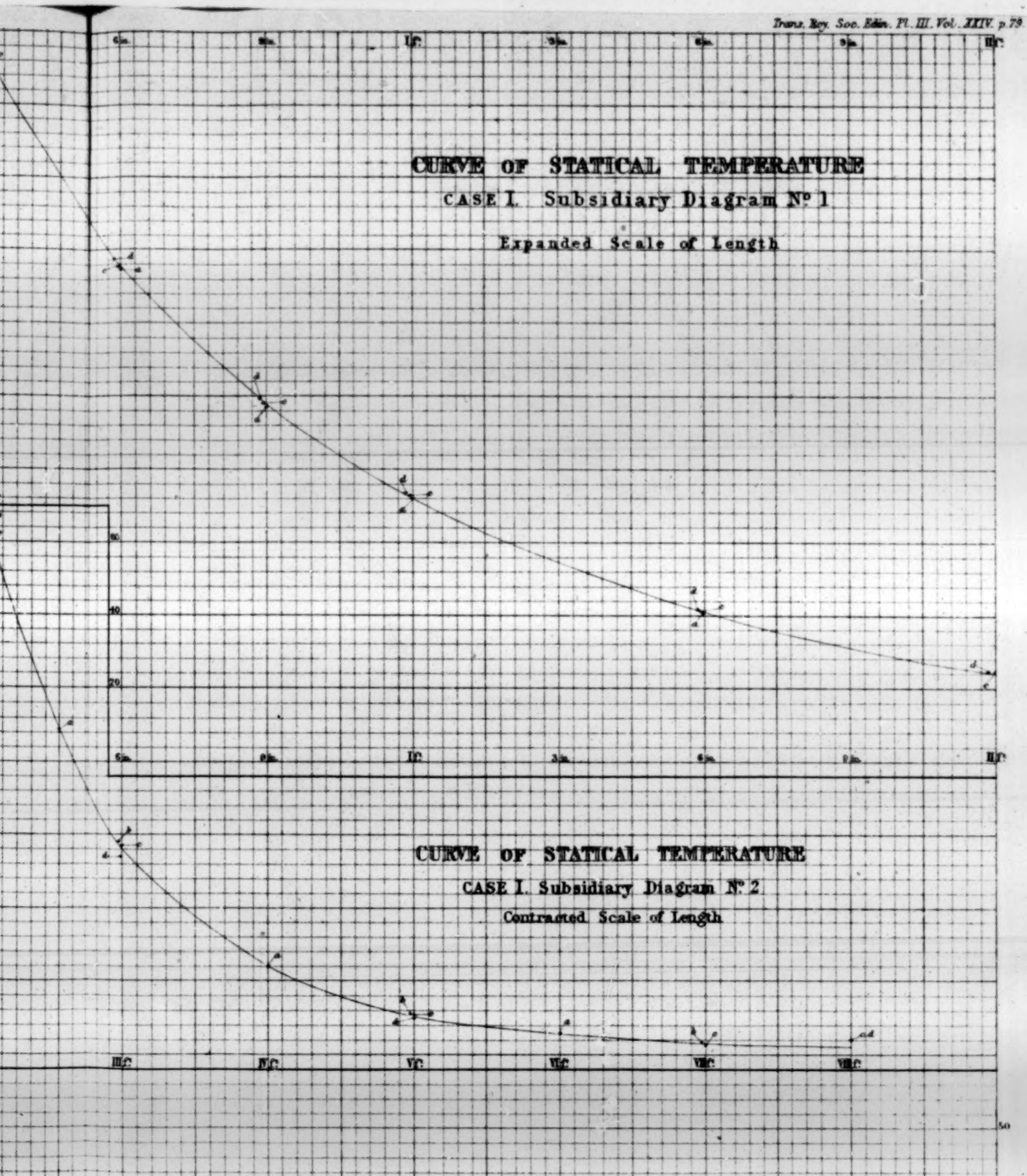
CASE I. 1¼ Inch Iron Bar - Naked

a *Observations of April 11. 1851, fundamental*
b „ „ *March 7.* „ } *interpolated*
c „ „ *March 8.* „ }
d „ „ *March 14.* „ }

The Dotted Curve adjacent shows the Curve of Statical Temperature for the same bar when covered with paper (Case II)

The shaded curve at the lowest part of the plate shows the Statical Curve of Cooling - Case I. (*See Art. 98*)

J. D. F. delt.



CURVE OF STATICAL TEMPERATURE

CASE I. Subsidiary Diagram No. 1

Expanded Scale of Length

CURVE OF STATICAL TEMPERATURE

CASE I. Subsidiary Diagram No. 2

Contracted Scale of Length

(April 15) with the last of the same bar naked (April 11), which were made in almost similar circumstances, we notice the prodigious effect of the increased radiation due to the paper casing. Though the heat at the origin may be considered as the same, at a distance of only three inches the temperature in the second case was less by nearly 30° Cent.; at thirty inches distance, the proper heat of the bar was but *one-half* of what it was in Case I.; at four feet, *one-third*; and, at eight feet, it had vanished in the second case, while it was still sensible in the first. *Fourthly*, it may be remarked that in Case I. the bar scarcely fulfilled, as to length, the implied condition of the experiment, which assumes that the extreme end of the bar shall be sensibly of the temperature of the air. As 0°·3 of heat remained at eight feet, and as the bar extended only a few inches beyond that point, it would appear that the conducted heat was not absolutely dissipated by radiation. The effect, which is to render the decrement of heat in the extreme holes rather too slow, is, however, barely appreciable in the deductions.

56. *Graphical Interpolation of the Statical Experiments.*—As it was desirable to combine the results of the independent experiments in each of the three Cases, and to deduce the most probable temperature for any point of the bar, a graphical method was adopted as follows:—Large sheets of drawing-paper were provided covered with engraved squares one-tenth of an inch in the side. A horizontal line was taken to represent the distances reckoned along the bar on a scale of four inches to a foot, and at the proper intervals the observed temperatures (or rather excesses of temperature) were set off as vertical ordinates on a scale of 10° to 1 inch. The general arrangement of the observations in this way is shown in Plate III. on a reduced scale for Case I.

57. It is plain, however, that this primary projection could only apply to a single and comparable series of observations under each Case of Table I., since the temperature of the origin might vary from one experiment to another. One set under each Case was assumed as a standard series to which the others were to be referred. In Case I., April 11; in Case II., April 15; in Case III., January 11 (*b*). But as it is plain that for one and the same bar the curve of temperature has the same form, though it may deviate in position to the right or left along the bar, each of the other days' observations was separately projected on transparent cloth, and then laid on the engraved squares over the first projection. By moving the system of projected points to the right or left (taking care to keep the line of abscissæ in each case accurately coincident), a position was easily found where the points to be interpolated accommodated themselves best to the general curvature of the fundamental series.

58. This method of interpolating independent series belonging to different fundamental temperatures has very great advantages. Had circumstances allowed me to continue these observations, I should have applied it more extensively. A clear instance of its utility will be seen by comparing the first and second row

of figures in Case II. of Table I. The observations of April 16 were made with melted solder as a source of heat, which fuses at a much lower temperature than lead. The result is, that the temperatures in the 3, 6, 9 inch holes and those which follow, are *intermediate* between the temperatures shown in those holes in the other experiment. Thus, by varying the temperature of the source of heat, we may multiply indefinitely points in the curve without increasing the number of holes with which the bar is pierced, which is evidently undesirable. It would have been very serviceable for the interpolation of the numbers in Case I., had the temperature of the origin been expressly varied for this purpose.

59. The general agreement of the independent interpolated observations has been highly satisfactory, as may be seen from Plate III., where the several sets of temperatures belonging to Case I. are distinguished by marks.

60. A continuous curve was next to be drawn through the extremities of the ordinates, so as best to conciliate the whole of the observations. To draw this curve was a matter requiring great nicety and judgment, owing to the limited number of ordinates disposable. It is well known to every one who has used such projections, that to draw an interpolating curve advantageously requires that the rate of increment of the two variables shall not be *excessively* unequal. In curves like those of Plate III., which rise very rapidly at one end, and become almost or quite asymptotic at the other, it is indispensable to make subsidiary projections of different parts of the curve, in which the relative scale of the vertical and horizontal co-ordinates shall be altered. For the part of the curve between 0 and 2 feet, the temperatures had to be contracted in scale and the abscissæ expanded; while for the right-hand branch of the curve the contrary was done, even to the extent of magnifying the vertical scale of degrees tenfold, whilst the horizontal scale of feet was diminished fourfold, compared with the first projection. For each of the three Cases (Art. 48), the statical curve was thus subdivided and partially projected on four different scales, three of which are exhibited on the engraved Plate. The result of this close analysis and comparison has been highly favourable to the assurance of accuracy in the final results, since the interpolated temperatures for any abscissæ are the result of two, if not three, projections of the observations on different relative scales. No numerical or other casual error could thus possibly escape detection.

61. I believe that these curves, as now obtained with the ordinates immediately to be given, are favourable specimens of numerical accuracy and geometrical definition, considering the difficulties attending the experiments. Throughout a great part of the curves (and that by far the most important for the results), the temperature excesses of the bar, or vertical ordinates, may, I hope, be esteemed correct within a very small fraction of their amount. The following table (which may be regarded as summing up the whole Statical data) contains the ordinates of the curves corresponding to the three Cases of Art. 48.

TABLE II.—STATIONARY EXCESSES OF TEMPERATURE (FROM ALL THE PROJECTIONS) ADOPTED.

Distance from Origin at Crucible.	Excess of Temperature (Centigrade) of Bar above Air.		
	Case I. 1¼ inch Bar, naked.	Case II. 1¼ inch Bar, covered.	Case III. 1 inch Bar, naked.
0 inch	275·5 c.	260·5 c.	282·2 c.
1	242·9 c.	221·7 c.	243·2 c.
2	214·8 c.	189·5 c.	210·2 c.
3	190·5	162·9	182·2 c.
4	168·8*	140·0*	159
5	150·6*	121·5	137·5*
6	134·7	105·9	120·5*
7·5	114·1*	86·6	99·8
9	97·3	71·3	82·8*
10·5	84·0*	59·0	68·6
I. foot 0 inch	72·0	49·2	57·1
3	53·6	34·5	40·9*
6	40·8	24·6	29·5
9	31·0	17·7	21·6
II. feet 0	24·2	13·0	15·65*
3	18·9*	9·45	11·5*
6	14·8	7·0	8·55*
III. „ 0	9·33	3·8	4·95*
6	6·15*	2·1*	2·78*
IV. „ 0	4·0	1·28	1·56
V. „ 0	1·8	0·47	0·55
VI. „ 0	0·9	0·165‡	0·13*
VII. „ 0	0·50	...	...
VIII. „ 0	0·28†	...	...

The numbers marked c. are derived from calculation. See Art 68 below.

The numbers marked thus * belong to points in the curve not closely adjacent to points of observation; and, therefore, are less certain than the others.

† 0·32 by mean of 7 observations. ‡ Mean of 4 observations.

Adjacent to the principal curve of Plate III. is a dotted curve, which exhibits the remarkable change of character in the curve when the bar is coated with a highly radiating surface of paper (Case II., Art. 48).

62. *Formulæ of Interpolation for the Statical Curves.*—It was not originally my intention to have entered on the thorny enterprise of seeking equations to satisfy the statical curves of temperature. My original plan (Art. 6 of former paper) was to deal with Curves alone, or almost entirely. And when we do not wish to exceed the limits of direct observation, it is perhaps the safest, as well as by far the easiest plan. I wished, however, to throw all possible light on the problem, for the benefit of those who may hereafter extend these observations. I also wished to obtain the greatest amount of information from the data at my disposal; and by means of formulæ, to extend the results somewhat (though not far) beyond the limits of observation. It will

be seen farther on (Art. 71) that a formula, however empirical, enables us to execute promptly and with confidence the otherwise tentative and uncertain process of drawing tangents to the curve in its higher part, in other words, of obtaining values of $\frac{dv}{dx}$ upon which the final deductions mainly depend. (See Arts. 6 and 31.)

63. It had been acutely perceived by Lambert nearly a century ago,[*] that the temperatures of a bar heated in the manner of our experiment would diminish in a regular geometrical progression. A more rigorous analysis led Biot[†] and Fourier[‡] to the same result, according to the physical assumptions with which they started. Biot and Despretz[§] subjected various metallic bars to experiment, but they assumed the logarithmic law to be true, and endeavoured to accommodate their numerical results to it, as well as they might. Biot, in particular, applied to his (apparently excellent) observations, the method of least squares to enable him to draw a logarithmic curve through his points of observation, giving no attention to the fact, that the temperatures found did not conform themselves by any means to the *a priori* geometrical law, and that the laws of Probability could not be applied to them without ascribing extravagant and improbable errors to a large part of the curve of observation.[‖]

64. That the temperatures deviate systematically from the law of continued progression, will appear from the following table of the ratios of successive ordinates, taken three inches apart in the three Cases of Table II.

MEAN RATIO[¶] BETWEEN TWO CONSECUTIVE ORDINATES 3 INCHES APART, FROM THE NUMBERS IN TABLE II.

INTERVALS.	CASE I.	CASE II.	CASE III.
3 to 6 inches, . . .	·707	·650	...
6 to 9 „	·722	·673	·687
9 inches to I. foot, . .	·733	·690	·690
I. foot to I. foot 6 inches,	·753	·707	·719
I. foot 6 inches to II. feet,	·770	·727	·728
II. to III. feet, . . .	·787	·735	·750
III. to IV. „	·809	·762	·755
IV. to VI. „	·830	·774	·731

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[*] Pyrometrie. Berlin, 1779, p. 185.

[†] Traité de Physique, vol. iv. p. 669.

[‡] Théorie Analytique de la Chaleur. 1822.

[§] Traité Elémentaire de la Physique. 1836, p. 197.

[‖] Compare Note to Art. 3 of this paper.

[¶] By "mean ratio," I intend to express, that where more than one 3-inch space is included in the Interval specified in the first column, the number which follows is the average decrement throughout that space. Thus, in Case I. the whole interval from II. to III. feet, shows a decrement from 24°·2 to 9°·33, which would result from the mean ratio of 0·787, four times multiplied into itself.

65. It will be observed that in every instance, with the single exception of the final number of the Table under Case III., the decrement of temperature becomes materially slower as we recede from the heated end of the bar. The exception is of little weight, as it depends on the residual temperature of the one-inch bar, six feet from the crucible, at which point the warmth was barely perceptible. The statical temperatures of the bar therefore increase more rapidly than in a geometrical progression. I have found that there is a sufficient analogy between the curve of statical temperature which we are here investigating, and that of the tension of steam at different temperatures, to afford some assistance in the selection of empirical formulæ in the present instance; being in each case a modified geometrical progression, though here the progression of ordinates is more rapid than a simple continued proportion, while in the tension of steam it is less rapid.

66. The most complete discussion of this class of formulæ, and the methods calculating from them, is to be found in M. REGNAULT'S *Relation des Expériences sur la Vapeur*. The available formulæ are reducible to three; YOUNG'S,* ROCHE'S,† and BIOT'S.‡ The two former contain three constants, the latter five. The last has been found the most satisfactory for the empirical representation of the elasticity of steam; and it is the only one of the three which can be regarded as applicable throughout the entire limits of experiment. The same is, I believe, true for the Conduction-curves with which we are now occupied. With five constants, five points of the curve may be accurately represented, and the intermediate deviations are of course inconsiderable. As none of the formulæ (except the simple logarithmic which is found to be inapplicable) have any foundation in principle, the whole matter is purely one of convenience. For simplicity's sake, I used only the formulæ of YOUNG and ROCHE, but I now think that the greater labour involved in the application of BIOT'S formula would have been repaid by the directness and certainty of the results. M. REGNAULT has given rules to facilitate its numerical calculation.

67. I have found the formula,—

$$\log v = \mathrm{A} - \frac{bx}{1 + cx} \quad . \quad . \quad . \quad . \quad . \qquad \text{Eq. (1.)}$$

(where v is the excess of temperature above the air at a point of the bar, whose abscissa, *in feet*, is x, and A, b, and c are constants), to represent tolerably well the temperature curve of Case I., as represented by the numbers in Table II., throughout the whole extent of the experimental curve.* But in order to follow the observations more closely, it seemed desirable to divide the curve into two parts, one between 0 and 1·5 feet, and the other beyond 1·5 feet, and to employ distinct constants for each. A like process was applied to the numbers of Table II., for Cases II. and III. In these, the variations of temperature along the bar being more rapid, the approximation of the formulæ was less exact than in the first case, and a formula with three constants was insufficient to represent the curve throughout its whole extent. I found it advisable, in the case of the bar covered with paper (No. II.), to use modified formulæ for the upper, middle, and lower part of the curve. I may add that it was found convenient to adapt the formula (Eq. (1.) of this article) to the calculation of the lower temperatures, by changing the origin to an arbitrary point some feet to the right, and by reckoning the abscissæ in the opposite direction, thus rendering the second term of the equation positive instead of negative. To this end the equation was written,—

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* $p = \mathrm{A}(1 + at)^m$, where p is the elasticity, and t the temperature.

† $\log p = \log a + \dfrac{bt}{1 + ct}$.

‡ $\log p = a + b\alpha^t + c\beta^t$.

$$\log v = \mathrm{A} + \frac{bz}{1+cz},$$

when

$$z = n - x.$$

68. The coincidence of the various formulæ with the experimental numbers of Table II. is shown in the foregoing Table.

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* The formula in this case would be,—

$$\log v = 272.7 - \frac{\cdot 63374\,x}{1+\cdot 0956\,x}.$$

The following adaptation of Young's formula also represents the observations in Case I. very approximately.

$$v = (\cdot 43027 + \cdot 09539\,x)^{-6\cdot 65}.$$

x in feet.	v by Experimental Curve.	v by Formula $(a+bx)^p$.	Difference.	v by Formula $\log v = \log a - \frac{bx}{1+cx}$	Difference.
	°	°	°	°	°
0	...	272·66	...	272·7	...
0·25	190·5	190·5	0·0	191·0	+0·5
0·5	134·7	135·5	+0·8	135·9	+1·2
0·75	97·3	98·04	+0·74	98·23	+0·93
1·0	72·0	72·0	0·0	72·0	0·0
1·25	53·6	53·6	0·0	...	...
1·5	40·8	40·41	−0·39	40·21	−0·59
2·0	24·2	23·75	−0·45	23·53	−0·67
2·5	14·8	14·52	−0·28	...	...
3·0	9·33	9·18	−0·15	9·08	−0·25
4·0	4·0	4·0	0·0	4·0	0·0
5·0	1·8	1·91	+0·11	1·96	+0·16
6·0	0·9	1·00	+0·10	...	...
8·0	0·28	0·31	+0·03	0·36	+0·08

TABLE III.—SHOWING THE COMPARISON OF THE GRAPHICAL CURVES OF STATICAL TEMPERATURE WITH DIFFERENT FORMULÆ OF INTERPOLATION.

Distance from Origin in		CASE I.				CASE II.					CASE III.			
Inches and Feet.	Feet and Decimals $= x$.	v by Experim. Curve.	Form. A.	Form. B.	Difference.*	v by Curve.	Form. C.	Form. D.	Form. E.	Difference.*	v by Curve.	Form. F.	Form. G.	Difference.*
0 inch	0	...	275·5	...	...	...	260·5	...	...	...	...	282·2	...	...
1 „	0·08̇3	...	242·9	...	...	...	221·7	...	...	...	...	243·2	...	...
2 „	0·16̇6	...	214·8	...	...	...	189·5	...	...	...	...	210·2	...	...
3 „	0·25	190·5	190·5	...	0·0	162·9	162·9	162·9	...	0·0	...	182·2	...	...
4 „	0·33̇3	168·8	169·3	...	+0·5	140·0	140·6	140·6	...	+0·6	159·0	158·4	...	−0.6
5 „	0·41̇6	150·6	150·9	...	+0·3	121·5	121·9	121·9	...	+0·4	137·5	138·2	...	+0·7
6 „	0·5	134·7	134·7	...	0·0	105·9	105·9	[106·0]	...	0·0	120·5	120·8	...	+0·3
7·5 „	0·625	114·1	114·2	...	+0·1	86·6	86·52	[86·53]	...	−0·08	99·8	99·36	...	−0·44
9 „	0·75	97·3	97·3	...	0·0	71·3	71·32	[71·14]	...	+0·02	82·8	82·18	...	−0·62
1 ft. 0 „	1·0	72·0	71·6	...	− 0·4	49·2	[49·48]	49·08	...	−0·12	57·1	57·19	[56·8]	+0·09
3 „	1·25	53·6	53·6	53·6	0·0	34·5	...	[34·7]	34·4	−0·1	40·9	40·67	[40·9]	−0·23
6 „	1·5	40·8	40·77	40·75	−0·04	24·6	...	[25·1]	24·64	+0·04	29·5	29·52	[29·61]	+0·02
2 ft. 0 „	2·0	24·2	[24·61]	24·17	−0·03	13·0	...	...	12·94	−0·06	15·65	...	15·76	+0·11
6 „	2·5	14·8	...	14·80	0·0	7·0	...	...	7·00	0·0	8·55	...	8·55	0·0
3 ft. 0 „	3·0	9·33	...	9·33	0·0	3·8	...	...	3·89	+0·09	4·95	...	4·72	−0·23
4 ft. 0 „	4·0	4·0	...	4·00	0·0	1·28	...	...	1·29	+0·01	1·56	...	1·52	−0·04
5 ft. 0 „	5·0	1·8	...	1·87	+0·07	0·47	...	...	0·47	0·0	0·55	...	0·52	−0·03
6 ft. 0 „	6·0	0·9	...	0·95	+0·05	0·165	...	...	0·185	+0·02	0·13	...	0·189	+0·06
7 ft. 0 „	7·0	0·50	...	0·51	+0·01	...	...	...	...	...	...	...	...	...
8 ft. 0 „	8·0	0·28	...	0·279	0·00	...	...	...	...	...	...	...	...	...

* The bracketed numbers are not used in taking the differences.

69. The formulæ used in the preceding calculations are the following:—

$$\text{Case I.}\left\{\begin{aligned}&(A)\ \log v=\log 275\cdot5-\frac{\cdot66184x}{1+\cdot13093x}\\[4pt]&(B)\ \log v=\log \quad 4\cdot0+\frac{\cdot3472z}{1-\cdot0556z},\ \text{where } z=4-x.\end{aligned}\right.$$

$$\text{Case II.}\left\{\begin{aligned}&(C)\ \log v=\log 260\cdot5-\frac{\cdot85265x}{1+\cdot1819x}\\[4pt]&(D)\ \log v=\log 259\cdot08-\frac{\cdot83855x}{1+\cdot1606x}\\[4pt]&(E)\ \log v=\log \quad 0\cdot47+\frac{\cdot4217z}{1-\cdot0405z},\ \text{where } z=5-x.\end{aligned}\right.$$

$$\text{Case III.}\left\{\begin{aligned}&(F)\ \log v=\log 282\cdot2-\frac{\cdot7872x}{1+\cdot1275x}\\[4pt]&(G)\ \log v=\log \quad 0\cdot52+\frac{\cdot4521z}{1-\cdot0282z},\ \text{where } z=5-x.\end{aligned}\right.$$

70. With reference to the preceding numerical Table, I may remark, *First*, that the differences shown are not in all cases deviations from *direct* observations, but between the formulæ and the graphical interpolation of the data. There is a difficulty (which will be understood from Art. 57) in comparing compendiously the formulæ with the *single* data of Table I. When the points of the curve are somewhat distant from points of observation, the numbers in the preceding Table, obtained from the formulæ, may be, and probably are more reliable than those assigned from the curve. *Secondly*, the curve of Case I. appears to be the most reliable in all respects. And in particular I consider the portion of the curve which includes the highest temperatures, or those corresponding to points on the bar between 0 and 3 inches, to be very nearly accurate. From numerous independent calculations, I conclude that the value of v at the origin, or in contact with the crucible, is pretty exactly 275°·5 Cent., as there assigned. If we add to this 12°·5 for the approximate temperature of the apartment, we have 288° for that of the bar where it enters the crucible, and is supposed to have very nearly the temperature of melting lead. This is a considerably lower temperature than is usually attributed to melting lead.*

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* Usually stated at from 320° to 330° Cent., 608° to 626° Fahr. Biot, indeed, gives it as only 260°, inferentially derived from his conduction experiments (Traité de Physique, iv. 677); but this is on the supposition of the logarithmic law prevailing. Crichton, junior, gives 606°·5 Fahr. (T. Thomson); Daniell, 612°; Kupffer, 633°. Supposing any of these last numbers to be correct, the inference must be, that in the conduction experiments described in the present paper, the temperature of melting lead did not extend to the outside of the iron crucible when the origin of the co-ordinates has been taken, but must be sought somewhere in the interior. This conclusion is strengthened by some other, though indirect considerations.

71. I cannot too distinctly repeat that the formulæ adopted in the preceding Table are only to be regarded as a means of more conveniently grouping the observations. The most important use to be made of these formulæ, however, yet remains to be mentioned. It will be seen by reference to Arts. 6, 28, &c., of the former part of this paper, or to § IV. of the present paper, that it is not the ordinates themselves of the statical curve of cooling which are to be used in obtaining the conductivity of the bar, but the values of the differential coefficient $-\frac{dv}{dx}$ for each part of the bar. In other words, we must be able to draw a tangent to the curve of statical temperature at any point of the curve. This may be roughly done mechanically, or it may be done by dividing the curve into short elementary portions, and treating each portion as if it were part of a logarithmic curve (see below, Art. 82, on the Analogous Treatment of the Dynamical Curve); or, finally, it may be obtained from the equations above given. The two last methods have been used in the reductions, and especially the last of all, which is the only satisfactory one for the higher parts of the statical curve. The general form of the empirical equation being,

$$\text{Tab. log } v = A - \frac{bx}{1+cx}$$

when reduced to Napierian logarithms, gives

$$0{\cdot}4343 \text{ hyp. log } v = A - \frac{bx}{1+cx}$$

and

$$\frac{dv}{dx} = -2{\cdot}3036\frac{bv}{(1+cx)^2}$$

whence the numbers which will be given in § IV. of the present paper are computed, the values of *b* and *c* being taken from the formulæ of Art. 69.

§ II. *Experiments on Cooling.*

72. *The Apparatus.*—It will be seen, by reference to the former part of this paper (Arts. 5, 24), that, in order to interpret the indications of the permanent temperature of a bar, and to deduce its conductivity, we must have an independent set of observations on the cooling of a similar bar, or a portion of a similar bar. For this purpose, the apparatus shown in fig. 2 of Plate I. was employed. The same short bar, L M, which has been already referred to (Art. 51), as being used in the statical experiment for determining the temperature the bar would have had independently of the heat applied at one end, was supported on the props N, O, after being duly heated. It is now to be used to ascertain the rate of loss of heat from a bar having the Section and Surface proper to each of the three Cases of Art. 48, in terms of the scale of the thermometer P, inserted at or near its middle point.

73. I have so fully described, in Art. 24, the manner of performing the

Cooling experiment, that I need here do little more than refer to the figures by which it is now illustrated, and give the corrected results as to the "law of cooling."

74. Fig. 2 of Plate I. shows the small iron bar employed, which in Case I. and Case II. (Art 48.) was 20 inches long and 1¼ inch square, first naked and polished, and afterwards covered with paper; it was marked C. In Case III. it was a polished (or at least a bright) bar, 20 inches long, 1 inch square, and marked E. Each bar had a ring at each end, *l*, *m*, and could be handled by seizing either end by the hook Q, fig. 3. Having been covered with several folds of stout paper to prevent a sudden chill of the metal bath into which it was to be introduced, it was lowered vertically and lengthwise into the cylindrical iron vessel shown at fig. 3, and in section in Plate II. fig. 2. It consists of a stout iron tube T V, about two feet long, with a bottom at V, and a handle at T. It rests by means of two iron pins, *o*, *p*, on the upper edge of a cylindrical iron chimney R S, supported by three feet, of which two are seen at *q* and *r* over the gas furnace U, the powerful flame of which, playing between the two cylinders, keeps a quantity of solder or of "fusible metal" in the interior one, not only melted, but heated considerably above the melting point. The bar under experiment, after being coated with several folds of paper, having usually also been well warmed over a hot-air stove, was introduced by the hook Q into the metal bath, then turned end for end several times, until it was believed that the heat had well penetrated its entire thickness. It was then withdrawn, shaken, the paper covering rapidly cut off, the bar wiped with a cloth,* and placed horizontally on the two ivory-topped props N, O (Plate I. fig. 2), the thermometer P inserted in the central hole,† into which heated mercury had already been placed, and the reading of the thermometer from minute to minute immediately commenced, the times being given by an assistant. The free temperature was determined by a thermometer sunk in a cold bar in the neighbourhood, or by one suspended in the air, or by both.

75. *The Observations.*—As in the statical observations there are three cases.

Case I. Iron bar, 1¼ inch square, roughly polished.
Case II. Do. do. covered with paper.
Case III. Do. 1 inch square, roughly polished.

76. Two independent sets of observations of the law of cooling on different days have been obtained for each case. Moreover, as more than one thermometer was observed in the holes of each bar (as in the example which follows), except for the very highest temperatures, use has from time to time been made of these auxiliary series. The whole of these observations have been most carefully corrected for the

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* The *wiping* of the bar I believe to have been unnecessary and injurious. It lowered the temperature, and interfered with the distribution of the heat in the bar.

† The 1¼ inch bar had a central hole, and others 1·5 inch distant, right and left. The 1 inch bar had only two holes equidistant from the centre of the bar.

scale errors and for the temperature of the stem. Where the temperature of the air of the room has not been quite steady, the variations have been interpolated and allowed for in deducing the excess of temperature of the bar above that of the room.

77. I shall give the details of one experiment as a specimen (all reductions being first made).

TABLE IV.—COOLING OF SHORT 1¼-INCH BAR, C.—26TH MARCH 1851.

Hour.	Hole (2), Centre. Corrected Excess.	Hole (1), to the Left. Corrected Excess.	Hole (3), to the Right. Corrected Excess.
h. m. s.	°	°	°
12 59	170·7		
1 0	167·4		
1	164·0		
2	160·85		
3	157·6		
4	154·5		
5	151·45		
6	148·45		
12	131·7		
13	129·2		
14	126·6		
15	124·2		
16	121·7	119·85	
17			
18	116·95		
19			114·25
20	112·65		
21		110·85	
22	108·35		
23			106·0
24	104·5		
25		103·05	
26	100·65		
29			95·10
30	93·75		
31		92·05	
32	90·25		
33		88·9	
34	87·0		
35			85·5
36	84·10		
37			82·5
38	81·25		
39		79·95	
40	78·50		
2 0	56·10		
2 1		55·20	
2	54·15		
3			53·2
4	52·4		
5		51·65	
6	50·6		
7			49·75
8	49·0		
9		48·35	
10	47·4		
21	40·1	40·15	
22	39·35		39·40
24	38·25	38·20	
26	37·10		37·10
28	36·0	36·05	
48	26·9	26·95	
50	26·15		26·2
52	25·4	25·4	
54	24·6		24·6
3 31	14·95	14·92	
34	14·35	14·32	
36	14·0		13·92
38 30	13·40		13·25
4 10	8·98	9·00	8·92
15	8·48	8·50	8·41
20	7·98	7·9	7·8
25	7·53	7·55	7·44
6 0*	2·79	2·6	
10*	2·54	2·4	
20*	2·15	2·1	
30*	2·0	1·9	
8 10*	0·9	0·7	
20*	0·9	0·7	
30*	0·85	0·7	

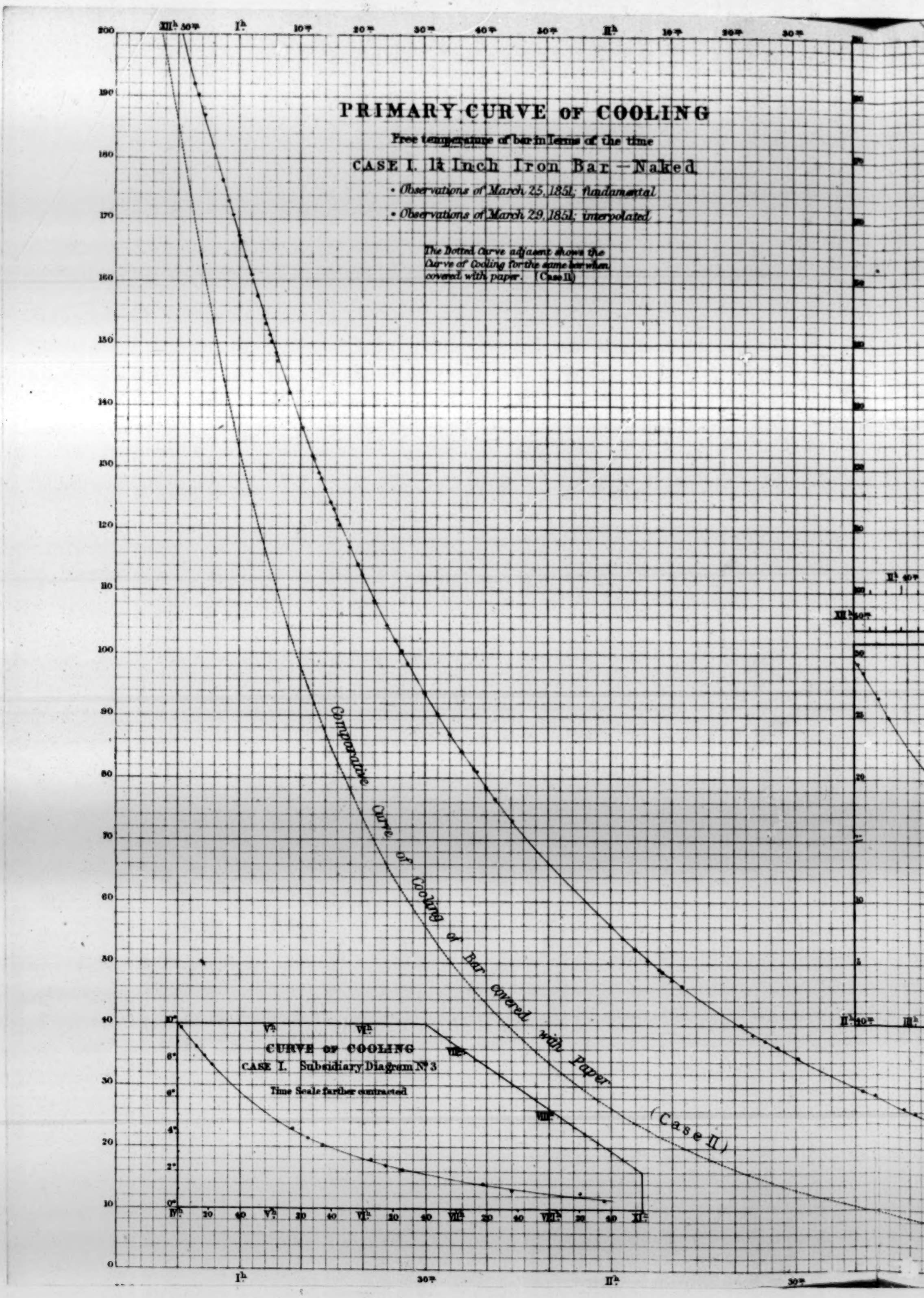
* Read by an assistant. The scale of the thermometer in hole (3) being liable to mistake in reading, its results are omitted.

78. *Graphical Interpolations.*—The observed excesses of temperature (as obtained, for example, in the preceding experiment for the central hole) were projected in a curve of which the times were taken as abscissæ, and the independent temperatures of the bar as ordinates. When more than one series of observations (on the same bar at different times) were to be combined, a procedure exactly similar to that described in Art. 57 for the stationary temperatures was employed; that is to say, one series being first projected on the engraved paper as fundamental, any other series was next similarly projected on tracing cloth, and the system of points thus obtained was moved to the right or left over the first, until the points in the two curves appeared to be superposed satisfactorily. The interpolated observations were then pricked through, and a curve drawn through the whole.

TABLE V.—CURVES OF COOLING (IN TERMS OF TIME).

Time from Arbitrary Origin.	CASE I. 1¼ inch Bar.		CASE II. 1¼ inch Bar, covered.	CASE III. 1 inch Bar.
	March 26.	March 29.		
−10 Min.	...	*242·3	...	...
− 5	...	*221·4	*263·9	...
− 2·5	...	*211·5	*243·6	...
0	...	*201·9	*225·0	*258·5
2·5	...	192·0	*207·7	*243·3
5	...	183·5	*191·8	*228·9
7·5	...	174·8	177·0	*215·3
10	167·3	166·5	163·6	202·4
12·5	159·1	158·2	150·6	190·05
15	151·4	150·6	139·1	178·7
20	137·0	136·5	119·25	157·55
25	124·2	124·2	102·95	138·9
30	112·6	112·8	88·8	122·45
35	102·6	102·5	77·1	108·6
40	93·7		67·0	96·45
50	78·6		50·95	77·25
60	65·9		39·4	62·2
70	55·8		30·65	50·5
80	47·3		24·25	41·3
90	40·5		19·2	33·95
100	34·9		15·27	28·2
125	24·2		8·9	18·0
150	17·1		5·45	11·95
175	12·3		3·42	8·1
200	8·95		2·15	5·55
300	2·95		0·4	1·4
400	1·12		...	0·4

The numbers marked thus * are deduced from the Equations of Art. 88.


PRIMARY CURVE OF COOLING
Free temperature of bar in Terms of the time
CASE I. 1¼ Inch Iron Bar — Naked
• Observations of March 25, 1851; fundamental
• Observations of March 29, 1851; interpolated
The Dotted Curve adjacent shows the Curve of Cooling for the same bar when covered with paper. (Case II)
Comparative Curve of Cooling of Bar covered with Paper
(Case II)
CURVE OF COOLING
CASE I. Subsidiary Diagram No. 3
Time Scale farther contracted
200
190
180
170
160
150
140
130
120
110
100
90
80
70
60
50
40
30
20
10
0
XIIh 50m
Ih
10m
20m
30m
40m
50m
IIh
10m
20m
30m
30m
10°
8°
6°
4°
2°
0°
IVh
20
40
Vh
20
40
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40
VIIh
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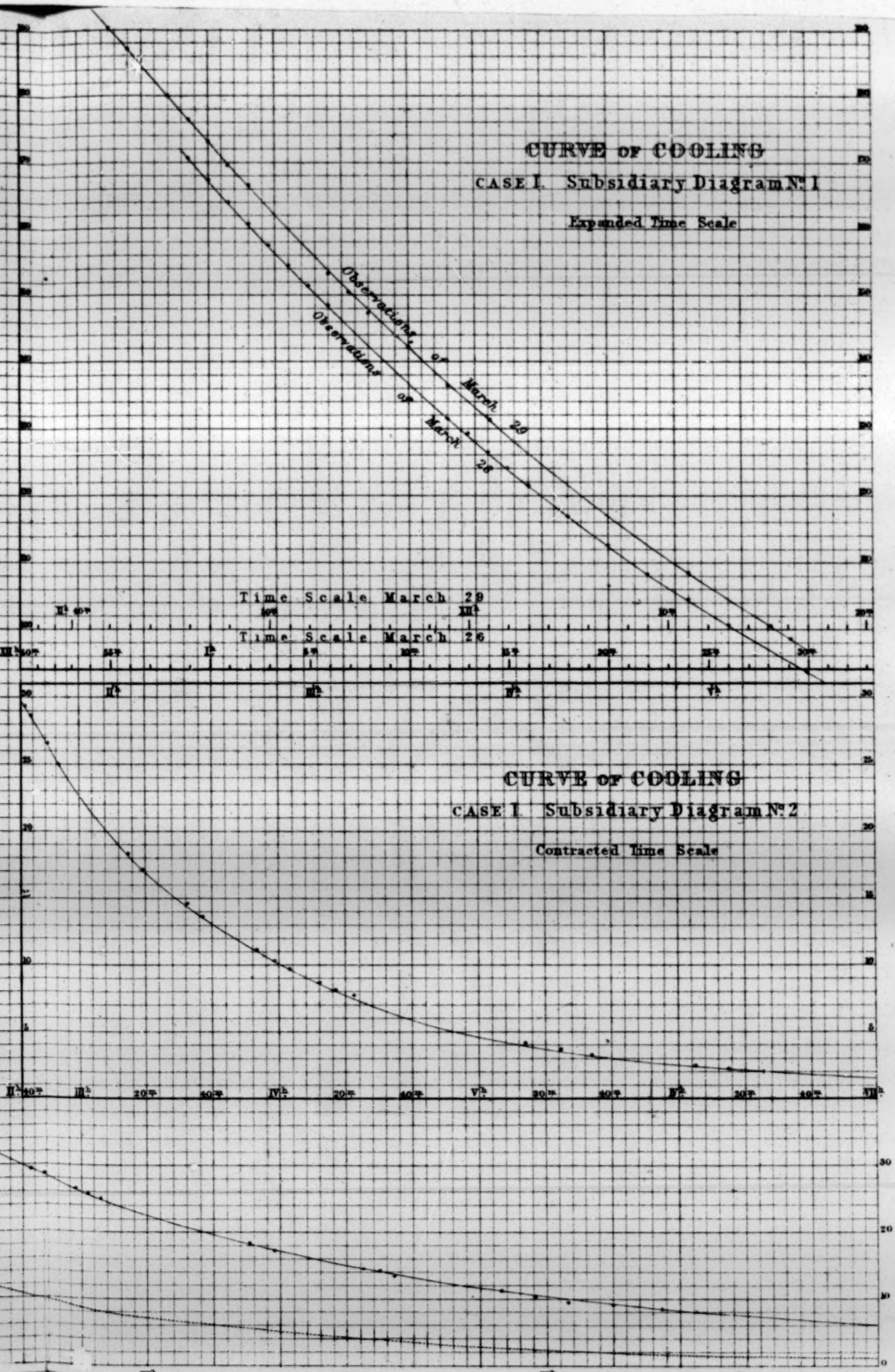
VIIIh
IIh 40m
IIIh



CURVE OF COOLING

CASE I. Subsidiary Diagram No. 1

Expanded Time Scale

CURVE OF COOLING

CASE I. Subsidiary Diagram No. 2

Contracted Time Scale

SECONDARY CURVES OF COOLING

"Rate of Cooling" in terms of Free Temperature (art. 81)

$-\frac{dv}{dt}$ in terms of v

- *Points in the curves deduced from Primary Curves of Cooling by Logarithmic Law — (art. 82)*
- *Points in Curve from Equation to Primary Curve (art. 88)*

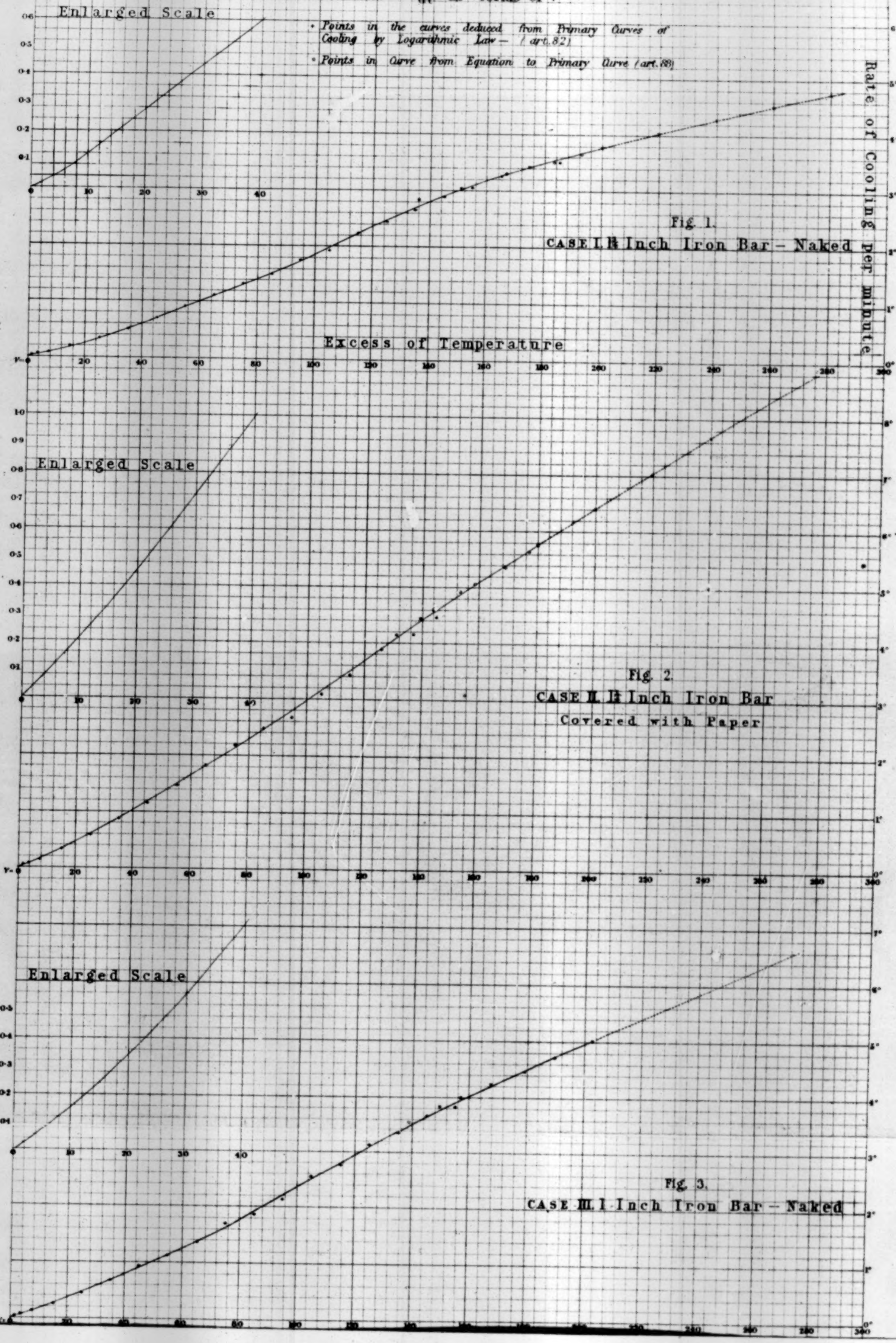


Fig. 1.
CASE I. 1½ Inch Iron Bar — Naked

Fig. 2.
CASE II. 1½ Inch Iron Bar
Covered with Paper

Fig. 3.
CASE III. 1 Inch Iron Bar — Naked

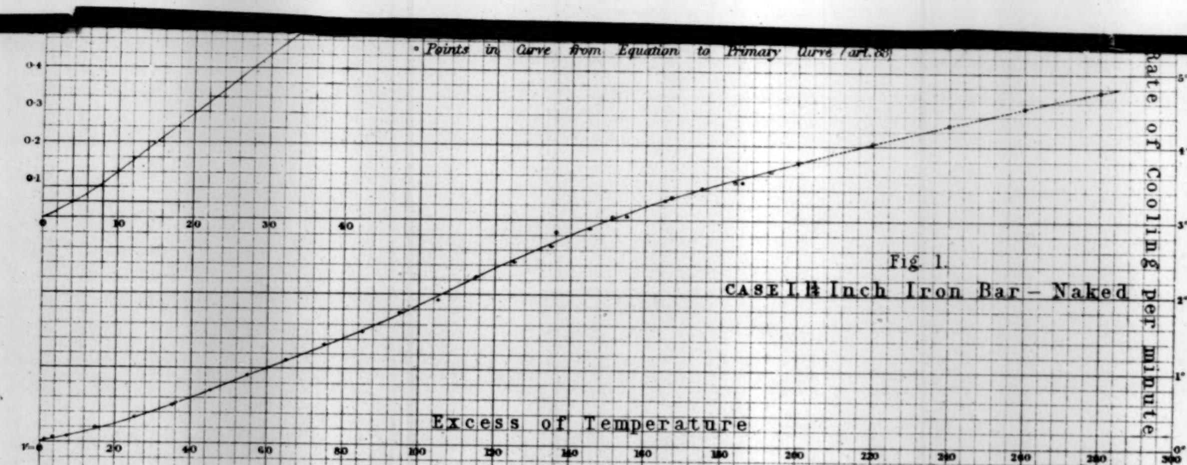


Fig. 1.

CASE I. 1¼ Inch Iron Bar – Naked

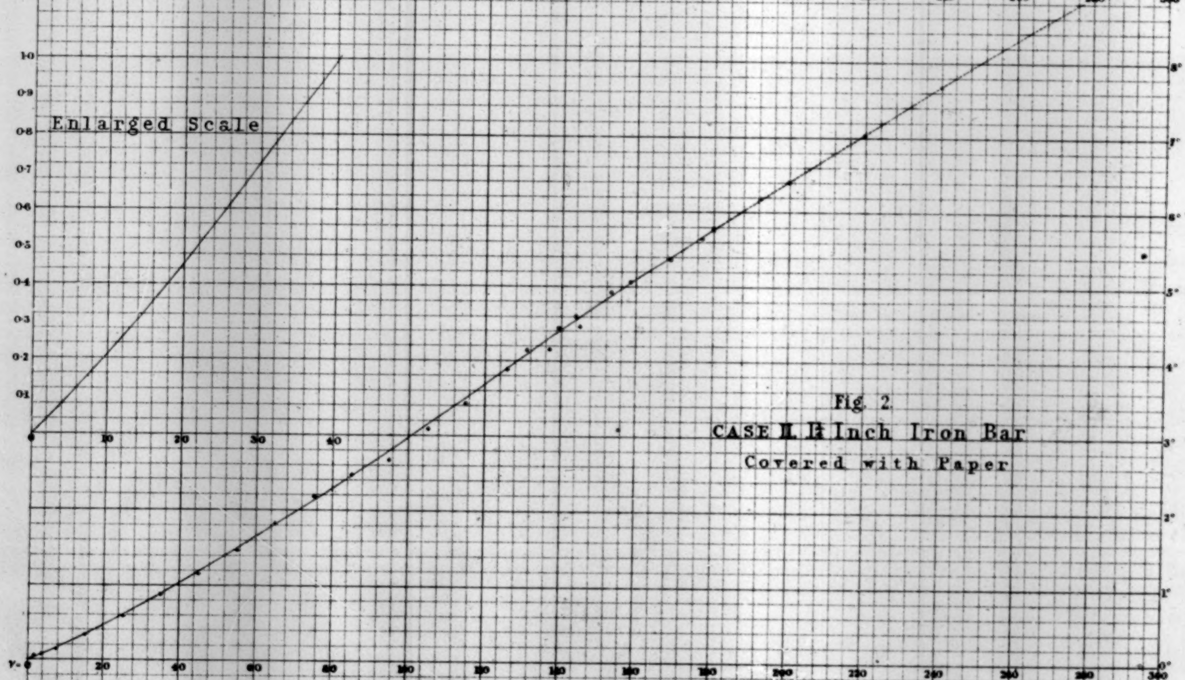


Fig. 2.

CASE II. 1¼ Inch Iron Bar

Covered with Paper

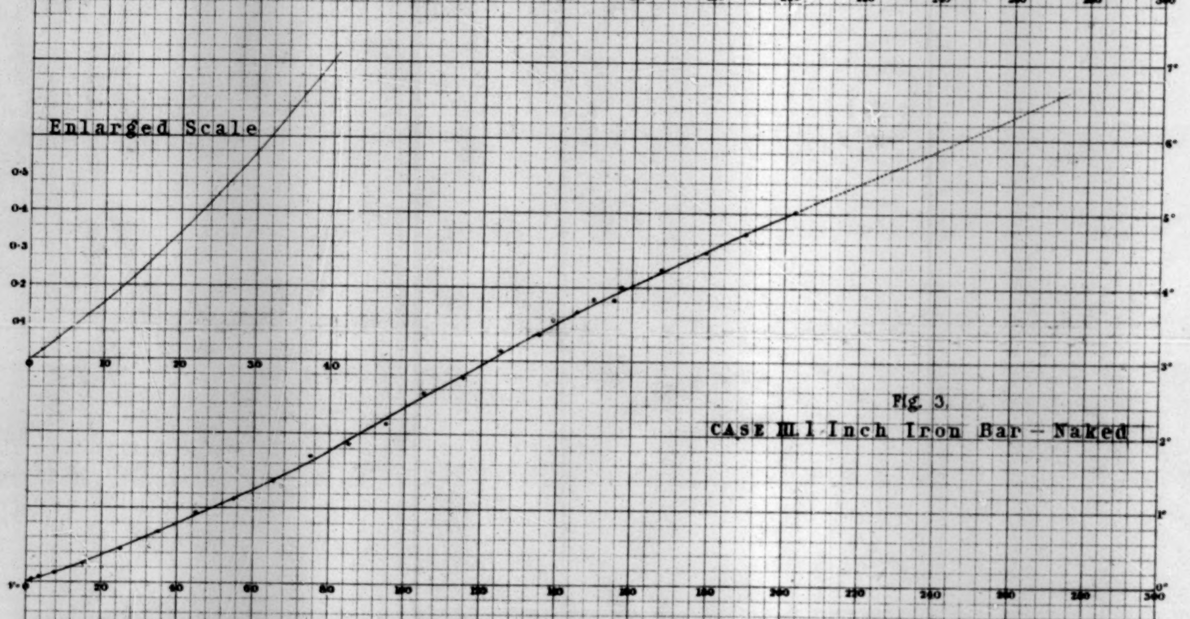


Fig. 3.

CASE III. 1 Inch Iron Bar – Naked

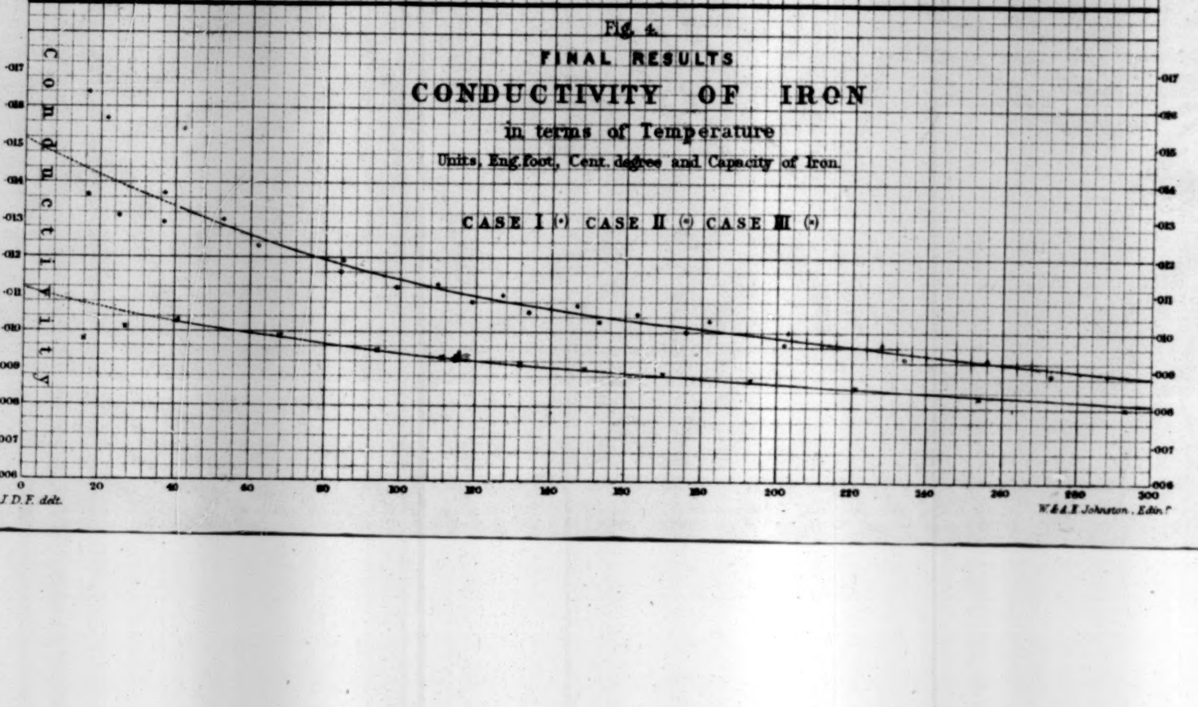


Fig. 4.

FINAL RESULTS

CONDUCTIVITY OF IRON

in terms of Temperature

Units, Eng. Foot, Cent. degree and Capacity of Iron.

CASE I (·) CASE II (·) CASE III (·)

J. D. F. delt.

W. & A. K. Johnston, Edin.r

79. A specimen of the Curves of Cooling is given in Plate IV. The subsidiary curves in the same plate show different sections of the main curve projected on different scales (as in the case of the Statical Curves, Art. 60), for convenience of interpolation. The main curve corresponds to Case I. The dotted line adjacent to the main curve in the Plate shows the modification of the law of cooling introduced by covering the bar with paper, as in Case II. The results of the whole are shown in the preceding Table. The origin of the abscissæ (the times) is of course wholly arbitrary in each case.

80. The continuity of the curves thus obtained was in general satisfactory, though in one or two instances it seemed desirable to project part of two curves as distinct, as in Case I.

81. It is of little use, however, to possess merely a knowledge of the free temperature of the bar in terms of the time. The valuable information which we require in the deduction of conductivity is the "rate of cooling," or the proportional momentary loss of heat corresponding to a given excess of temperature. This is expressed mathematically by $\frac{dv}{dt}$, and might be directly obtained by discovering the equation to the primary curve of cooling, and then differentiating it.

82. There is, however, not less difficulty in finding a formula of interpolation to represent the curve of Cooling throughout its extent, than we have already found in the case of the curve of Statical Temperature, and it would evidently require the introduction of as many constants. I therefore preferred, in the first instance, (seeing that from the multiplied observations of cooling, the ordinates of this curve are more perfectly known than in the other case), to subdivide it into elementary arcs, and treating each of these as a portion of a logarithmic curve (to which it approximates), to find the value of $\frac{dv}{dt}$, or the "rate of cooling," corresponding to successive values of v,[*] and by projecting these in curves to study their inflections in detail in each of the three forms of experiment already often referred to.

83. The three upper figures of Plate V. represent the "rates of cooling" of each bar in terms of its temperature-excess. From the study of these the peculiarities of the law of cooling above adverted to will become evident, and the harmony of the three cases is exhibited to the eye.

84. *First*, For very small excesses of temperature, the rate of cooling is comparatively slow, but increases much more rapidly than the temperature. To illus-

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[†] By the formula $\frac{dv}{dt} = 2{\cdot}3026 \, \frac{\log v' - \log v}{t - t'} \times \frac{v' + v}{2}$, where v and v' are the excesses of temperature corresponding to the times t and t'. $\frac{v' + v}{2}$ is the mean ordinate to which the result corresponds. The logarithms are tabular.

trate this, a portion of each curve near its origin has been drawn separately on an exaggerated scale in a subsidiary figure, where its deviation from a straight line is abundantly manifest. In each case it may be adequately represented for the first 4° or 5° by an arc of a common parabola—more accurately perhaps by a semicubical parabola. However, taking the former as the simplest, I find the following equations to represent the part of the three curves nearest their origin:—

$$\text{Case I.}\quad \frac{dv}{dt} = {\cdot}00836\,v + {\cdot}000615\,v^2$$

$$\text{Case II.}\quad \frac{dv}{dt} = {\cdot}01626\,v + {\cdot}00065\,v^2$$

$$\text{Case III.}\quad \frac{dv}{dt} = {\cdot}01046\,v + {\cdot}00091\,v^2$$

85. *Secondly,* The concavity upwards of these curves of the "rate of cooling"—showing that the cooling increases faster than the temperature rises—gradually diminishes; and in all the three curves we find between 110° and 120° (centigrade), a space nearly straight, indicating a point of contrary flexure. Above 150° the curve is in all the three cases slightly convex upwards, showing a rate of cooling slower in proportion than the rise of temperature.

86. *Thirdly,* This last circumstance appeared to me to be deserving of an elaborate verification. I therefore applied the same formula of interpolation which I had used with success to represent considerable arcs of the statical curve of temperature (see Art. 67, Eq. (1.)), being what has been called ROCHE's formula, to represent the temperatures of the cooling bar in the higher parts of the primary curve of cooling.

87. In this I was successful, and I deem the matter of sufficient importance to show the coincidence between the original thermometric observations and the formulæ employed in each of the three cases. The times (t) are in each case reckoned from an arbitrary origin; and v is the excess of temperature above that of the air actually observed. [See Table VI.]

88. *Fourthly,* It will be seen that, within the limits of these tables, the observations are, upon the whole, well represented by the equations. Moreover, they confirm a result at which I had previously arrived from the projections, as to the law of cooling at higher temperatures, namely, that above 140° or 150° there is a gradual falling off in the rate of cooling, compared to the measure of temperature. For it is to be observed, that the equations employed to represent the primary curve of cooling coincide with the simple geometrical or logarithmic law, when the co-efficient of t in the *denominator* of the fraction (c of Art. 67) vanishes. When this co-efficient is positive, the progression is faster than geometrical; when negative, it is slower. Now, in each of the three cases c is negative,

TABLE VI.

CASE I.—$1\frac{1}{4}$-INCH BAR, NAKED.				CASE II.—$1\frac{1}{4}$-IN. BAR, COVERED.				CASE III.—1-INCH BAR, NAKED.			
Formula; $\log v = 2{\cdot}30471 - \frac{{\cdot}008133t}{1-{\cdot}00262t}$.				Formula; $\log v = 2{\cdot}35215 - \frac{{\cdot}01385t}{1-{\cdot}00007t}$.				Formula; $\log v = 2{\cdot}41255 - \frac{{\cdot}01051t}{1-{\cdot}00113t}$.			
t	v observed.	v calc.	Diff.	t	v observed.	v calc.	Diff.		observed.	v calc.	Diff.
minutes.	°	°	°	minutes.	°	°	°	minutes.	°	°	°
2·5	192·1	192·45	+0·35	7·5	177·0*	177·1	+0·1	9	207·4	207·5	+0·1
3	190·3	190·55	+0·25	8	174·3	174·3	0·0	10	202·4	202·4	0·0
4	187·15	187·0	−0·15	9	168·7	168·8	+0·1	11	197·45	197·4	−0·05
5	183·5*	183·45	−0·05	10	163·6	163·5	−0·1	12	192·65	192·6	−0·05
6	179·95	179·95	0·0	11	158·25	158·4	+0·15	13	187·9	187·9	0·0
7	176·55	176·55	0·0	12	153·4	153·4	0·0	14	183·25	183·3	+0·05
8	173·1	173·15	+0·05	13	148·4	148·5	+0·1	15	178·7	178·7	0·0
9	169·7	169·65	−0·05	14	143·75	144·0	+0·25	17·5	167·5*	167·8	+0·3
10	166·55	166·4	−0·15	15	139·3	139·3	0·0	20	157·55	157·55	0·0
14	153·45	153·7	+0·25	16	135·0	135·0	0·0	21	153·75	153·5	−0·25
15	150·45	150·55	+0·1	17	130·9	130·8	−0·1	22	149·9	149·7	−0·2
16	147·65	147·5	−0·15					23	146·1	146·0	−0·1
								25	138·9*	138·7	−0·2
* From interpolating curve.				* From curve.				* From curve.			

consequently the progression is slower than geometrical, and the curve of the "rate of cooling," in terms of v, is convex upwards, as already stated.

89. *Fifthly*, By satisfying the observations by equations, we have farther these advantages—(1.) We can, with approximate accuracy, extend the law of cooling somewhat beyond the limits of observation, though with caution; (2.) We can also obtain the values of $\frac{dv}{dt}$ in a ready and continuous manner. The higher parts of the curves in Plate V. have been deduced in this way, and thus the "rate of cooling" has been tabulated for temperatures higher than those actually observed; but such numbers, being more or less hypothetical, are distinguished by asterisks in the following Table, which in other respects includes the results obtained from the observations treated as has been already described.

TABLE VII.—Showing the "Rate of Cooling," $-\frac{dv}{dt}$ for Different Excesses of Temperature (v).

v.	Case I. 1¼ inch Bar, naked.	Case II. 1¼ inch Bar, papered.	Ratio to I.	Case III. 1 inch Bar, naked.	Ratio to I.
1	0·009	·017	1·74	0·0115	1·28
2	·019	·035		·0245	
3	·031	·054		·0395	
4	·043	·075		·056	
5	·057	·096		·072	
10	0·124	·203	1·64	·158	1·27
20	·275	·44	1·60	·34	1·24
30	·43	·72	1·67	·55	1·28
40	·60	1·01	1·68	·78	1·30
50	·80	1·30	1·62	1·01	1·26
60	1·01	1·62	1·60	1·25	1·24
70	1·21	1·95	1·61	1·52	1·26
80	1·42	2·27	1·60	1·77	1·25
90	1·63	2·60	1·59	2·04	1·25
100	1·84	2·95	1·60	2·33	1·27
120	2·27	3·67	1·62	2·92	1·28
140	2·80	4·40	1·57	3·50	1·25
160	3·18	5·08	1·60	4·03	1·27
180	3·48	5·75	1·65	4·50	1·29
200	3·78	*6·38	1·69	4·95	1·31
220	*4·04	*7·00	1·73	5·40	1·34
240	*4·29	*7·65	1·78	*5·85	1·36
260	*4·52	*8·28	1·83	*6·30	1·39
280	*4·75	*8·90	1·88	*6·72	1·42

The numbers marked thus * being the results of calculation, are to be regarded as more or less hypothetical, and increasingly so at the higher temperatures.

90. *Sixthly*, I will not attempt to account for the inflections of the curves of Plate V. on physical principles, farther than to remark that the rapid increase in the velocity of cooling with temperature in the lowest part of the scale is perhaps owing to the separate effects of cooling by radiation, and cooling by convection. It seems probable that a certain excess of temperature of the bar above the air is necessary to determine efficient atmospheric currents, and thus to accelerate the rate of cooling; that, in fact, there is an amount of viscosity in air, which it requires a certain elevation of temperature properly to overcome. I would also observe, that the cooling in Case I. is (at higher temperatures) less regular than in the two other cases, while in Case III. the logarithmic law is almost accurately observed at those temperatures. This is no doubt to be ascribed to the greater mass of the Bar No. I., compared to its radiating power, occasioning probably

sensible irregularities, depending on the primitive distribution of heat in the bar, and on the want of uniformity in the temperature of its transverse section. The nearer that we approach to the ideal of an infinitely slender bar, the more shall we escape those periodical irregularities (see Art. 25 of the former part of this paper), arising from the primitive distribution of heat in its substance, which no doubt gives rise to some of the peculiarities of the inflections in the curves of "rates of cooling." In particular, we may naturally ascribe, in part at least, to the fact that the bar is heated first of all to a uniform temperature throughout in the fusible metal bath, the relatively diminished rate of cooling observed at the highest temperatures. At the same time I would repeat the caution, that the hypothetical or dotted portion of those curves cannot be relied on as expressing an actual fact, at least to more than a little way beyond the range of experiment.

§ III. *On the Proportion of Heat dissipated from the Bar by Radiation and Convection.*

91. Although not of direct importance to the determination of conducting power, I will indicate shortly how the numbers in Table VII., may be used to ascertain the relative amount of heat lost by radiation and convection at any or all points of the surface of the bar in Cases I. and II. The method was originally due to Sir John Leslie, but was stated more clearly by Dalton (System of Chem. Philosophy, p. 115), and was happily applied by Dulong and Petit. Suppose the total "rate of cooling" of the same bar to be ascertained in air, first, when it is naked, and, secondly, when covered with paper, and let the ratio of the first case to the second be as $1:p$. Next, by comparing after the manner of Leslie's canister-experiments the "emissive power" of the same two surfaces, iron and paper, let it be as $1:q$. Let the required ratio of the heat lost by convection to that lost by radiation be as $1:x$ in the first case; then, of course, it will be in the proportion of $1:qx$ in the second. But as the heat dissipated in each case is the sum of the effects due to convection (which is always $=1$), and that due to radiation, we have

$$1:p=1+x:1+qx$$

and

$$x=\frac{p-1}{q-p}$$

92. I have given in Table VII. the ratios of cooling, at different temperatures, for Cases I. and II., that is, for the same bar covered with paper and naked iron; and though the ratios vary somewhat,* yet they agree pretty nearly within the

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* Since this was written, I have observed that a like diminution of the ratios of cooling from glass and silver up to a certain point, and afterwards an increase, was noticed by Dulong and Petit, in their admirable Memoir on the Law of Cooling, page 102.—*Mem. Acad. Sci. Par.*

safe limits of observation. In fact, if we compare the average ratio from 10° to 100° Cent., and again from 100° to 200° Cent., we shall find them to be almost identical. They give for the value of p, the number 1·6023. This represents the proportion in which the papered bar dissipates its heat more rapidly than the naked bar.

93. For the direct radiating or emissive power of the two surfaces, I had recourse to the kind aid of Mr BALFOUR STEWART, not having had recently conveniences for making the experiment myself. He used the thermo-electric pile, and he found the experiment to be attended with considerably greater difficulty than is commonly attributed to it. I believe that Mr STEWART is not yet satisfied as to the reliability of his methods of observation; but the four best series of experiments made in February and March 1864, gave the emissive power of paper compared to iron as 5·8 to 1.* The value of q is therefore 5·8.

94. Hence by the previous investigation—

The value of x, the heat dissipated by radiation from naked iron (the dissipation by convection being always$=1$) is $x=\frac{\cdot 6023}{5\cdot 8-0\cdot 60}=0\cdot 116$. In the case of the paper surface, x is 5·8 times greater, or$=0\cdot 673$. In other words, of the heat dissipated from the bar in Case I., nearly $\frac{9}{10}$ths are lost by convection, and $\frac{1}{10}$th by radiation. In the paper-covered bar (Case II.), only $\frac{6}{10}$ths are lost by convection, and $\frac{4}{10}$ths by radiation.†

95. From this it appears that the principal agent in the dissipation of heat in these experiments is Convection and not Radiation; nay, that the effect of the latter is comparatively almost insensible, when naked metallic bars are used. This of itself tends to explain the systematic deviation of the statical curve of temperature (Art. 64) from the logarithmic law. The experiments of DULONG and PETIT show that the dissipation of heat due to Convection increases not as the excess of temperature simply, but as its $\frac{5}{4}$th power nearly (more exactly 1·233). This accords so far with what has been said of the variation in the rate of cooling in Art. 84; but it gives no adequate explanation of the inflections of the curves of Plate V. at higher temperatures. Were it not for the unquestionable precision of DULONG's admirable experiments, in which the law of cooling due to the contact of air was verified as high as 260° Cent., one might have not unreasonably supposed that the energy of convection was *relatively* less at higher temperatures.

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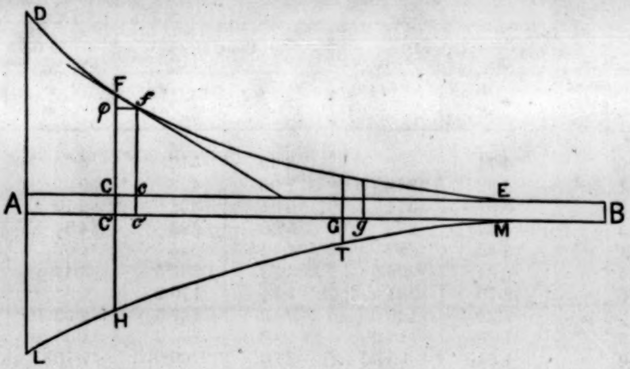
* This corresponds nearly to the relative emissive power of glass and polished silver used by DULONG.

† For in Case I. the whole heat lost from a point having a given temperature being represented by the number 1·116, that due to Convection is 1, that due to Radiation is ·116. In Case II. the total loss is 1·673, whereof 1 is due to Convection, and ·673 to Radiation.

§ IV.—*The "Statical Curve of Cooling." Recapitulation and Application of the Method of Deducing the Conductivity.*

96. It will be convenient here to recapitulate, from Arts. 5, &c. of the former part of this paper, the use which is to be made of the data obtained from the two fundamental experiments described in the previous sections, namely, the determination of the Curve of Statical Temperatures (Table II. Art. 61), and the rate or velocity of Cooling of the bar at any temperature (Table VII. Art. 89).

97. Let A B be the bar, kept hot at the extremity A, and left to assume a permanent temperature at its various points under natural causes. Let the upper curve, or DFE, represent by its ordinates (as FC) these temperatures. All the heat that enters the bar at A, and is propagated along it, has to be accounted for. Since the bar is so long that at the end B the heat has become insensible, the entire heat entering the bar at A has been dissipated from its surface in various proportions, according to its temperature, between the point A and some remote point E where the elevation of temperature is practically insensible. In like manner, if we take any point C in the bar, the heat transmitted from the hotter end, by conduction across the transverse section of the bar at C, is dissipated by the cooling of the bar between C and E. To know the quantity of heat passing across this transverse section, we have therefore to ascertain the aggregate loss of heat from the surface of the bar to the right hand of C.



98. To do this, we must construct what I call the *Statical Curve of Cooling*, which is represented in the same figure by the curve LHM, beneath the bar AB. The ordinate C′H represents the heat lost by the bar *per minute*, from the portion CC′, whose temperature is represented by CF in the upper curve. This loss or C′H, is found from Table VII., by entering it with the thermometer reading CF, which again is known from Table II. in terms of the position of the point C in the length of the bar. Thus all the ordinates of the Statical Curve of Cooling, LHM, can

be constructed. Dividing the length of the bar into sections, in the three experimental cases so often referred to, the ordinates of the curve of statical cooling, or values of $-\frac{dv}{dx}$, appropriate to every point of the bar, will be found as in the following Table:—

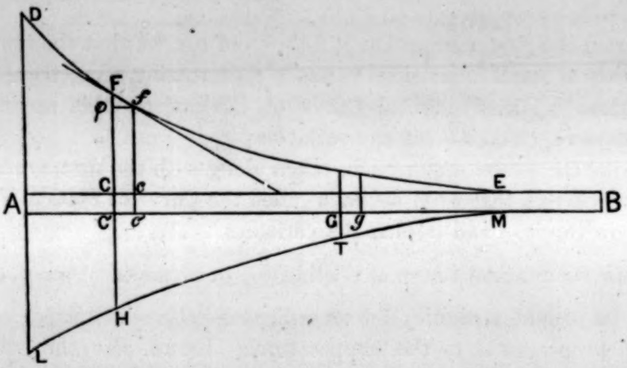
TABLE VIII.—SHOWING THE RATE OF COOLING PROPER TO EACH POINT OF THE LENGTH OF THE BAR (OR ORDINATES OF THE *Statical Curve of Cooling*), CONTAINING ALSO THE VALUES OF $-\frac{dv}{dx}$.

Distance from Origin in Feet and Inches.	CASE I.		CASE II.		CASE III.	
	$-\frac{dv}{dt}$	$-\frac{dv}{dx}$	$-\frac{dv}{dt}$	$-\frac{dv}{dx}$	$-\frac{dv}{dt}$	$-\frac{dv}{dx}$
Ft. In.	°	°	°	°	°	°
„ 0	4·71	420	8·30	512	6·75	512
„ 1	4·32	362	7·06	423	5·92	432
„ 2	3·97	314	6·05	351	5·19	366
„ 3	3·64	272	5·20	292	4·54	310
„ 4	3·32	237	4·41	245	4·00	264
„ 5	3·01	206·8	3·73	206·5	3·44	226
„ 6	2·70	181·0	3·14	175·2	2·93	193·6
„ 7·5	2·20	148·8	2·48	137·6	2·33	154·6
„ 9	1·80	123·0	1·98	109·0	1·85	124·1
I. 0	1·245	85·35	1·28	69·9	1·185	81·6
6	0·620	43·70	0·58	35·2	0·55	38·0
II. 0	0·342	24·47	0·282	16·3	0·258	19·64
III. 0	0·114	8·15	0·070	4·47	0·070	5·52
IV. 0	0·043	3·34	0·021	1·36	0·0185	1·69
VI. 0	0·008	0·57				

N.B.—The values of $-\frac{dv}{dx}$ are computed by the method explained in Art. 71. Only at some of the lower temperatures a mixed method of calculation and projection has been used.

99. It is evident that, if we can effect the quadrature of successive sections of the statical curve of cooling, continued until it vanishes in the direction of the cool end of the bar, we shall have got the "flux of heat" across the section of the bar at which the quadrature commences. The measure of the heat expressed by the area of the curve in question will have for unit the amount of heat required to raise unit of volume (1 cubic foot) of iron by 1° Cent. The shaded curve, in the lower part of Plate III., shows the Statical Curve of Cooling proper to Case I. The ordinates of the curve are related to those of the curve of statical temperature immediately above it, by the relation of $-\frac{dv}{dt}$ to v, shown in the secondary curve of cooling in the upper figure of Plate V.

100. The flux of heat is greatest in the hottest part of the bar, because the temperature of the bar varies most rapidly there, and the heat is more rapidly drawn towards the cold end. To give exact expression to the tendency of the heat to traverse the section of the bar at C, we will take C*c* to represent the thickness of a plate, bounded by imaginary parallel surfaces, situated transversely within the bar through which the flow of heat is to be considered. This is to be compared with the flow of heat across any other plate, G*g*, of equal thickness, in a different part of the bar. Then, according to FOURIER, the flow of heat across C*c* will be proportional to the small decrement of temperature Fφ, by which the side of the plate nearest to A is hotter than the farther side, and to the Conductivity jointly. The value of this decrement, Fφ, is evidently nothing else than the differential coefficient $\frac{dv}{dx}$, which has been given in the last Table, as derived from the equations to the curve of statical temperature in Art. 71.



101. Hence (in conformity with Arts. 7, 31, and 35 of the first part of this paper),

$$\text{Flux of heat, or area CFE} = -\frac{dv}{dx} \times \text{conductivity},$$

or

$$\text{Conductivity} = \frac{\text{Area CFE}}{-\frac{dv}{dx}}.$$

§ V.—*The Method of this paper applied, under the usual assumptions made in the Theory of Conduction, as a first approximation to the determination of Conductivity.*

102. The area of the statical curve of cooling to the right hand of any ordinate is therefore to be found. It will be convenient, for this purpose, to show what the nature of this curve would be were the *usual assumptions* of the mathematical theory of Heat adopted. These assumptions are (1.) That the superficial

loss of heat follows NEWTON'S law, or that the loss of heat in unit of time varies simply as the excess of temperature; (2.) That the same law holds for the internal communication of heat, or that the quantity of heat conducted is proportional simply to the difference of temperature of two adjacent elementary portions of a bar.

103. From the *first* assumption it follows, of course, that the temperature of a cooling body of small dimensions varies in a decreasing geometrical progression with the time. The dynamical Curve of Cooling on this assumption is a logarithmic curve, t and v being the variables.

104. From the *second* assumption, taken along with the first, we learn from a well-known analysis, that what we have called the Curve of Statical Temperature is also a logarithmic, x and v being the variables.

105. Now the Statical Curve of Cooling ($\frac{dv}{dt}$, in terms of x) must, on these assumptions, be also logarithmic; for its ordinates—the velocities of cooling—are everywhere proportional to the temperature. Hence also the subtangent to these two last curves* is the same. Let it be M. Then by a property of the logarithmic curve (M being the modulus) the area of the curve bounded by an ordinate y, and carried to infinity, is My. Also the flux of heat corresponding to the position of the ordinate y is (Art. 99)= My, y being, as we have seen, $=-\frac{dv}{dt}$. But, by Art. 102, $-\frac{dv}{dt}$ is everywhere *assumed* (for the present) to be proportional to v, or $-\frac{dv}{dt}=pv$. Also since the dynamical curve of cooling is a logarithmic (103), let its modulus be m. Then, by the property of the curve, $-\frac{dv}{dt}=\frac{v}{m}$. Hence, comparing the last two equations $p=\frac{1}{m}$. And,

$$\mathrm{F} = \text{Flux of heat} = \mathrm{M}y = -\mathrm{M}\frac{dv}{dt} = \mathrm{M}\frac{v}{m}$$

and (by Art. 101).

$$\text{Conductivity} = \frac{\mathrm{F}}{-\frac{dv}{dx}} = \frac{\mathrm{M}\cdot v}{-m\cdot\frac{dv}{dx}}$$

But the curve of statical temperature being also assumed to be logarithmic (104); and consequently

$$-\frac{dv}{dx}=\frac{v}{\mathrm{M}};$$

we finally get

$$\text{Conductivity} = \frac{\mathrm{M}v}{m\cdot\frac{v}{\mathrm{M}}} = \frac{\mathrm{M}^2}{m}$$

106. A first approximation to the conductivity of the bar may therefore be found *by dividing the square of the modulus or subtangent of the statical curve of*

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* Namely, the Curve of Statical Temperature and the Statical Curve of Cooling, being the two curves shown in the wood-cut of last page.

Temperature (assumed to be logarithmic) *by the modulus of the Dynamical Curve of Cooling.*

107. Thus, to illustrate this by a numerical example, were we to attempt to reduce the statical curves of temperature of Table II. to logarithmics after the manner of Biot, we should probably find the following approximate values of the subtangent M:—

	Case I.	Case II.	Case III.
M	0·9 foot	0·7 foot	0·8 foot

And from Table V. of the Dynamical Curves of Cooling, the subtangents might be nearly

m	60 min.	40 min.	50 min.

whence

$\frac{M^2}{m}$	·0135	·0122	·0128

which, it is seen, give nearly approaching values of the conductivity.

§ VI. *Final Determinations of the Conductivity of Iron at various Temperatures.*

108. The results given in the last section are in the highest degree rude, and are introduced merely to illustrate the general form of the method. The curves of Temperature and Cooling are neither of them sensibly logarithmic, and therefore we have found the necessity of dividing them into small portions, and taking their elements from point to point. Therefore, in continuation of what has been said in Art. 99, we must proceed to the quadrature of the Statical Curve of Cooling whose elements are given in Table VIII. This is a curve which though not logarithmic, may, like the other curves we have already discussed, be treated as if it had been, when divided into numerous elements bounded by parallel ordinates. Every one of these segments may have its area estimated by the simple formula proper to a logarithmic curve,* and for the infinite branch a similar formula must be adopted.

109. The following Tables contain the determination of the total Flux of Heat (F) across any section of the bar by the summation of the areas of the statical curve of cooling, commencing from the colder end of the bar, where this curve is (like the primary curve of temperatures) apparently asymptotic. In these Tables (corresponding to the three experimental Cases discussed in this Memoir, the chief uncertainty attaches to the two extremities of the curve. There are difficulties inherent in the precise determination of very small excesses of temperature of a bar, whether in a statical or a cooling condition, above the surrounding air, itself not absolutely constant in temperature. These difficulties have been previously referred to. Moreover, when we have to take the ratio of two quantities, both to be experi-

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* Namely, area between ordinates y and $y' = M\,(y' - y)$ where M the subtangent equals

$$\frac{0{\cdot}4343\ \&c.\ (x - x')}{\log y' - \log y}$$

mentally determined, and both in an almost evanescent state (as is the case in the extreme portion of the curve of statical temperature and of the statical curve of cooling), the quotient may be sensibly in error. To this I add, that in Case I. the length of the bar was certainly not quite sufficient to allow the conducted heat to be entirely spent by dissipation. Consequently there is, as it were, a slight *congestion* of heat towards the extremity—very slight indeed, but still sufficient to give to the subtangent there a too large value, and consequently to the decrement of the primary curve of temperature too small a one. Hence the ratio $\frac{F}{\frac{dv}{dx}}$ is somewhat too great, both in consequence of the numerator being too large and the denominator too small. But how little any such ambiguity can effect the general evaluation of the flux of heat in the succeeding lines of the Table, either in the case of this or of the two succeeding experiments, will be seen by noticing the minuteness of the areas representing the flux which correspond to the extreme portions of the curves. They are so small, that an error amounting to one-half their amount, would hardly affect by $\frac{1}{300}$th or $\frac{1}{400}$th part the measure of the conductivity in the middle and more important part of the Tables.

TABLE IX.—CASE I. $1\frac{1}{4}$-INCH IRON BAR, NAKED. CALCULATION OF AREA OF STATICAL CURVE OF COOLING (F), AND OF THE CONDUCTIVITY AT DIFFERENT TEMPERATURES.

Limits of Abscissæ.		Limits of Ordinates.		M=Subtangent.*	Area $M(y'-y)$.	Total Area F.	$-\frac{dv}{dx}$	Conductivity, $\frac{F.}{-\frac{dv}{dx}}$	Corresponding Actual Temp. Cent. $(v+13)$.
x.	x'.	y.	y'.						
Ft. Inch.	Ft. Inch.								°
∞	VI. 0	$\overset{\circ}{0}$·	$\overset{\circ}{0}$·008	1·662	0·0133	...	...	...	...
VI. 0	IV. 0	·008	·043	1·189	·0416	0·0549	3·34†	·0164	17
IV. 0	III. 0	·043	·114	1·026	·0728	·1277	8·15†	·0157	22
III. 0	II. 0	·114	·342	·9104	·2075	·3352	24·47	·0137	37
II. 0	I. 6	·342	·620	·8403	·2336	·5688	43·7	·0130	53
I. 6	I. 0	·62	1·245	·7175	·4484	1·0172	85·35	·0119	85
I. 0	0 9	1·245	1·80	·6777	·3762	1·3934	123·0	·0113	110
0 9	„ 7·5	1·80	2·20	·6233	·2493	1·6427	148·8	·0110	127
„ 7·5	„ 6	2·20	2·70	·6100	·3050	1·9477	181·0	·0107	147
„ 6	„ 5	2·70	3·01	·7669	·2378	2·1855	206·8	·0105	163
„ 5	„ 4	3·01	3·32	·8515	·2640	2·4495	237·1	·0103	182
„ 4	„ 3	3·32	3·64	·9047	·2895	2·7390	272·4	·0100	203
„ 3	„ 2	3·64	3·97	·960	·3168	3·0558‡	313·7	·0097‡	228
„ 2	„ 1	3·97	4·32	·986	·3451	3·4009‡	362·5	·0093‡	256
„ 1	„ 0	4·32	4·71	·965	·3764	3·7773‡	420·0	·0090‡	288
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)

* From the formula $0{\cdot}4343 \times \frac{x-x'}{\log y' - \log y}$

† From curve; the rest from equation. ‡ More or less uncertain.

TABLE X.—CASE II. 1¼-INCH IRON BAR, COVERED WITH PAPER. CALCULATION OF AREA OF STATICAL CURVE OF COOLING (F), AND OF THE CONDUCTIVITY AT DIFFERENT TEMPERATURES.

Limits of Abscissæ.		Limits of Ordinates.		M.*	Area M $(y'-y)$.	Total Area F.	$-\frac{dv}{dx}$	Conductivity, $\frac{F.}{-\frac{dv}{dx}}$	Corresponding Actual Temp. Cent. $(v+13)$.
x	x'	y	y'						
Ft. In.	Ft. In.	°							°
∞	IV. ,,	0·	·021	·980	·0206	...	...	...	...
IV. ,,	III. ,,	·021	·070	·8305	·0407	·0613	4·47†	·01372	17
III. ,,	II. ,,	·070	·282	·7177	·1521	·2134	16·3	·01310	26
II. ,,	I. 6	·282	·58	·6935	·2066	·4200	32·5	·01292	37
I. 6	I. ,,	·58	1·28	·6317	·4422	·8622	69·9	·01234	62
I. ,,	,, 9	1·28	1·98	·5728	·4020	1·2642	109·0	·01160	84
,, 9	,, 7·5	1·98	2·48	·5551	·2776	1·5418	137·6	·01120	99
,, 7·5	,, 6	2·48	3·14	·5301	·3499	1·8917	175·2	·01080	119
,, 6	,, 5	3·14	3·73	·4840	·2855	2·1772	206·5	·01054	134
,, 5	,, 4	3·73	4·41	·4978	·3385	2·5157	245	·01027	153
,, 4	,, 3	4·41	5·20	·5055	·3993	2·9150	292	·00998	176
,, 3	,, 2	5·20	6·05	·5500	·4675	3·3825‡	351	·00964‡	202
,, 2	,, 1	6·05	7·06	·5401	·5455	3·9280‡	423	·00929‡	234
,, 1	,, 0	7·06	8·30	·5147	·6383	4·5663‡	512	·00892‡	273
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)

* $M = 0{\cdot}4343 \frac{x-x'}{\log y' - \log y}$

‡ More or less uncertain.

† The values of $\frac{dv}{dx}$ are all from equations, and they all agree satisfactorily with projection.

TABLE XI.—CASE III. 1-INCH IRON BAR, NAKED. CALCULATION OF AREA OF STATICAL CURVE OF COOLING (F), AND OF THE CONDUCTIVITY AT DIFFERENT TEMPERATURES.

x	x'	y	y'	M	Area M $(y'-y)$.	Total Area F.	$-\frac{dv}{dx}$	Conductivity, $\frac{F.}{-\frac{dv}{dx}}$	Actual Temp. Cent $(v+11)$.
Ft. In.	Ft. In.	°							°
∞	IV. ,,	0·	0·0185	·820	·0152	...	...	...	...
IV. ,,	III. ,,	·0185	·070	·7515	·0387	·0539	5·52	·00977	16
III. ,,	II. ,,	·070	·258	·7667	·1441	·1980	19·64	·01008	27
II. ,,	I. 6	·258	·55	·6605	·1928	·3908	38·0	·01029	41
I. 6	I. ,,	·55	1·185	·6515	·4137	·8045	81·6	·00986	68
I. ,,	,, 9	1·185	1·85	·5612	·3733	1·1778	124·1	·00949	94
,, 9	,, 7·5	1·85	2·33	·5419	·2600	1·4378	154·6	·00930	111
,, 7·5	,, 6	2·33	2·93	·5456	·3274	1·7652	193·6	·00912	132
,, 6	,, 5	2·93	3·44	·5192	·2648	2·0300	226·0	·00898	149
,, 5	,, 4	3·44	4·00	·5526	·3094	2·3394	264·4	·00885	170
,, 4	,, 3	4·00	4·54	·6580	·3553	2·6947	310·5	·00868	193
,, 3	,, 2	4·54	5·19	·6229	·4048	3·0995*	366	*·00847	221
,, 2	,, 1	5·19	5·92	·6339	·4627	3·5622*	432	*·00824	254
,, 1	,, 0	5·92	6·75	·6349	·5270	4·0892*	512	*·00799	293
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)

* More or less uncertain.

110. Uncertainty, I have already said, attends the determinations of conductivity for the higher as well as those at the lowest temperatures. In fact, the former are (as will have been seen from the details already given) the results of analogies rather than of direct experiments. The experiments, whether Statical or Dynamical, rarely extended beyond a temperature of 200°, or at most 220° Cent. The results have been here carried out by the analogies afforded by the equations to the curves to nearly 300°. Nevertheless, the continuity of the law of conductivity diminishing with temperature, is consistently brought out by these approximations.

111. In the preceding Tables the conductivity is expressed in terms of the amount of heat as unity, which is required to raise the temperature of one cubic foot of iron, by one degree Cent. It expresses the amount of heat reckoned in such units which would traverse in one minute across an area of one square foot, a plate of iron one foot thick, with the two surfaces maintained at temperatures differing by 1° Cent.

112. If we now project the values of the conductivity of iron found in the last column but one of the three preceding Tables in terms of the thermometric temperatures (Centigrade) in the last columns, we are enabled to trace easily the connected results of the whole inquiry. (See Plate V. fig. 4.)

113. We find that in each case *the conductivity diminishes as the temperature increases;* and *that*, for the next part, in a progressive manner. The variation with temperature is clearly most rapid at the lower temperatures.

114. The two first series agree very closely in their numerical results, with the exception of certain irregularities in the part of the curve where the temperatures are lowest; which have already been in part accounted for (Arts. 55, 65, 109). These two series belong to one and the same bar, though cooling under very different circumstances, owing to the largely increased radiating power conferred upon it by coating it with paper. And the value of the striking coincidence in the numerical results in Tables I. and II. is enhanced by the consideration, that the numbers expressing the conductivity are obtained by taking the ratios of two different columns (7 and 8), which in the two Tables differ most widely, and the result cannot be even guessed at until the ratio is actually taken.

115. The third series (Table X.) leads to numbers very sensibly differing from the two first series, yet following the same general law, the conductivity decreasing with temperature (excepting at the lowest part of the scale, where we find an anomaly corresponding to that noted in an early part of this paper (Art. 65), showing that the lowest portion of the statical curve has not in this instance been satisfactorily determined). The conductivity in Table X. is smaller throughout than in the two former cases. It is believed that this can be satisfactorily accounted for by the different quality of the iron of which this

bar was made, which came from a different manufactory, and was probably inferior in quality.[*]

116. Tracing an interpolating curve through the projected observations of Cases I. and II., which run nearly parallel and at no great distance, at temperatures superior to 40° and do not diverge even in the higher and more hypothetical part of the diagram, and doing the same separately for Case III., we obtain the following numbers, purely as results of observation:—In the first column of each division of Table XII., we have the ratio $\frac{F}{-\frac{dv}{dx}}$, which expresses the conductivity in terms of the heat required to raise a cubic foot of iron by one degree Centigrade. In the two following columns, we have the same reduced to the usual standard of conductivity in French and English measures respectively.[†]

TABLE. XII.

Temp. Cent.	Cases I. and II.			Case III.		
		Conductivity.			Conductivity.	
	$k=\frac{F}{-\frac{dv}{dx}}$	Units: Foot, Minute and Cent. Deg.	Units: Centimetre, Minute, Cent. Deg.	$k=\frac{F}{-\frac{dv}{dx}}$	Units: Foot, Minute, Cent. Deg.	Units: Centimetre, Minute. Cent. Deg.
0	·01506	·01337	12·42	·01117	·00992	9·21
25	·01391	·01235	11·48	·01062	·00943	8·79
50	·01288	·01144	10·63	·01014	·00904	8·37
75	·01205	·01070	9·94	·00974	·00865	8·04
100	·01140	·01012	9·40	·00940	·00835	7·76
125	·01088	·00966	8·98	·00916	·00813	7·56
150	·01052	·00934	8·68	·00895	·00795	7·38
175	·01018	·00904	8·39	·00877	·00779	7·23
200	·00987	·00876	8·14	·00860	·00764	7·10
225	·00958	·00851	7·90	·00844	·00749	6·96
250	·00930	·00826	7·67	·00826	·00736	6·84
275	·00902	·00801	7·44	·00815	·00724	6·72

117. The coincidence of the results in the second column with the results of the provisional reduction in the case of the 1¼-inch bar, made in 1852, and printed at Art. 33, page 144, of the former part of this paper, is both striking and satisfactory. For it shows, as I there anticipated (Art. 38), that the method is, to a great extent, independent of the ordinary instrumental errors,

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[*] Dr Matthiesson in his Experiments on the Electric Conductivity of Iron (Phil. Trans., 1863), has found nearly equally wide variations in different specimens.

[†] If the numbers in the first column of each division of the Table be called A, then A × ·888 will express the conductivity in water-measure for the foot, minute, and Cent. degree; and A × 825 gives the numbers in the third column,. where the centimetre is substituted for the foot.

and even of the laborious computations which have formed the basis of the present paper.

118. In the preceding Table I have completed the series for lower temperatures, where the observations were less accordant, in the following way:—I have assumed that the most trustworthy part of the observational curves are those between the actual temperatures of 40° or 50° and 150° or 160°, and that within moderate limits, the conductivities (k) may be represented in terms of the temperature (t), by such a formula as

$$k = A + at + bt^2$$

In the case of the $1\frac{1}{4}$-inch bar, I find for these constants

$$A = \cdot 01506 \qquad a = -\cdot 0000488 \qquad b = +\cdot 000000122$$

From which the conductivities corresponding to 0° and 25° have been interpolated. In the case of the 1-inch bar the constants are—

$$A \cdot 01117 \qquad a = -\cdot 0000235 \qquad b = +\cdot 000000058.$$

119. I must here observe, however, that the above form of relation between k and t, which has been applied by Dr Matthiessen, in his extensive and important researches on electric conductivity, does not satisfy the form of our conductive curves, Plate V. fig. 4, except through a limited range. I have reason, however, to think, that down to 0° of temperature it may be sufficiently exact. The "percentage decrement" of the conductivity between 0° and 100° is 24·5 for the larger bar of iron, and 15·9 for the smaller one. As in the case of Dr Matthiessen's electrical experiments, the "percentage decrement" diminishes with the conducting power, and in almost exactly the same proportion.* The numerical values in either case are, however, considerably smaller for heat than those obtained by Dr Matthiessen for electricity.

120. With this exception, however, there is an agreement in the character of the metals (so far as is yet known) in conducting heat and electricity. (See Art. 2 of this paper.)

§ VII.—*Concluding Remarks and Suggestions.*

121. In Art. (15) of the first part of this paper, I expressed my desire to afford to future experimenters every aid I possibly could to resume and extend my observations (confined, unfortunately, to only one metal—iron), and to furnish them with such advantages as my experience afforded, as well in methods of observation as of reduction.

122. It was especially with this view that I have spent what may perhaps appear an undue amount of labour on the reduction of the experiments considered

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* Phil. Trans. 1863, p. 380.

in the present paper. I do not, however, regard this labour as wasted, for the knowledge thus acquired of the nature of the remarkable curves of which it treats will enable a future observer to attack the question in a far more direct manner, and to obtain, with comparatively little trouble, numerical determinations of the conductivity of the metals under ordinary circumstances, and adapted to most purposes of theory or practice.

123. *Suggestions as to Experiments.*—After mature consideration, I do not think that the experimental methods require almost any modification. The independence of the results of any moderate error in the thermometers seems satisfactorily proved (Arts. 38 and 117); and if the object be merely to ascertain the conductivity and "percentage decrement" for a number of metals, it may easily be done without pushing the observations to the high temperatures used in my experiments, which are always a fertile source of difficulty and error. If, for instance, an extreme temperature of 120° or 140° Cent. only was aimed at, shorter bars might be used; the heat would be more manageable and more quickly attained; the thermometers would be more easily made, more easily used, and subject to far smaller corrections; and the dynamical experiments especially, would be freed from an anxious and troublesome source of error, arising from the irregularity of the primitive distribution of the heat in the cooling bar (Arts. 25, 26, and 90).

124. A more exact knowledge of the form of the statical curve of temperature in any case may be obtained by using sources of heat of progressively lower temperature, as explained in Arts. 27 and 58.

125. It is probable that very good results might be obtained by simply using boiling water as a source of heat at the hottest end of the bar, than which nothing can be more manageable. The duration of the statical experiments could thus be much reduced, and the temperature of the air of the apartment rendered more stable. The difficulties referred to in Arts. 65, 109, as to the determination of very small excesses of temperature next the cool end of the bar might thus be in a great measure removed. Indeed, it would be a worthy object of study, in a theoretical point of view, to determine the form of the Statical and Dynamical curves for those low temperatures more accurately than I have done. I cannot but suspect an anomaly in the conduction of heat when the temperature varies with extreme slowness from point to point, which my observations rather indicate than establish.*

126. Another experimental point of interest for the theory would be to estab-

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* I may be allowed to state here generally, that this anomaly would apparently assign a too great conducting power to iron at low temperatures than we can readily admit. [The case of the 1-inch bar might rather lead to an opposite conclusion, but I have less confidence in the observations made on it for very small excesses of temperature.] Both the statical curve and the curve of cooling deviate more and more from the logarithmic form as the temperature-excesses diminish.

lish, for a few points of a metallic bar, the difference between the superficial and the internal temperature of the bar in any transverse section. This might be done by thermo-electric methods, such as, I think, were used by the late M. LANGBERG of Christiania in his experiments on the conduction of heat in bars. I made some attempts (which were not unpromising) in a different way, by applying to the surface of the bar small portions of fusible alloys or other substances, liquefying at definite temperatures. There did not appear to be much difficulty by gently sliding these proof-pieces along the bar from the cooler towards the hotter part, of ascertaining with considerable precision the co-ordinate of the superficial point, corresponding to the fusing temperature of the alloy or other substance used. The five following substances, in a descending scale, were found to have tolerably definite fusing-points, and to be sufficiently suitable for the experiment:—Tin; solder (tin 9 parts, lead 5 by weight); fusible metal (consisting of bismuth 2 parts, lead 1 part, and tin 1 part by weight); napthalic acid; and bees-wax. The fusing temperatures of the *three* first were carefully ascertained by direct experiment to be—

Tin,*	.	229°·0 Cent.	=444°·2 Fahr.
Solder,	.	181°·6 ,,	=358°·9 ,,
Fusible metal,		94°·15 ,,	=201°·4 ,,

The fusing points of the others were not ascertained by me.

127. The experiments which I made in this manner were entirely tentative and preliminary. The following is a specimen:—Statical experiment; 1851, March 14. $1\frac{1}{4}$-inch bar, naked [see Table I., page 78 of this paper.] " At 1^{h} 40^{m} I tried the following experiment to test the difference of temperature of interior and exterior of bar. Taking small sharp-pointed pieces of tin [and] fusible metal (prepared on purpose, bismuth 2, lead 1, tin 1 by weight), I rubbed them gently on the surface of the iron bar till I found the melting point, keeping them gently in motion so as not to allow the surface to heat beneath them. I fixed these points with very considerable exactness, in the case of the fusible metal (the best observation), to perhaps within $\frac{1}{20}$th inch. I did not find the position sensibly [to] vary on the centre of the top, and on the centre of the side of the bar, nor even towards the angle of the bar (with the fusible metal). These experiments deserve repetition.

" 1^{h} 40^{m} Tin melted when rubbed on the centre of one side of the bar, at . . . 0 ft. 0·65 in. } from origin at the edge of the crucible.
Fusible metal, . . 0 10·45 ,, ,, ,, ,, ,,
Bees-wax, . . 1 4·7 ,, ,, ,, ,, ,,

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* The melting point of tin seems to be one of the best determined of the higher temperatures. According to CRICHTON, Senior (of Glasgow), it is 442° Fahr. [T. THOMSON]; KUPFFER, 446°; DANIELL, 441°. On the melting point of lead, see Art. 70.

128. *Suggestions as to Reductions.*—Were any one desirous of pursuing the subject of the theory of conduction into its details, I should be disposed to recommend the employment of BIOT's formula of 5 constants (used to express the elasticity of steam), instead of ROCHE's, containing 3 constants, which we have here used, see Art. 66. The method of calculation (which is necessarily laborious), is given in REGNAULT's large treatise on the Theory of the Steam Engine.* For any merely practical purpose, however, this is not required. An experimenter desiring to compare the conductivity and "percentage decrement" of different metals, may reasonably confine his attention between the useful limits of 20° and 120°, or at most 140° Centigrade. For that interval, ROCHE's formula will suffice. And the chief use of the formula is to obtain readily and accurately the differential co-efficient $\frac{dv}{dx}$ (see Arts. 71 and 78), on the determination of which the value of the conductivity mainly depends.

129. Though I would not recommend the attempt to proceed by graphical methods *alone*, they are an invaluable help, and also serve as a check to the calculations. Where these are not made throughout in duplicate, the use of curves ensures the detection of any material error of the computer. The check by taking first and second differences should also not be disregarded. The curves of cooling may be treated in a similar way.

130. I believe, however, that very fair results might be rapidly and approximately obtained by graphical methods alone. The curves of Statical Temperature and of Cooling being first projected in the usual way, tangents might be drawn mechanically for ordinates successively differing by 10°. The ordinate divided by the subtangent found would give the numerical values of $\frac{dv}{dx}$ and $\frac{dv}{dt}$. They would no doubt be somewhat irregular from the clumsiness of the graphical process; but being projected in terms of x and v respectively, and equalizing curves drawn through them, fair results would be obtained.† The "statical curve of cooling" is then constructed without any calculation whatever; and for evaluating its area up to any limiting ordinate, it might be sufficient that the curvilinear space it encloses should be defined on writing paper and cut out with scissors: the successive portions being weighed, would represent the flux of heat in known

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* I ought perhaps to mention the formula which Professor Rankine has applied with success to express the elasticity of steam at all temperatures (Edin. Phil. Journ. 1849, vol. xlvii. p. 28, and Philos. Mag. 1854, vol. viii. p. 530). It is as follows:—

$$\log P = A - \frac{B}{\tau} - \frac{C}{\tau^2}$$

where P is the elasticity of vapour, and τ the temperature reckoned from an absolute zero (−274° cent). In applying the formula to the temperature of a bar, there can be no natural zero from which the lengths are reckoned along the bar; and therefore the constants, instead of three in number, may be reckoned as four; putting v instead of P in the above formula, and, instead of τ, writing $x+D$, D being some fourth constant. (See article 67.)

† This method was used by me in 1852.

units. I have little doubt that the results would come out within one or two hundredths of those obtained by elaborate calculations.

131. I have only to add, that the greater part of the computations in this paper were executed by Mr ALEXANDER PIRIE of St Andrews. Every part of the projections and graphical interpolations was performed by my own hand; and my thanks are especially due to the Messrs JOHNSTON for the unusual care with which they have been reduced in scale, and transferred to copper, as seen in the Plates.

ST ANDREWS, *April* 1865.

IX.—*Some Observations on the Cuticle in relation to Evaporation.* By John Davy, M.D., F.R.S. London and Edinburgh.

(Read 1st May 1865.)

Though it is generally admitted that the cuticle performs an important part in retarding and regulating evaporation from the surface of the body, yet I am not aware of any inquiry hitherto made to determine the fact with exactness.

On account of the importance of the subject, I have been induced to engage in it. The experiments instituted for the purpose have all been of a very simple kind and easily made. They were on the similar parts of dead animals, detached immediately or very soon after the animals had been killed. From one specimen in each instance, the cuticle with the cutis, or the cuticle alone, was removed; whilst from the other these parts of the integuments were left entire. Each was carefully weighed, and then suspended, exposed to the air, side by side. Day after day, with occasional interruptions, or hour after hour, the weighing was repeated, and the result in the loss sustained was noted down. The experiments were made, when not otherwise mentioned, as just described, in a room in which, except in the height of summer, there was commonly a fire by day, its temperature during the day and night varying from about 50° of Fahr. to 55° and 58°. The animals affording the subjects of the trials were the trout, frog, toad, hare, rabbit, pig, thrush, common fowl, blue tit. Even at the risk of tediousness, I shall give the results of the weighing in some detail,—exactness in such trials being the first thing necessary.

1. The Common Trout (*Salmo fario*). Two trouts were selected of the same size. From one (No. 1) the greater part of the skin was removed, when it weighed 112 grs. The skin of the other (No. 2) was left on entire; it weighed 267 grs. This was on the 20th October.

October	21.	No.	1	had lost	32·0	grs., or	28·5	per cent.
„	„	„	2	„	31·7	„	19·0	„
„	22.	„	1	„	55·3	„	50·6	„
„	„	„	2	„	66·0	„	39·0	„
„	24.	„	1	„	80·0	„	71·4	„
„	„	„	2	„	108·0	„	64·6	„
„	27.	„	1	„	82·8	„	74·0	„
„	„	„	2	„	120·8	„	72·0	„
„	28.	„	1	„	83·0	„	74·2	„
„	„	„	2	„	121·3	„	72·6	„
„	30.	„	1	„	83·1	„	74·28	„
„	„	„	2	„	121·8	„	72·90	„

Weighed again on the 31st, there was no further loss. Both were dry and rigid, and free from any unpleasant smell.

Another trial was made in the following manner:—a trout, just after it had been taken, October 26th, was divided in the line of the spine. From one moiety (No. 1) the skin was removed, when it weighed 117·7 grs. On the other (No. 2) the skin was left; it weighed 83·7 grs. The head had previously been detached and the fish eviscerated.

October	27.	No.	1	had lost	59·7	grs., or	50·7	per cent.
„	„	„	2	„	43·2	„	50·4	„
„	28.	„	1	„	88·7	„	75·3	„
„	„	„	2	„	63·2	„	75·5	„
„	30.	„	1	„	90·0	„	76·4	„
„	„	„	2	„	63·6	„	75·9	„

On the following day the weight of No. 2 was the same, that of No. 1 was ·1 gr. less; both were dry and rigid.

2. The Frog (*Rana temporaria*). A female on the 7th April was killed by decapitation. After the application of a ligature to each thigh, just at the junction with the pelvis, they were detached. From one (No. 1) the integuments were removed; it weighed 55·9 grs. On the other (No. 2) they were left; it weighed 69·9 grs.

April	8.	No.	1	had lost	17·2	grs., or	30·7	per cent.
„	„	„	2	„	20·4	„	29·2	„
„	9.	„	1	„	34·0	„	61·9	„
„	„	„	2	„	47·3	„	60·5	„
„	10.	„	1	„	39·0	„	70·0	„
„	„	„	2	„	48·2	„	69·0	„
„	11.	„	1	„	39·5	„	70·6	„
„	„	„	2	„	49·6	„	70·9	„
„	12.	„	1	„	39·7	„	71·0	„
„	„	„	2	„	50·0	„	71·5	„
„	13.	„	1	„	40·0	„	71·4	„
„	„	„	2	„	50·2	„	71·8	„

After this they sustained no further loss; on the contrary, the air being damper they gained slightly in weight. In the dry and rigid state to which they were reduced they were put into water. Taken out after three hours and wiped to remove adhering water, No. 1 had gained 10·8 grs., No. 2, 14·8 grs. Immersed again and left in twenty-four hours, each had recovered its original weight.

3. The Toad (*Bufo vulgaris*). A similar trial was made with the lower extremities of a large toad, killed on the 11th July, when in full vigour. The extremity (No. 1), deprived of its integuments, weighed 80 grs.; the other, with the integuments on (No. 2), 82·3 grs.

July 12.	No. 1	had lost 39·0	grs., or 48·7	per cent.
„ „	„ 2	„ 37·60	„ 45·5	„
„ 13.	„ 1	„ 51·90	„ 64·9	„
„ „	„ 2	„ 54·00	„ 65·2	„
„ 17.	„ 1	„ 53·07	„ 66·3	„
„ „	„ 2	„ 56·85	„ 69·0	„

Neither sustained any further loss from exposure. The thermometer during the time ranged from 65° to 68°.

4. The Rabbit (*Lepus cuniculus*). From a wild one recently killed the skin was removed from the under surface of one ear and left on the upper; it (No. 1) weighed 34·5 grs. Of the other (No. 2) the skin was left on both surfaces; this weighed 43·1 grs. They were placed on a stove, side by side, the under surface of each uppermost; a thermometer close to them was 93°.

In 1 hour 35 minutes	No. 1	had lost 15·5	grs., or 44·9	per cent.	It has become rigid.
„ „ „ „ „	„ 2	„ 3·4	„ 7·8	„	
„ 3 „ 35 „	„ 1	„ 19·1	„ 55·2	„	
„ „ „ „ „	„ 2	„ 6·0	„ 13·9	„	It is rigid only at its edge.
„ 10 „ 35 „	„ 1	„ 20·9	„ 60·5	„	
„ „ „ „ „	„ 2	„ 8·5	„ 19·7	„	
„ 24 „ 0 „	„ 1	„ 22·3	„ 64·6	„	
„ „ „ 0 „	„ 2	„ 16·2	„ 37·6	„	

No. 1 sustained no further loss; No. 2 continued to lose weight, gradually diminishing in flexibility up to 289 hours, when it had lost 28·1 grs., or 65·2 per cent., and had become hard and rigid.

5. The Hare (*L. timidus*). On the 5th November, from a hare recently killed, one ear (No. 1) was immersed in boiling water for a minute, after which, when cold, the integument was easily removed, this from the outer surface only; it weighed 81·8 grs. On the other ear (No. 2) the integuments were left entire; it weighed 109·7 grs. They were placed on paper on a stove, when the temperature was about 100°.

In 20 hours	No. 1	had lost 57·0	grs., or 69·6	per cent.	It had become shrivelled and hard.
„ „ „	„ 2	„ 7·7	„ 7·0	„	
„ 24 „	„ 1	„ 57·8	„ 70·6	„	It sustained no further loss.
„ „ „	„ 2	„ 37·5	„ 34·2	„	
„ 48 „	„ 2	„ 47·7	„ 43·5	„	
„ 72 „	„ 2	„ 58·9	„ 53·7	„	Is still tolerably soft and supple.
„ 96 „	„ 2	„ 61·5	„ 56·	„	Suppleness much impaired.
„ 192 „	„ 2	„ 67·6	„ 61·6	„	It is now hard and little flexible.

It should be mentioned that it was only by day that the temperature was kept up to about 100°.

6. The Pig (*Sus vulgaris*). As soon as killed, March 24th, a ligature was applied to one ear, and the portion included cut off; it (No. 1) weighed 128·5 grs. From the other ear a portion similarly included (No. 2) was cut off; after the application of boiling water and the removal of the cuticle, it weighed 159 grs. They were suspended fully exposed to the air.

March	30.	No. 1	had lost	8·1	grs., or	6·6	per cent.	
„	„	„ 2	„	56·0	„	36·6	„	
„	31.	„ 1	„	18·1	„	14·0	„	
„	„	„ 2	„	107·0	„	67·2	„	It is shrunk and rigid.
April	3.	„ 1	„	36·0	„	28·1	„	
„	„	„ 2	„	114·3	„	71·7	„	It sustained no further loss.
„	9.	„ 1	„	61·0	„	47·4	„	Its softness and suppleness diminishing.
„	12.	„ 1	„	71·6	„	55·6	„	Is now rigid.

7. The Barn-door Fowl (*Gallus domesticus*). As soon as killed, the wattles of a cock two years old were cut off, a ligature having been previously applied at the base of each. One (No. 1), from which the cuticle was scraped off, weighed 62·2 grs.; the other (No. 2), on which the cuticle was left, weighed 79·5 grs.

October	21.	No. 1	has lost	25·3	grs., or	40·6	per cent.	
„	„	„ 2	„	14·8	„	18·6	„	
„	22.	„ 1	„	35·6	„	57·2	„	It is hard and rigid.
„	„	„ 2	„	26·4	„	33·2	„	
„	24.	„ 1	„	41·7	„	67·0	„	
„	„	„ 2	„	42·5	„	53·4	„	
„	27.	„ 1	„	44·7	„	71·8	„	
„	„	„ 2	„	52·8	„	66·4	„	Excepting margin, still retains some flexibility.
„	30.	„ 1	„	45·2	„	72·3	„	
„	„	„ 2	„	55·6	„	70·0	„	Is quite rigid.

8. The trial was repeated on the legs of a fowl on the 18th May, when the temperature of the room was 65°, without a fire. They were separated at both their junctions, viz., femur and tarsus. One (No. 1), stripped of its integuments, weighed 835·5 grs.; the other (No. 2) its integuments on, the skin drawn over each stump and secured by a ligature, weighed 889 grs. This without the feathers, which had been removed.

May	19.	No. 1	has lost	115·5	grs., or	13·8	per cent.	
„	„	„ 2	„	8·5	„	·9	„	
„	20.	„ 1	„	201·5	„	24·0	„	
„	„	„ 2	„	14·5	„	1·6	„	
„	21.	„ 1	„	266·0	„	31·8	„	
„	„	„ 2	„	20·0	„	2·2	„	
„	22.	„ 1	„	320·0	„	38·3	„	Is free from any unpleasant smell.
„	„	„ 2	„	25·5	„	2·8	„	Is becoming putrid.
June	5.	„ 1	„	81·5	„	10·0	„	Not again weighed, owing to its putrid state.
„	„	„ 2	„	487·0	„	58·0	„	
„	17.	„ 2	„	522·	„	62·4	„	No further loss.

Comparing the results of the one covered with integument with those of the other deprived of it, apart from the vastly greater loss of water by evaporation, the other changes were strikingly contrasted. No. 1, that deprived of integument, excepting its loss of water and its hard, rigid state in consequence, seemed little altered; when moistened it was quite free from any putrid taint, and its muscles exhibited their striated structure with undiminished distinctness. No. 2, on the contrary, that on which the integument was left—that still retaining its toughness and little changed—was found, when an incision was made into the contained muscles, to be undergoing the putrefactive change, denoted by the sickening, putrid smell, the softening of fibre and the loss of striated structure, with the appearance of many crystals, chiefly four-sided prisms, which were pretty readily dissolved in dilute acetic acid.

9. The Martin (*Hirundo urbica*). A young bird, fledged, on the 14th July, was found dead, thrown out of its nest by the female bird, which had been forsaken by her mate, probably killed. The nestling weighed 230 grs. One of its thighs (No. 1), stripped of integuments, weighed 3·6 grs. ; the other limb (No. 2), the entire lower extremity with integuments on, weighed 7·6 grs.

July	15.	No. 1	had lost	2·3	grs., or	63·8	per cent.	It had no further loss.
„	„	„ 2	„	1·4	„	18·4	„	
„	16.	„ 2	„	2·3	„	30·2	„	
„	19.	„ 2	„	3·6	„	47·3	„	
„	22.	„ 2	„	3·9	„	51·3	„	
„	25.	„ 2	„	4·0	„	52·6	„	It sustained no further loss; proportionally it was so much less than that of No. 1, from having less muscle, more bone, &c.

10. The Thrush (*Turdus musicus*). A male was shot on the 15th July. Its leg, deprived of its integuments (No. 1), weighed 32·2 grs.; the other leg, with foot, the integuments left on, but without the feathers (No. 2), weighed 42·1 grs.

July	16.	No. 1	had lost	13·2	grs., or	41·0	per cent.	
„	„	„ 2	„	2·8	„	6·8	„	
„	17.	„ 1	„	19·2	„	59·6	„	
„	„	„ 2	„	4·6	„	10·9	„	
„	19.	„ 1	„	20·7	„	64·2	„	
„	„	„ 2	„	12·6	„	30·0	„	
„	27.	„ 1	„	21·0	„	65·1	„	No further loss.

It was rigid, and the muscles were well preserved; perfectly free from putrid taint. No. 2 was not weighed after the 19th; then examined, it was found full of magots of the flesh-fly, twenty-seven in number, all of about the same size, about ·4 inch in length; their weight was 16·6 grs. The muscles of the leg were entirely devoured; what remained, namely, skin and bone, weighed 12·9 grs. The larvæ were partially distended with putrid matter of muscle, in a semi-fluid state, of a reddish hue, and like chyme in appearance. As seen under the microscope, it was found to consist of extremely minute granules, amongst which were dispersed oil globules of different sizes, and some crystals, mostly prismatic. In these larvæ we have a striking example, it may be remarked, of highly organised beings, structurally consisting of striped and unstriped muscles, of tracheæ, of nerves and various glands, &c., formed in so short a time by assimilation of dead putrid matter.

11. Of another thrush, killed on the 27th July, one thigh and leg (No. 1), deprived of integuments, weighed 50·1 grs.; the other leg and foot, stripped of

feathers, and without the thigh (No. 2), weighed 42·6. The integuments were left on, and to prevent access to the muscles and the deposition of the ova of the flesh-fly, sufficient skin from the thigh was drawn over the stump and secured from ingress by a ligature of fine silk.

July 28.	No. 1	had lost 22·2	grs., or 44·3	per cent.	
„ „	„ 2	„ 2·7	„ 6·0	„	
„ 29.	„ 1	„ 31·6	„ 63·0	„	
„ „	„ 2	„ 4·5	„ 10·0	„	
„ 31.	„ 1	„ 33·4	„ 66·6	„	
„ „	„ 2	„ 6·75	„ 15·8	„	
Aug. 4.	„ 1	„ 33·55	„ 66·9	„	No further loss.
„ „	„ 2	„ 14·6	„ 34·0	„	
„ 7.	„ 2	„ 21·6	„ 50·0	„	
„ 10.	„ 2	„ 25·2	„ 59·1	„	

In both instances, in the dried state to which they were reduced, little change had taken place. Even when examined now, after eight months, the muscles are found to retain their striated structure. In the instance of No. 2, this was probably owing to the pretty rapid drying from the small bulk of the limb.

12. I will mention one example more, a trial made in winter, between January the 4th and March the 14th, in a room the temperature of which seldom exceeded 50°. The Blue Tit (*Parus cœruleus*) was the subject of the experiments. One, deprived of its skin (No. 1), weighed 169 grs.; another (No. 2), deprived merely of its feathers, weighed 122·3 grs. Without giving the details of the weighing at short intervals, it may suffice to state that No. 1 had lost in twenty-six days 105 grs., or 62 per cent.; whilst No. 2 had lost 51·4 grs., or only 37 per cent. The first had become quite rigid and hard, and sustained no further loss; the second continued to lose weight, but so very slowly, that on the 14th March it was not thoroughly desiccated. It had lost 83·1 grs., or 60 per cent.

Whilst the results which have been described sufficiently show the powerful influence of the integuments in moderating evaporation, if we compare those obtained in the experiments on different animals a marked difference is notable. The moderating or retentive power of the integuments of the frog and toad is seen to be lowest, that of the trout next, that of the mammalia higher, and that of birds highest.

In the instance of the common fowl the thigh showed a much more retentive power than that of the wattle; and it can hardly be doubted that were trials made of different parts of any other animal, a variety of moderating influences would be witnessed, according to the degree of thickness of the covering and difference of physical structure.

As the cuticle is considered anorganic, may not the part it performs in relation to the checking of evaporation in the living body be held to be much the same as in the dead body?

In the majority of the experiments described, the cutis was removed with the cuticle. The results might appear more satisfactory if the cuticle alone had been abstracted—this a difficult matter, so difficult, that I rarely attempted it—but, inasmuch as the cutis does not seem to exercise any limiting power on evaporation, may it not be regarded as inoperative or impassive, and to have no material vitiating effect on the results?

Viewing the function of the cuticle physiologically, must it not be considered as intimately connected with animal heat? Thus, where its retentive, moderating power is lowest, as in the instance of the batrachians, is it not operative in preserving these comparatively cold-blooded animals cool; and *vice versâ* in the instances in which its power is highest, in birds, is it not conducive to the preservation of the elevated temperature for which they are remarkable?

A more important function, it may be inferred, is performed by it, associated with the preceding, namely, of preventing a too rapid loss of water from the system, and especially from the blood, thus preserving this vital fluid of a proper degree of dilution, and the solid parts of a proper degree of moisture and flexibility. In cases of extensive burns, when a large surface of integument has been destroyed, the loss of the aqueous portion of the blood is remarkable. In those cases which have been fatal, the blood has been found by M. BARADUC dark and inspissated, and the viscera surprisingly dry, with an absence of fluid in all the serous cavities. This inquirer, indeed, considers the gravity of burns in proportion to the amount of abstraction of fluid or the drying; and accordingly in the treatment he holds it to be a principle to counteract this as much as possible by keeping the patients many hours in a bath daily, aided by the use of diluents, and by the dressing of the burnt parts with cerate.

In the experiments on the limb of the fowl and of the thrush, as well as in all the others, the results show how much the rapid drying of the parts deprived of their integuments checks and prevents putrefaction at a certain temperature, and *vice versâ*, how a retardation of drying, from the integuments being left on, favours putrefaction. And this it may be inferred, much in the same manner as an atmosphere loaded with moisture promotes the same change.[*]

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[*] A simple experiment illustrates this. Two portions of recently killed lamb were selected. One (No. 1) weighing 168·5 grs. was suspended by a thread, freely exposed to the air of a room varying in temperature from 60° to 65°, *i.e.*, the day and night temperature. The other (No. 2), weighing 160·1 grs., was suspended hanging free in a small glass receiver, in which was a little water, and was so covered as to allow ingress of air, and yet almost to prevent any evaporation. The results were strongly marked. No. 1 lost weight rapidly, and soon became hard and dry without acquiring any putrid taint. No. 2, on the contrary, softened and actually liquefied, at the same time becoming extremely putrid. In an experiment similarly conducted over water—but the water exhausted of air and in vacuo—the muscle escaped putridity. This, from the 11th July to the 20th August, illustrating in addition, I may remark, the difference as regards tendency to putrefaction between muscle and blood; the latter, as I have shown elsewhere (p. 25 of this volume), undergoing the putrid decomposition, even in vacuo, being impregnated with oxygen.

Why muscle deprived of its integuments should escape putrefaction, most other conditions favouring, is not very obvious to reason. Whether electricity is concerned in any way in the prevention is open to question. Be this as it may, the property is an important one, economically considered, and deserving, I cannot but think, of more attention than it has commonly received. The fact that meat, when cut into thin slices, can by drying be kept in a state fit for food even within the tropics, where putrefaction proceeds at so rapid a rate, is well known. The Boucaniers, we are informed, who, in the beginning of the last century, were such formidable pirates in the West Indies, depended very much for subsistence on meat thus prepared. Père Labat, in his abridged history of St Domingo, describes this meat, calling it by its popular name, "Viandes boucannés," as excellent; and he details the exact method of preparing it, as obtained from the wild hog, and from cattle run wild in the forests of that island.[*] Now, considering the qualities of such meat, free from the defects of salted meat, the concentrated nourishment it affords, and that of an agreeable kind, and easily cooked when softened by water, it seems peculiarly fitted for the army and navy in protracted campaigns and in long voyages, and also for the use of travellers in countries where subsistence is precarious. A method very similar to the preceding, I am informed by Sir John Richardson, is employed by the North American Indians, in summer and autumn, for preserving the flesh of deer; they, like the Boucaniers, bring in the aid of smoke, but chiefly for the purpose of protecting the meat from flies.[†] The dried meat powdered, mixed with lard or marrow, forms pemican, which has been of such inestimable value to arctic explorers.[‡] The same Indians are well acquainted with the effect of skinning an animal in retarding its putrefaction. It is a practice of theirs to remove the skin with as little delay as possible, eviscerating their game at the same time.

That thorough desiccation should have the effect of preserving meat from putrefactive change is obviously owing to an arrest of chemical action, the presence of a certain portion of water being essential to such action.[§]

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[*] Nouveau Voyage aux Isles de l'Amerique, &c. Par R. P. Labat. Tom. iii. p. 132.

[†] When dry, even muscle no longer attracts the flesh-fly; it is the moist putrefying flesh which allures it, that being alone suitable to the development of its ova.

[‡] A specimen of pemican (for which I was indebted to Sir John Richardson), about twelve months old, of the best quality, I found composed of—

84·88 fat,
11·77 muscular fibre chiefly,
3·35 water.

The fat consisted of oleine or elaine chiefly, and stearine. The muscular fibre, moistened, was found unaltered as to striated structure.

[§] In 1852 I put by, merely wrapped in paper, portions of pork, mutton, beef, fowl, common trout, pollack (*Gadus pollachius*). They were left in the drawer of a table, in a room in which, during three-fourths of the year, there was a fire. Examined in December 1864; in all of them, with one exception, the muscular fibre exhibited the original striated structure distinctly. The exception was that of the trout, in the muscular fasciculi of which the striæ were less distinct Pemican, which had been kept two years (a portion of that of which the composition is given in the preceding note), exhibited the striated muscle with perfect distinctness.

The same protection from change is witnessed, as is well known, in vegetables, from the removal of their aqueous portion. And in them, too, the cuticle appears to act a part in many instances similar to that of the animal cuticle in retarding evaporation. I may mention an instance or two in illustration, selecting a tuber, the potato, and a fruit, the apple, these being striking examples:—

On the 22d June, three potatoes of the kidney kind were taken from the ground: one (No. 1) had its skin left on—it weighed 295·2 grs.; another (No. 2) had its outer skin removed—it weighed 181·7 grs.; a third (No. 3) had its outer and very fine inner skin both entirely removed—it weighed 263·2 grs. They were placed on the chimney-piece, where the temperature throughout the year varied inconsiderably, ranging from about 50° to 60°. The three were weighed from time to time; the loss per cent., as found on each weighing, is given in the following table:—

June	24.	No. 1 lost	7·8	per cent.,	No. 2,	46·5,	No. 3,	54·0
„	27.	„	8·8	„	„	66·4,	„	70·2
„	30.	„	10·4	„	„	72·6,	„	73·3
July	6.	„	11·4	„	„	74·9,	„	75·8
„	11.	„	12·6	„	„	75·7,	„	77·8
„	22.	„	13·1	„	„	75·8,	„	78·6
August	5.	„	13·9	„	„	75·9,	„	79·8[*]
September	10.	„	15·8	„	No further loss.		„	85·7[†]
December	1.	„	21·0[‡]	„			„	86·2
February	2.	„	27·5[§]	„			No further loss.	
March	14.	„	32·7[‖]	„				

In these instances it is seen, that not only is the loss of water, owing to evaporation, retarded by the cuticular covering, but also that the vitality or germinating power of the tuber is destroyed by its removal; and further, that the removal of the outer delicate cuticle has much the same effect in promoting

To what extent these portions of meat and fish might otherwise be altered, and their nutritive quality impaired, is a question I am not prepared to answer. I may mention that white of egg, which, on thorough desiccation at a low temperature, is again for most part soluble, appears, if long kept, to become insoluble, judging from a trial of some put by after desiccation in 1852, and recently examined.

[*] Now, August 5, No. 1, where least exposed to light, has become greenish; where most exposed, brownish green.

No. 2 has become dark brown, almost black, and has acquired a crescentic form, contracted, without being shrivelled.

No. 3 has become of a light brown, and is much shrunk and shrivelled.

[†] Now, September 10, on No. 1, three small greenish sprouts have appeared tipped with black; general hue the same. Nos. 2 and 3 of the same colour and appearance as before.

[‡] Now, December 1, the sprouting buds of No. 1 have grown a very little, showing a very feeble vitality, and very slow progress. In Nos. 2 and 3 no apparent change.

[§] Now, February 2, the bud of No. 1 has grown into a stalk, with terminal greenish leaflets, and three lateral long roots, with delicate spongioles.

[‖] Now, March 14, the tuber No. 1 is slightly shrunk; the stem from it is ·9 inch in length, and ·3 inch in diameter where thickest; is of a dark purple, and is surmounted by several small green leaflets; the roots from its side, numbering five, vary in length; the longest is 1·7 inch. Weighed again on the 28th of May, the tuber was only a little more shrunk; it was reduced to 161·5 grs. The growth from it, the stem, leaflets, and other offshoots, had all a healthy appearance.

evaporation as the removal of the whole of the integuments, and is as fatal to life; the colour, too, which the tuber acquires, with change of form from its removal, are circumstances which seem worthy of note.*

On the 26th September, two apples of the same kind, not sweet, were selected; one unpeeled (No. 1), weighed 933·5 grs.; the other peeled (No. 2), weighed 1051 grs. They were suspended by their stalk in a room, the temperature of which varied from about 60° to 55° and 50°, during the time of trial. The results of the weighing from time to time are given in the following table, viz., the loss per cent.:—

October	1.	No. 1 had lost	1·8	per cent.,	No. 2,	13·9
,,	11.	,,	8·1	,,	,,	61·5
,,	30.	,,	14·2	,,	,,	80·8
November	6.	,,	17·7	,,	,,	82·4†
,,	12.	,,	18·9	,,		
,,	20.	,,	21·8	,,		
,,	26.	,,	24·2	,,		
December	2.	,,	26·5	,,		
,,	14.	,,	30·1	,,		
,,	30.	,,	33·7	,,		

On the 30th December, No. 1 was shrivelled; it retained its colour, a greenish hue, and, cut into, was found free from decay. No. 2 had become very much shrunk, had acquired a brown colour, and a slight degree of sweetness.

I need not dwell further on the remarkable manner in which the dessication of vegetable substances preserves them from chemical change, and in many instances for a long period; but I must express regret that a process so simple, and in other countries, especially France and the United States of America, so much used, is not more employed in Great Britain. By means of it, the families of the labouring class might secure to themselves throughout the year a greater variety of food at a cheap rate; the apple for instance; several vegetables, such as the carrot, potato, &c.—a variety equally recommended by two qualities, which happily are seldom disjoined, those of agreeableness and wholesomeness.‡

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* In another experiment, begun on 27th October 1863, the results were much the same, with this difference, that, on the 17th March 1864, the unpeeled potato was removed from the light into a dark cupboard, and covered with a small inverted porcelain jar. There it has vegetated; it has shot out many branches, all but the largest of which are white; it is of a light purple; attached to them are many well-formed tubers. Now, March 15, 1865, the weight of the potato is reduced from what it was at first, viz., 900·5 grs., to 331 grs. It has no terminal leaflets. There are seventeen small tubers connected with it; all are of an oval form, like the parent tuber; the largest is ·4 inch in length. All of them are throwing out shoots, and they are most easily detached.

† Its weight afterwards fluctuated a little, according to the hygroscopic state of the atmosphere.

‡ Sliced apples exposed to the air dry rapidly, as do also sliced potatoes and carrots; and if put up in paper bags in a dry place, they will keep fit for use for a long time. No vegetable that I am acquainted with undergoes change more rapidly than the sweet potato (*Batatas edulis*), yet when sliced and dried, as I have found by experience, it may be kept for years unaltered. I have some thus preserved, which I brought from the West Indies in 1848.

X.—*On the Contact of the Loops of Epicycloidal Curves.* By Edward Sang, Esq.

(Plates VI. to XII.)

(Read 3d January 1865.)

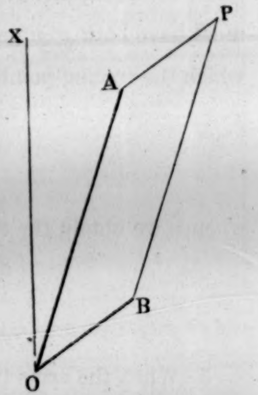
During the summer vacation, Mr Henry Perigal of London proposed to me the following problem :—

"To determine the proportions of an epicycloid of which the loops touch each other."

The solution of this problem contains some points of interest to the general analyst, and exhibits relations between certain trigonometric formulæ and algebraic equations. I, therefore, offer an outline of it to the attention of the Royal Society.

Mr Perigal had obtained the solution, in a considerable variety of cases by the method of trial, aided by mechanical appliances, and has exhibited them in his beautiful series of machine-engraved epicycloids.

1. If we suppose two radii OA and OB to turn on a common centre O with uniform velocities, in the manner of the two hands of a watch, and if, at each instant, we complete the parallelogram OAPB, the opposite corner P describes an *epicycloid.* This curve may be obtained by causing the line AP to turn on A as a centre, while A itself describes a circle round O ; or by causing the arm BP to turn on B as a centre, while B moves round the fixed centre O. These are the ordinary arrangements by wheel-work.



There are other arrangements by help of which epicycloids may be produced, but they all result in giving, for the equation of the curve referred to rectangular co-ordinates, the formulæ

$$x = \mathrm{A} \,.\, \cos \alpha t + \mathrm{B} \,.\, \cos \beta t$$
$$y = \mathrm{A} \,.\, \sin \alpha t + \mathrm{B} \,.\, \sin \beta t$$

in which A and B represent the length of the arms, α and β their angular velocities, and t the time elapsed since both arms were in the direction OX, so that αt and βt are the angles XOA and XOB respectively.

2. β being supposed to be the greater of the two angular velocities, if the arm OB were minute as compared with OA, the curve described by P would be nearly circular and slightly undulated ; as OB is augmented the waves become deeper, as shown in figs. 13 and 14, and when OB reaches the magnitude determined

by the proportion $\beta^2 : a^2 :: \mathrm{A} : \mathrm{B}$, the curve becomes flat at certain points, that is, the radius of curvature there becomes infinite; this phase is exemplified in figs. 15, 16. When OB increases beyond this value the curve becomes sinuous, its concavity being turned inwards and outwards alternately, as is seen in figs. 17, 18; and when OB becomes so great as to satisfy the condition $\beta : \alpha :: \mathrm{A} : \mathrm{B}$, the curve becomes cusped, and assumes that form to which the name *epicycloid* is sometimes restricted; this form is exemplified in figs. 19, 20. When the arm OB is made still longer the epicycloid is looped, the loops being arranged at regular intervals, as in figs. 21, 22; and if OB be made sufficiently long, the loops come to touch each other, as in figs. 23, 24. Mr PERIGAL's problem is to determine the conditions under which this contact of the loops takes place. The loop may touch those adjacent to it on either side, or, if OB be made sufficiently long, those separated from it by two, three, or more intervals; so that the problem may have more than one solution.

3. From the very genesis of the curve it follows that the contact of the loops must occur either on the *major* or on the *minor radius-vector;* now the angular motions of the arms may be either in the same or in opposite directions, wherefore there are four cases to be examined.

4. When the arms turn in the same direction, and when the contact is to be on the major radius-vector, we may use the formulæ of Article 1, unchanged; and since, at the instant of contact, the curve must touch the radius-vector OX, we must have both $y=0$, and its derivative $\frac{dy}{dt}=0$; hence, if T denote the time at which the tracing-point is in this position, we must have

$$\begin{aligned} 0 &= \mathrm{A} \,.\, \sin \alpha\mathrm{T} + \mathrm{B} \,.\, \sin \beta\mathrm{T} \\ 0 &= \alpha\mathrm{A} \,.\, \cos \alpha\mathrm{T} + \beta\mathrm{B} \,.\, \cos \beta\mathrm{T} \end{aligned}$$

whence we obtain the two proportions

$$\begin{aligned} \alpha : \beta &:: \tan \alpha\mathrm{T} : \tan \beta\mathrm{T} \\ \mathrm{A} : -\mathrm{B} &:: \sin \beta\mathrm{T} : \sin \alpha\mathrm{T} \end{aligned}$$

5. When the arms turn in the same direction, and when the contact is to be on the minor radius-vector, we have to change the sign of B in the preceding formulæ, which are thereby converted into

$$\begin{aligned} 0 &= \mathrm{A} \,.\, \sin \alpha\mathrm{T} - \mathrm{B} \,.\, \sin \beta\mathrm{T} \\ 0 &= \alpha\mathrm{A} \,.\, \sin \alpha\mathrm{T} - \beta\mathrm{B} \,.\, \sin \beta\mathrm{T} \end{aligned}$$

whence

$$\begin{aligned} \alpha : \beta &:: \tan \alpha\mathrm{T} : \tan \beta\mathrm{T} \\ \mathrm{A} : \mathrm{B} &:: \sin \beta\mathrm{T} : \sin \alpha\mathrm{T} \end{aligned}$$

6. If the arms turn in opposite directions, we have to change the sign of β in the preceding formulæ. By this we merely reverse the order of the occurrence on the major or on the minor radius, and thus the general condition of the contact of two loops is contained in the proportion

$$\alpha : \beta :: \tan \alpha\mathrm{T} : \tan \beta\mathrm{T};$$

in other words, we have to find two arcs αT and βT in the ratio of α to β, and having their tangents in the same ratio.

7. If we put x for the tangent of the arc T, the above proportion becomes,

$$\beta \frac{\frac{\alpha}{1} x - \frac{\alpha}{1} \frac{\alpha-1}{2} \frac{\alpha-2}{3} x^3 + \&c.}{1 - \frac{\alpha}{1} \frac{\alpha-1}{2} x^2 + \frac{\alpha}{1} \frac{\alpha-1}{2} \frac{\alpha-2}{3} \frac{\alpha-3}{4} x^4 - \&c.}$$

$$= \alpha \frac{\frac{\beta}{1} x - \frac{\beta}{1} \frac{\beta-1}{2} \frac{\beta-2}{3} x^3 + \&c.}{1 - \frac{\beta}{1} \frac{\beta-1}{2} x^2 + \frac{\beta}{1} \frac{\beta-1}{2} \frac{\beta-2}{3} \frac{\beta-3}{4} x^4 - \&c.}$$

On developing and subtracting the common term $\alpha\beta x$ from each side, the result is divisible by x^3, and we obtain an equation into which only the even powers of x enter. When α and β are prime to each other and both odd, the order of the equation, x^2 being regarded as the unknown quantity, is $\frac{1}{2}(\alpha + \beta - 4)$; and when one of them is even the equation is of the order $\frac{1}{2}(\alpha + \beta - 3)$; and it is to be observed that when $\alpha + \beta$ is even there is always the solution $\mathrm{T} = 90^\circ$, $\mathrm{B} = \mathrm{A}$, which corresponds to a contact at the centre of the epicycloid.

8. A table of the values of T and B (A being taken as unit) for all cases up to $\beta = 10$, is subjoined. These values were readily obtained by the process which I published in 1829 (Solution of Algebraic Equations of all orders); the values of x^2 having been taken to ten places of decimals; and those of T and B having been thence computed by the ordinary seven-place tables.

It may be noticed that the only case in which the ratio of A to B can be expressed by integer numbers is that of $\alpha : \beta :: 1 : 5$; this rationality being connected with the fact that 1 and 5 are the only two component parts of the perfect number 6 which express a ratio in its lowest terms. And farther, that the ratio of A to B is the same whether the arms turn in the same or in opposite directions; this fact is exhibited in the accompanying figures.

α	β	A	B	Log B	T		
1	3	1	1·000 0000	0·000 0000	90°	00′	00″·00
1	4	1	·915 5587	9·963 1069	54.	05	41·40
1	5	1	·800 0000	9·903 0900	37	45	40·50
1	5	1	1·000 0000	0·000 0000	90	00	00·00
1	6	1	·696 8284	9·843 1257	43	19	48·35
1	6	1	·964 9338	9·984 4975	74	33	38·71
1	7	1	·613 0718	9·787 5114	37	02	50·10
1	7	1	·898 7860	9·953 6563	63	42	32·40
1	7	1	1·000 0000	0·000 0000	90	00	00·00
1	8	1	·545 4784	9·736 7775	32	21	17·20
1	8	1	·828 5136	9·918 2997	55	38	21·84
1	8	1	·980 4880	9·991 4423	78	34	08·48
1	9	1	·490 4087	9·690 5581	28	43	36·00
1	9	1	·762 6565	9·882 3290	49	23	43·62
1	9	1	·938 9196	9·972 6284	69	44	20·53
1	9	1	1·000 0000	0·000 0000	90	00	00·00
1	10	1	·444 9490	9·648 3102	25	48	58·54
1	10	1	·703 5140	9·847 2728	44	25	02·25
1	10	1	·889 8366	9·949 3102	62	42	16·54
1	10	1	·987 5693	9·994 5676	80	53	15·78
2	5	1	·837 3864	9·922 9259	55	47	59·35
2	7	1	·972 6638	9·987 9628	37	59	16·80
2	7	1	·758 9075	9·880 1889	66	24	11·65
2	9	1	·858 4477	9·933 7138	29	07	49·32
2	9	1	·984 0117	9·993 0002	50	15	45·72
2	9	1	·617 9169	9·790 9301	71	52	23·40
3	5	1	1·000 0000	0·000 0000	90	00	00·00
3	7	1	·876 5924	9·942 7977	40	43	39·83
3	7	1	1·000 0000	0·000 0000	90	00	00·00
3	8	1	·977 5973	9·990 1600	34	22	28·47
3	8	1	·799 5282	9·902 8338	76	32	18·95
3	10	1	·986 4601	9·994 0800	26	42	01·12
3	10	1	·699 6930	9·825 8758	47	02	27·70
3	10	1	·879 2330	9·944 1040	80	01	10·72
4	7	1	·962 3744	9·983 3441	62	39	57·36
4	9	1	·828 3048	9·918 1901	32	10	43·41
4	9	1	·981 0198	9·991 6778	70	37	31·13
5	7	1	1·000 0000	0·000 0000	90	00	00·00
5	8	1	·968 0850	9·985 9134	57	44	43·17
5	9	1	·910 5817	9·959 3189	48	02	17·95
5	9	1	1·000 0000	0·000 0000	90	00	00·00
7	9	1	1·000 0000	0·000 0000	90	00	00·00
7	10	1	·975 5140	9·989 2335	61	43	23·76

9. When one of the radii, say OA, becomes indefinitely large the epicycloid merges into the cycloid produced by carrying the centre of a revolving wheel along a straight line; and the extension of Perigal's problem leads naturally to this one:—

"To construct a cycloid of which the loops may touch each other."

10. If we suppose the centre of the revolving circle to be carried along the axis Y with a linear velocity v, while the radius B turns with an angular velocity β, the co-ordinates of the tracing-point are

$$x = \mathrm{B} . \cos \beta t ; \qquad y = vt + \mathrm{B} . \sin \beta t$$

and for the point of contact we must have

$$\begin{aligned} 0 &= v\mathrm{T} + \ \ \mathrm{B} . \sin \beta\mathrm{T} \\ 0 &= \ \ v + \beta\mathrm{B} . \cos \beta\mathrm{T} \end{aligned}$$

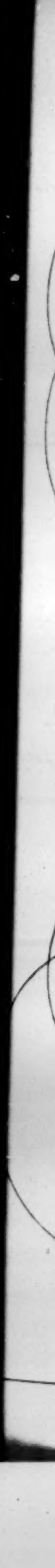
wherefore all such points are determined by the solution of the trigonometrical equation

$$\beta\mathrm{T} = \tan \beta\mathrm{T} ;$$

that is to say, we must discover all those arcs which are equal in length to their own tangents.

11. If we put $v = \beta = 1$, we obtain the following solutions for the first ten cases, the first of these being that of the common or cusped cycloid.

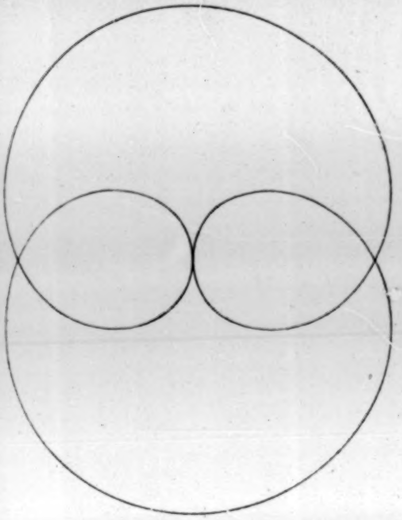
T			Log sec T	B
°	′	″		
0	00	00·00	0·000 0000	1·000 00
257	27	12·24	0·663 0732	4·603 34
442	37	27·57	0·891 5209	7·789 70
624	45	36·54	1·039 4093	10·949 88
805	56	00·77	1·149 2717	14·101 71
986	40	35·75	1·237 7832	17·289 54
1167	11	22·88	1·309 5424	20·395 88
1347	33	55·30	1·371 8187	23·540 67
1527	51	08·52	1·426 2642	26·684 82
1708	04	43·65	1·474 6296	29·828 38



EPICYCLOIDS

$\alpha : \beta :: 1 : 3$

Fig. 1

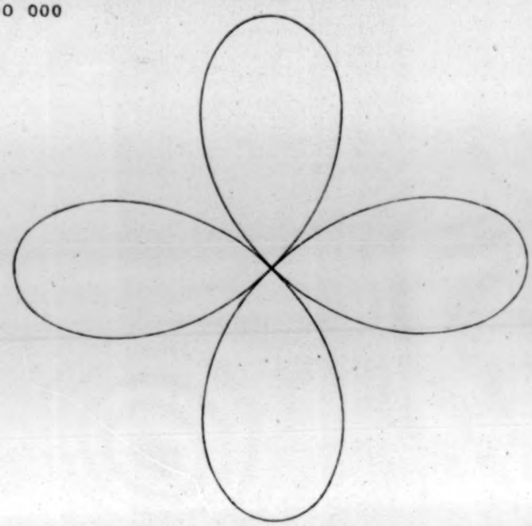


A = .500 000

B = .500 000

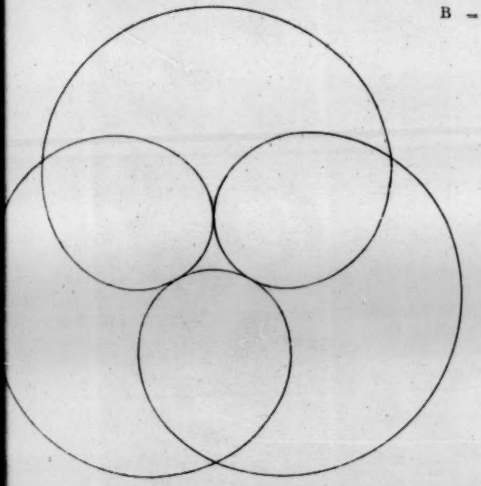
$\alpha : \beta :: 1 : -3$

Fig. 2



$\alpha : \beta :: 1 : 4$

Fig. 3

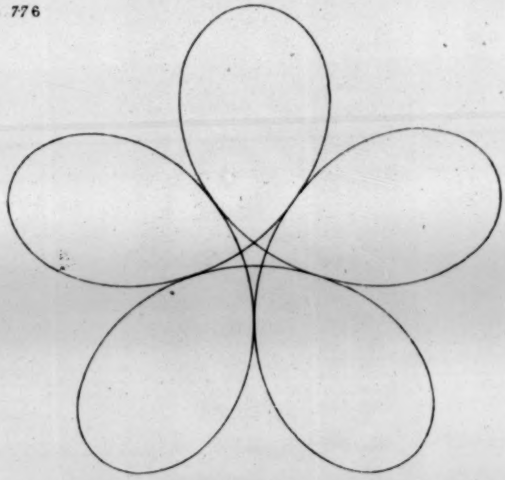


A = .521 224

B = .478 776

$\alpha : \beta :: 1 : -4$

Fig. 4



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EPICYCLOIDS

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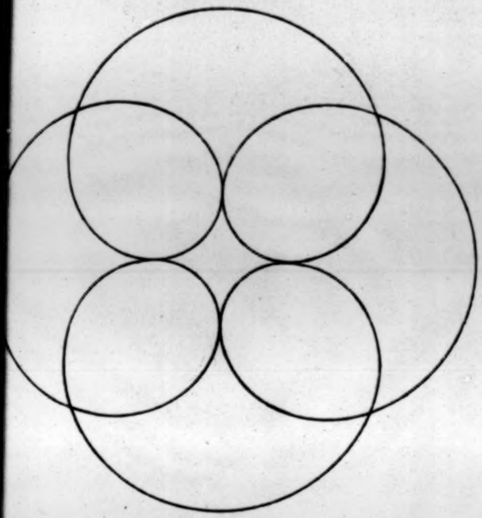
A = .555 556
B = .444 444

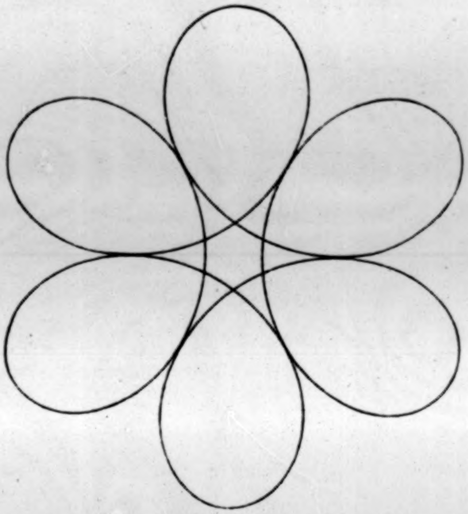
$\alpha : \beta :: 1 : 5$

Fig. 5



$\alpha : \beta :: 1 : -5$

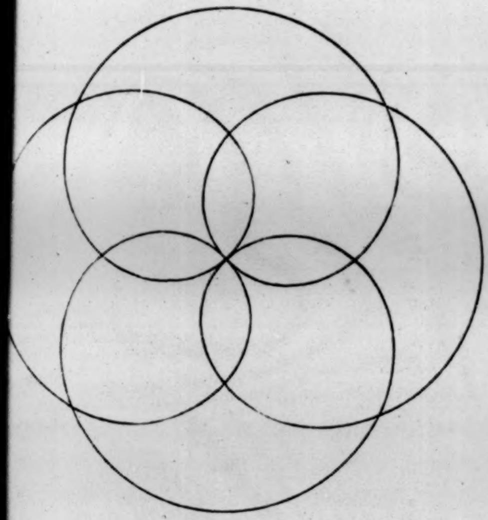
Fig. 6



A = .500000
B = .500000

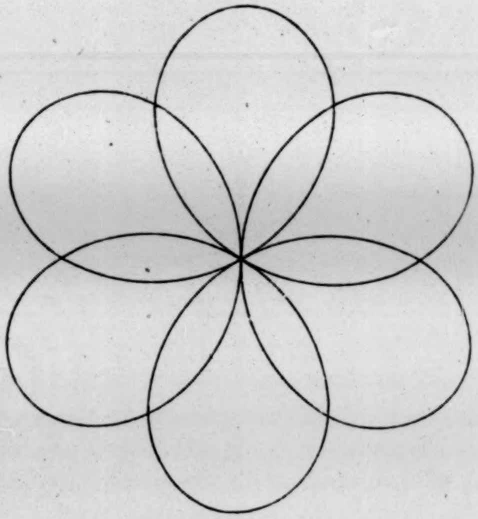
$\alpha : \beta :: 1 : 5$

Fig. 7



$\alpha : \beta :: 1 : -5$

Fig. 8



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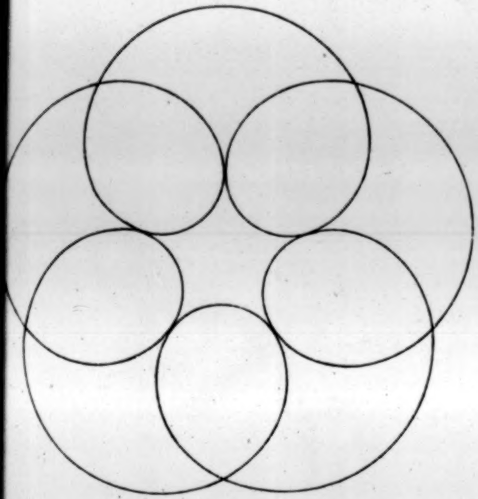
$\alpha : \beta :: 1 : 6$

Fig. 9

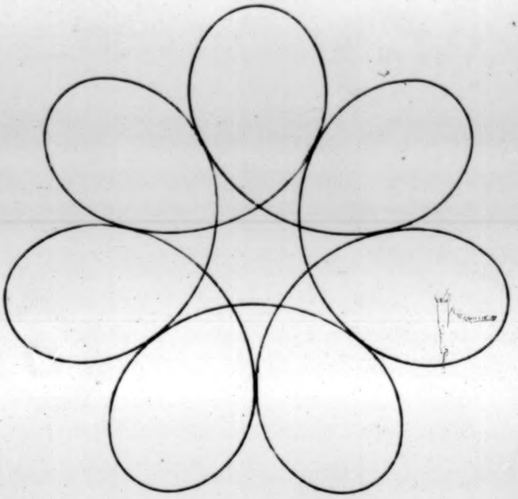


A – .589 335

B – .410 665

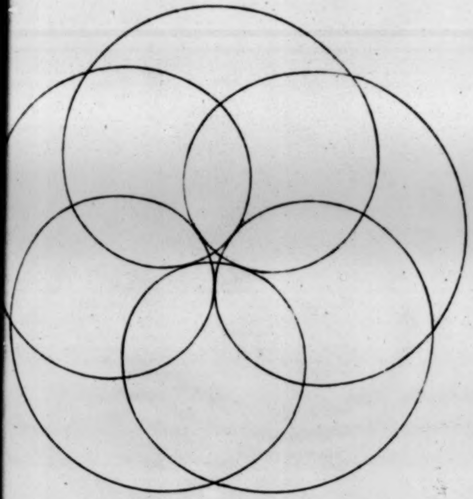
$\alpha : \beta :: 1 : -6$

Fig. 10



$\alpha : \beta :: 1 : 6$

Fig. 11

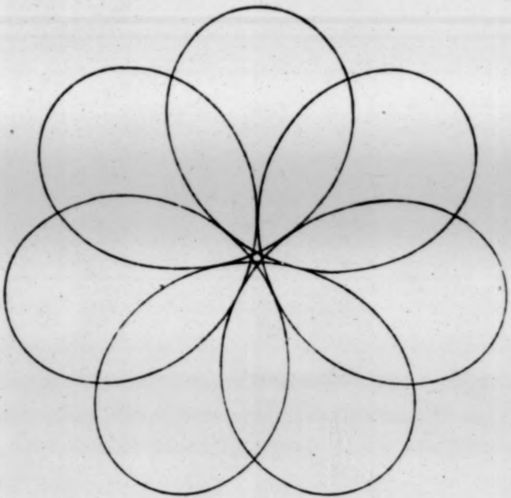


A – .508 923

B – .491 077.

$\alpha : \beta :: 1 : -6$

Fig. 12



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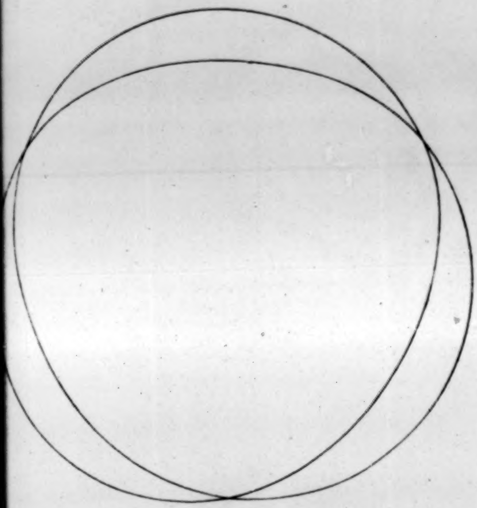
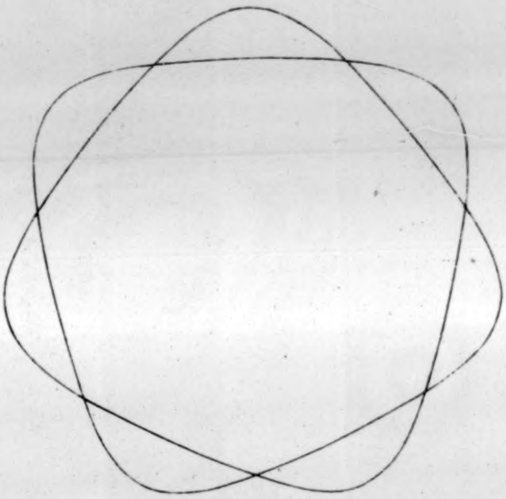
EPICYCLOIDS

$\alpha : \beta :: 2 : 5$
Fig. 13

A — . 900 000
B — . 100 000

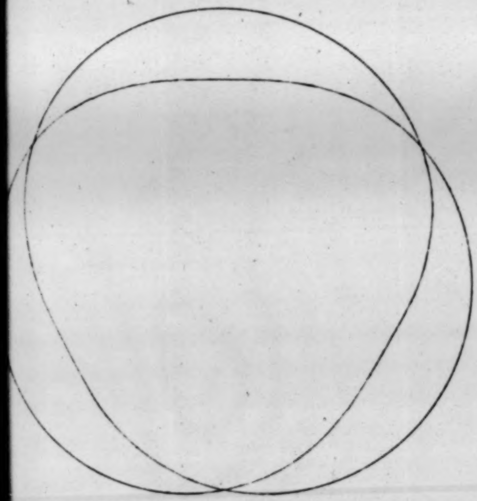


$\alpha : \beta :: 2 : -5$
Fig. 14

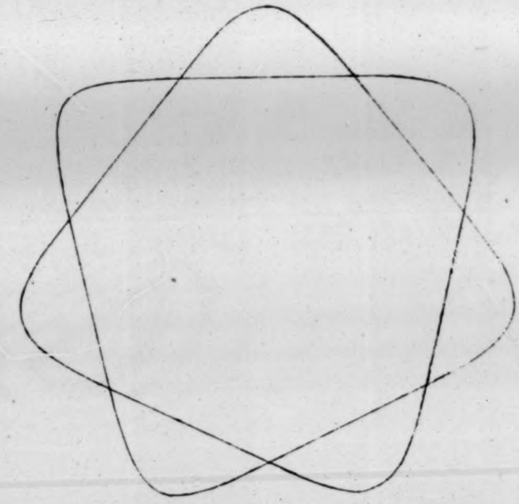


A — . 862 069
B — . 137 931

$\alpha : \beta :: 2 : 5$
Fig. 15



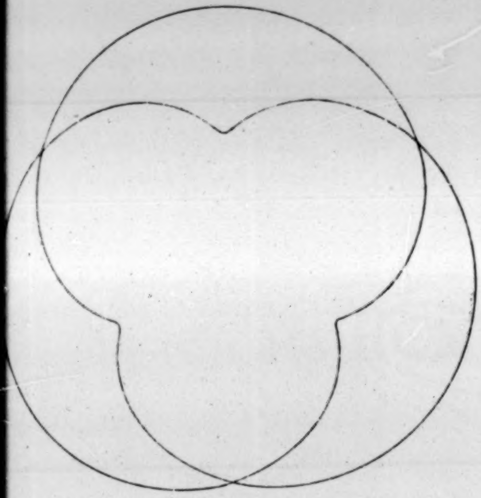
$\alpha : \beta :: 2 : -5$
Fig. 16



A = .750 000
B = .250 000

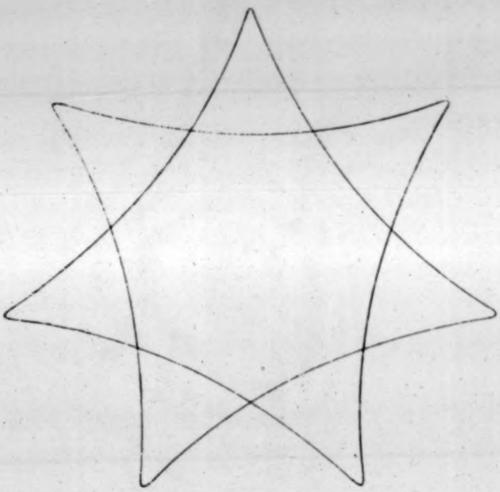
$\alpha : \beta :: 2 : 5$

Fig. 17



$\alpha : \beta :: 2 : -5$

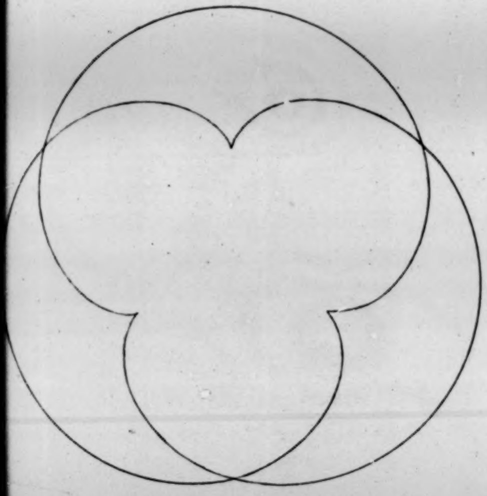
Fig. 18



A = .714 286
B = .285 714

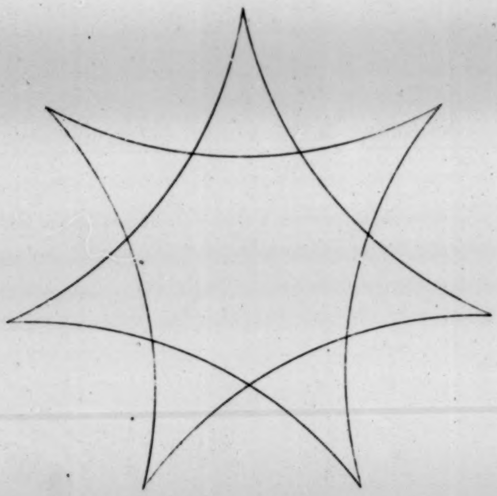
$\alpha : \beta :: 2 : 5$

Fig. 19



$\alpha : \beta :: 2 : -5$

Fig. 20



W & A K Johnston, Edinburgh.

EPICYCLOIDS

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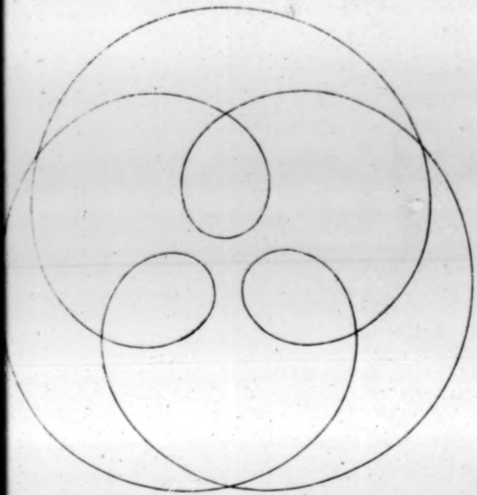
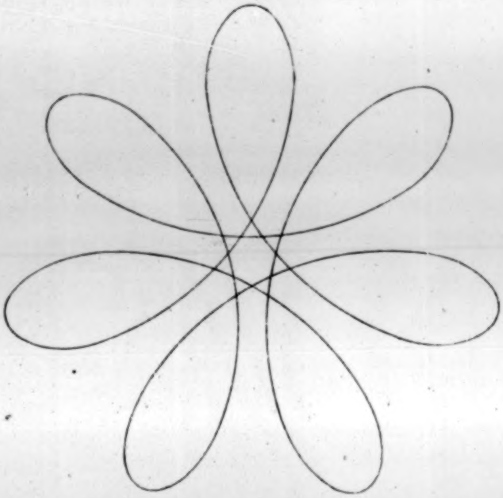
A = .540 000
B = .460 000

$\alpha : \beta :: 2 : 5$
Fig. 21

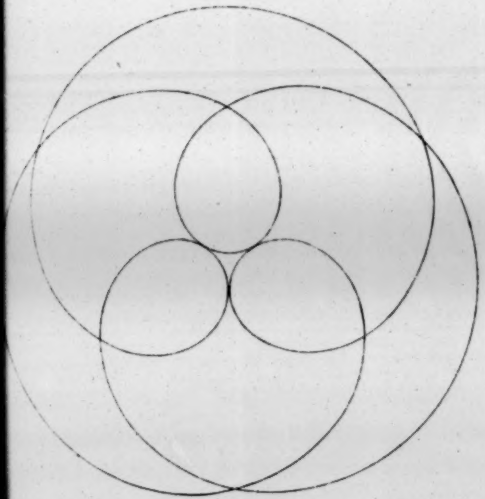


$\alpha : \beta :: 2 : -5$
Fig. 22

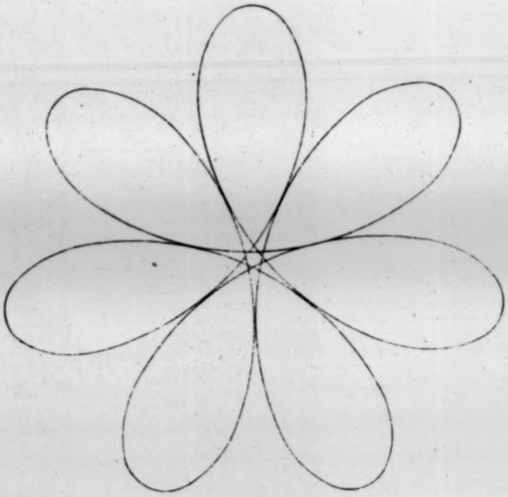


A = .515 100
B = .484 900

$\alpha : \beta :: 2 : 5$
Fig. 23



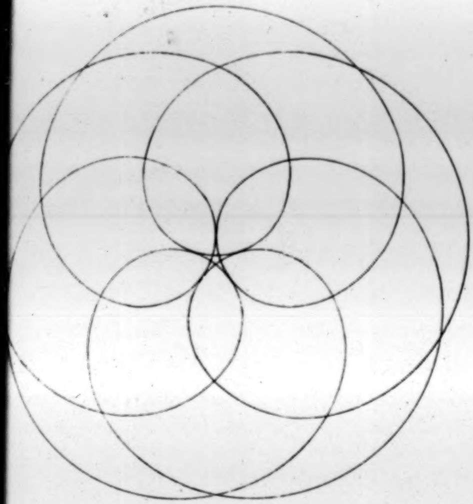
$\alpha : \beta :: 2 : -5$
Fig. 24



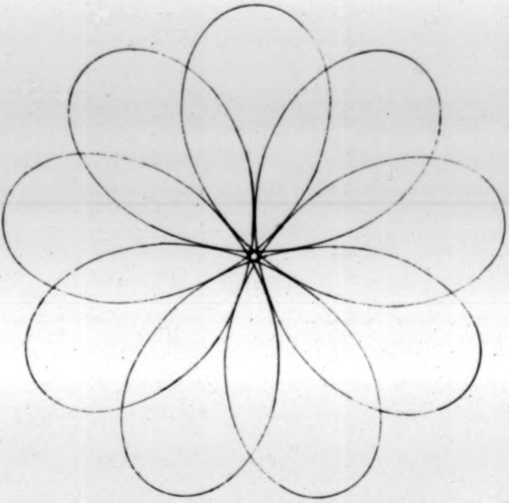
W & A K Johnston, Edinburgh

A = ·506 929
B = ·493 071

$\alpha : \beta :: 2 : 7$
Fig. 25

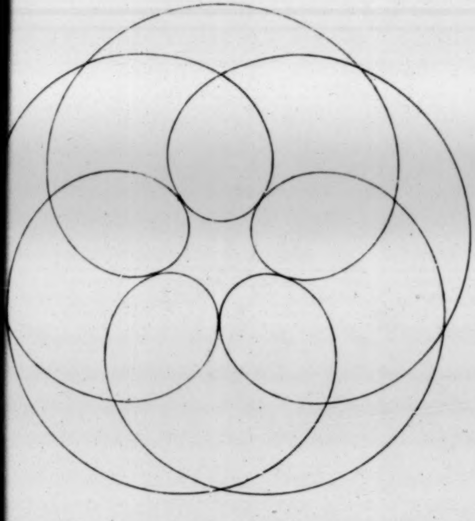


$\alpha : \beta :: 2 : -7$
Fig. 26

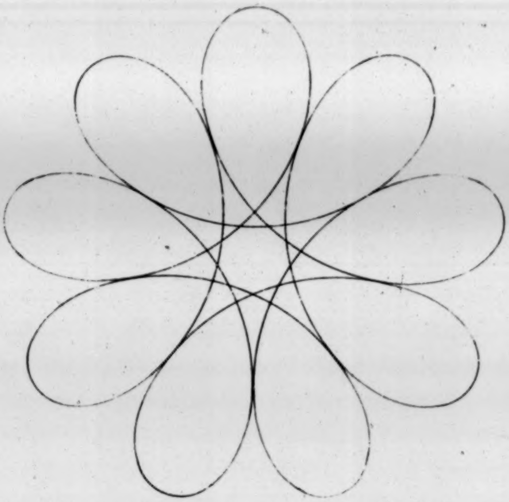


A = ·568 535
B = ·431 465

$\alpha : \beta :: 2 : 7$
Fig. 27



$\alpha : \beta :: 2 : -7$
Fig. 28



XI.—*Researches on Malfatti's Problem.* By H. F. Talbot, Esq.

(Read 20th March 1865.)

The problem which bears the name of the Italian geometer **Malfatti**, by whom it was first proposed and solved, has long attracted the attention and exercised the ingenuity of mathematicians, and has been made the subject of many careful and elaborate researches.

The great attention which has been bestowed upon this problem has arisen partly from its intrinsic difficulty, but chiefly on account of the extreme simplicity of the solution finally obtained by **Malfatti**, which seemed to open new views of geometrical research, and gave reason to hope that simple solutions might in like manner be found of many other geometrical problems usually accounted very difficult or insoluble.

The problem of **Malfatti** offers another singularity. Although it is a question of elementary geometry which can be solved by a simple and elegant geometrical construction, yet no *geometrical* proof has ever been given, as far as I am aware, of the truth of this construction. It has been established hitherto only by a very elaborate use of algebraic analysis, in the course of which, however indisputable the result may be, all *geometrical* perception of its truth is lost. And yet there can be little doubt, it should seem, that a *geometrical* reason must exist for any simple series of facts belonging to elementary geometry.

The necessity of calling in the aid of analysis can only arise from the true connection of the geometrical principles involved in the problem being imperfectly understood.

I now offer to the Royal Society a purely geometrical solution of the problem; and, for the sake of clearness, I have divided it into several parts, which I have called Lemmas. Some of these are well deserving of attention for their own sake, and irrespective of **Malfatti**'s problem. When these theorems have been established, their combination affords a lucid proof of the truth of the solutions which mathematicians have hitherto only obtained by the help of analysis.

History of the Problem.

In the year 1803, a distinguished Italian geometer, Signor **Malfatti**, proposed the following problem in the Memoirs of the Italian Society of Sciences, vol. x. part 1:—*

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* See **Gergonne**, vol. i. p. 347.

"In a given triangle to inscribe three circles touching each other, and each of them touching two sides of the triangle."

He gave at the same time a remarkably simple geometrical solution which he had discovered, but unaccompanied by any *geometrical* demonstration of its truth. He contented himself with showing that, if we calculate the algebraic values of the three radii which result from the above-mentioned geometrical construction, these three values, when substituted in the analytical equations deduced from the original conditions of the problem, do in fact satisfy them, and are therefore demonstrated to be true. But he gives no indication of any process of reasoning by which he arrived at the knowledge of these values.

In the year 1810, GERGONNE proposed this problem for solution in the "Annales des Mathématiques," vol. i. p. 196, without knowing that it had been previously solved by MALFATTI; and no solution of it being sent to him by his correspondents, he took up the inquiry himself.* He makes the following preliminary statement:—

"Il y a plus de 10 ans que ce difficile problême s'est offert pour la première fois aux rédacteurs de ce recueil, mais bien qu'ils l'aient attaqué un grand nombre de fois ils n'ont pu pendant longtemps parvenir à le resoudre ni même à s'assurer s'il était resoluble par la ligne droite et le cercle. . . . .

"Ils ont cru devoir faire encore de nouvelles tentatives, et plus heureux cette fois que les précédentes ils sont parvenus sinon à trouver une construction du problême, du moins à l'abaisser au premier dégré."

Then follows an analytical investigation, which finally gives an algebraic value for the radius of any one of the circles in terms of known quantities. But this value does not lead to any *simple* geometrical construction, nor is it easy to show that it agrees with that previously found by MALFATTI, which, however, must necessarily be the case.

Having succeeded to this extent, M. GERGONNE did not believe the problem to be susceptible of much further simplification, when he first became acquainted with the previous researches of MALFATTI. Having procured and perused the memoir of that author, he found that it threw no light upon the point of chief interest, viz., the mode of investigation by which a result so unexpectedly simple had been obtained.†

Nothing further appears to have been done till the year 1820, when M. LECHMÜTZ, of Berlin, published a memoir in GERGONNE'S Annales, vol. x. p. 289, in which he succeeded for the first time in solving the problem, by a course of *à priori* reasoning. His investigation is algebraical, and his results coincide with those of MALFATTI. In the year 1826, STEINER, a distinguished geometer of Berlin, threw an entirely new light upon the problem, by giving, in CRELLE'S Annals (vol. i. p.

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* See GERGONNE, vol. i. p. 343.

† Ibid. vol. ii. p. 60.

178), a geometrical construction of singular simplicity, but entirely unaccompanied with proof. His solution is as follows:—

Let ABC be the triangle in which it is required to inscribe three circles touching each other, and each of them touching two sides of the triangle. Bisect the angles A, B, C, by three lines AO, BO, CO, which will meet in the point O. In the three triangles AOB, BOC, COA, inscribe three circles, touching the sides of the given triangle in the points D, E, F, which letters will serve to denote those circles respectively. From the point of contact D, draw a line DG, touching the circle E on its inner side. Similarly from the point of contact E, draw a line EG touching the circle D, on its inner side. And let DG, EG, intersect in G. Then we have a trapezium BDEG, and STEINER affirms, *first*, that a circle can always be inscribed in this trapezium; and, *secondly*, that the circle so inscribed will be one of the three required circles.

But no doubt a difficulty will be observed immediately. STEINER directs that the line DG shall be tangent to the circle E; and plainly there is no reason why the circle E should be selected rather than the circle F. But the reply to this is, according to STEINER, that if the line DG touches the circle E, it will also *necessarily* touch the circle F.

But of this most remarkable theorem he gives no demonstration whatever, although there is assuredly no theorem in the whole of geometry which has less claim to be considered as an axiom. Moreover, he affirms that the same line, DG, touches two of the required circles of the problem, at the point where they touch each other. This being admitted, the construction of the problem follows at once, as it is only requisite to describe a circle touching AB, BC, two sides of the given triangle, and also the known line DG, and this circle will be one of the three circles required, the others being found with equal facility.

STEINER'S solution, therefore, would have left nothing more to be wished for, if it had been accompanied with a demonstration. But of such his memoir contains not a single syllable. He says, indeed (page 178), that this solution of a difficult problem shows "the fruitfulness of the preceding theory;" but the critical researches of subsequent inquirers have not failed to discover the singular circumstance, that there is no connection whatever between this solution of MALFATTI'S problem and the theories set forth in the preceding part of STEINER'S memoir.

The great simplicity and elegance of this solution discovered by STEINER rendered a demonstration of it very desirable, which was at length accomplished by ZORNOW of Konigsberg, in CRELLE'S Annals for 1833 (vol. x. p. 300). The demonstration of ZORNOW is remarkably elegant, but it chiefly depends upon some very dexterous algebraic transformations, in the course of which, however, all perception of a geometric proof of the construction necessarily disappears.

In the following year, PLÜCKER of Bonn resumed the subject in CRELLE'S "Annals," xi. p. 121. His memoir, which bears date October 1831, throws a great deal of new light upon the subject. His object was, like that of ZORNOW, to demonstrate the truth of STEINER'S construction, in which attempt he succeeds up to a certain point, by a well-conducted train of geometrical reasoning; but beyond that point he cannot proceed without the help of analysis. In fact, he shows geometrically that there exists a certain point O within any given triangle ABC (see former figure), which possesses the property, that if AO, BO, CO, are joined, and three circles are inscribed in the three triangles AOB, BOC, COA, as in that figure, then the line DG will touch both the circles E, F, and also two of the required circles. It remained to discover what point of the triangle the point O was, and to verify STEINER'S assertion that it was the centre of the inscribed circle, or that the lines AO, BO, CO, respectively bisect the three angles of the given triangle. But of this capital point PLÜCKER was unable to find any geometrical proof. He has recourse, therefore, to a very free and prolix use of trigonometry and algebra, through which I doubt whether any of his readers have had the courage to follow him, but which finally conducts him to the conclusion that STEINER'S assertion is true. It will be observed that PLÜCKER'S memoir, though published subsequently to that of ZORNOW, preceded it in point of date; that of ZORNOW being dated in October 1832. He was, therefore, the first who succeeded in demonstrating STEINER'S theorem. PLÜCKER concludes his memoir with the following remarks upon the mode in which STEINER has treated the question :—*

"The construction which I have given is essentially the same with that proposed by STEINER in vol. i. of this journal, p. 178. There is, however, at that place no indication of a demonstration. The introductory words of the author—'*to show the fruitfulness of the theorems set forth in paragraphs* 1, 2, 3, *by a suitable example, we add the geometrical solution, and also a greater generalisation of Malfatti's problem, omitting the proof*,'—might cause a person who (as I must confess to be my own case) has no idea how the construction of that problem can depend upon the well-known theorems explained in the above quoted paragraphs concerning points of similitude, &c., &c., to think that the given construction is not proved."†

I have said that PLÜCKER'S recourse to a difficult and very prolix analysis in order to justify the assertion that O is the centre of the inscribed circle, is the weak point of his able investigation. He has admitted this himself, for he says (p. 126), "So soon as this theorem is brought into its proper connection there can be no doubt that an easier proof will be found of it." And he adds a wish for "a simple geometrical proof."

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* Vol. xi. p. 126.

† Die einleitenden Worte des Verfassers; "*Um die Fruchtbarkeit*," &c. &c., könnten demjenigen, der, wie ich von mir bekennen muss, keine Idee davon hat, wie die Construction, &c., &c. . . . . . den Gedanken aufdrängen, dass die gegebene Construction nicht bewiesen sei.

In the demonstration which I now submit, I shall follow PLÜCKER's geometrical proof in a general way, up to the point where he breaks away from geometry into the regions of analysis, and then give a proof, by geometry alone, of the remaining portion of the investigation.

Lemma 1.—If a circle is inscribed in a triangle, the difference of the sides equals the difference of the segments of the base.

This is evident. D, E, F, being the points of contact, AB — AC = BE — CF = BD — DC.

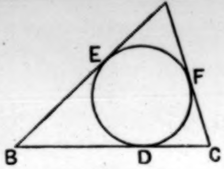

Fig. 1.

Lemma 2.—Let two circles touch each other at O, and let BFGC be their common external tangent, and AOD their common internal tangent. Let A be any point in the tangent AOD, and let AEB, AHC, be drawn touching the circles; then if a circle be inscribed in the triangle ABC, it will touch the base BC at D.

Demonstration.—We have manifestly the equal tangents AE = AH, DF = DG, BE = BF, and CG = CH. Therefore AB — AC = BE — HC = BF — GC = BD — DC. Therefore by Lemma 1 the inscribed circle touches the base in D.

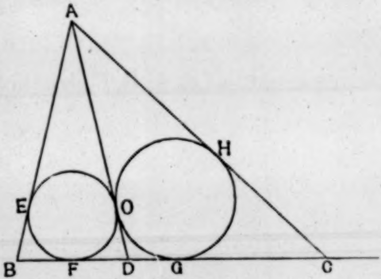

Fig. 2.

Lemma 3.—This is only another case of Lemma 2, when the point A is taken so near to O that the tangents AE, AH, diverge from the base BC, but their prolongations AB, AC, intersect the base at B, C.

In this case also we have the equal tangents AE = AH, DF = DG, BE = BF, and CG = CH. Therefore AB — AC = BE — HC = BF — GC = BD — DC.

Lemma 3 is the case which occurs in the solution of MALFATTI's problem, but as the same demonstration applies to Lemma 2, I have given both of them.

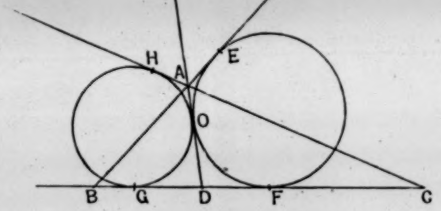

Fig. 3.

Lemma 4.—If tangents of equal length are drawn to a circle, the locus of their extremities is a circle concentric to the first.

Lemma 5.—Let there be two circles A, B, and let RS and its equal ŔŚ be their two common internal tangents; then if OP, OQ are two tangents drawn from any point O not situated either in RS produced, or in ŔŚ produced, OQ – OP is not equal to RS.

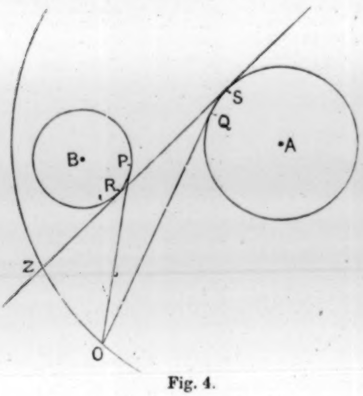

Fig. 4.

Demonstration.—Let the points A, B, be the centres of the given circles. From centre A with radius AO describe a circle cutting RS produced in Z. Then by Lemma 4, OQ = ZS, whence OQ – OP = ZS – OP. But OP is not equal to ZR, because O lies on the circumference ZO, which is not concentric to the circle B. Therefore OQ – OP is not equal to ZS – ZR; therefore it is not equal to RS. *Q. E. D.*

Corollary.—If OQ – OP is equal to RS, O must either lie in RS produced or in ŔŚ produced.

Lemma 6.—Let A, B, C, be three circles. Let DE be the internal common tangent of A and B; FG of B and C; and HI of C and A. Then if these three tangents, when produced, meet in a single point O,

$$DE = FG + IH$$

or the greatest common tangent equals the sum of the two others.

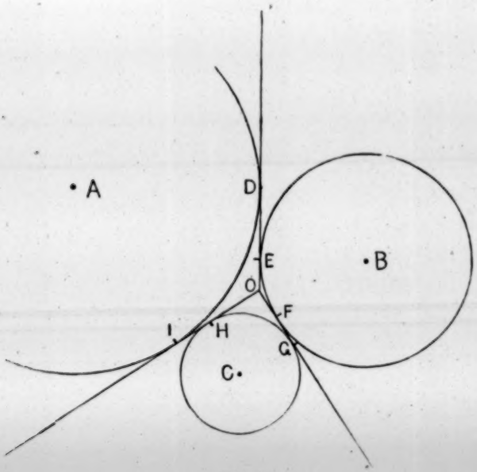

Fig. 5.

For,

$$IH = OI - OH = OD - OG = DE - FG.$$

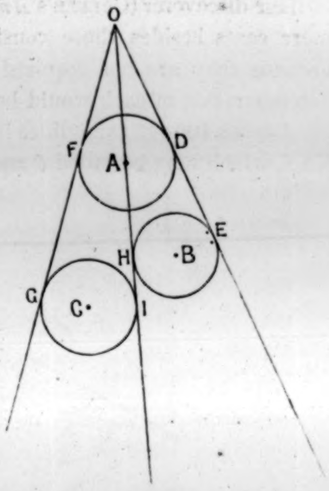

Fig. 6.

Lemma 7.—Let A, B, C, be three circles. Let DE be the external tangent of A and B. Let FG be the external tangent of A and C; and let HI be the internal tangent of B and C. Then if these three tangents, when produced, meet in a single point O,

$$FG - DE = HI$$

For,

$$FG - DE = OG - OE = OI - OH = HI.$$

Lemma 8.—If three internal common tangents of three circles meet in a point O, their other three internal common tangents meet in another point P.

Demonstration.—In figure 5, suppose the common tangents meeting in O to be effaced, and replaced by the three other internal common tangents, it is required to show that these also meet in a point.

Let two of them, viz. those touching the circle A, meet in the point P. Denoting the new tangents by the same letters as before, but accentuated, we have of course

$$DE = \acute{D}\acute{E},\ FG = \acute{F}\acute{G},\ HI = \acute{H}\acute{I}$$

Now we proved in Lemma 6 that

$$DE = FG + HI, \text{ or } ED - HI = FG$$

therefore we have

$$\acute{E}\acute{D} - \acute{H}\acute{I} = \acute{F}\acute{G}$$

The new common tangents meeting in P will be $P\acute{E}\acute{D}$, $P\acute{H}\acute{I}$; but these have a part which is equal in each, namely $PE' = PH'$, which are tangents to the same circle A. Therefore subtracting this part, we have

$$P\acute{E}\acute{D} - P\acute{H}\acute{I} = \acute{E}\acute{D} - \acute{H}\acute{I}$$

which we proved to be equal to $\acute{F}\acute{G}$.

Therefore P is a point from which tangents PD', PI' have been drawn to the circles B and C, and their difference has been found equal to $\acute{F}\acute{G}$ the internal common tangent of B and C. Therefore by the corollary to Lemma 5, P is a point in $\acute{F}\acute{G}$ produced. *Q. E. D.*

Lemma 9.—If two external and one internal common tangents of three circles meet in a point O, the three other *corresponding* tangents meet in another point P.

The demonstration of this is the same as the last, employing Lemma 7 instead of Lemma 6. These theorems may be called PLÜCKER's tangents, from the name of their discoverer (CRELLE's *Annals*, tom. xi.). It is evident that there are several more cases besides those considered in Lemmas 6, 7, 8, 9; but I omit them, because they are not required for the solution of MALFATTI's problem. The demonstration of each would be nearly in the same words.

Lemma 10.—If three lines issue from a point A, and contain the angles DAX, EAX, which may be called θ and ϕ; and if two circles, with centres B and C, are inscribed anywhere in these angles, touching the outer sides at D and E; then if DE is joined, the intercepted chords DF, GE are in a contant ratio; namely, in the ratio of $\tan \frac{1}{2}\theta$ to $\tan \frac{1}{2}\phi$.

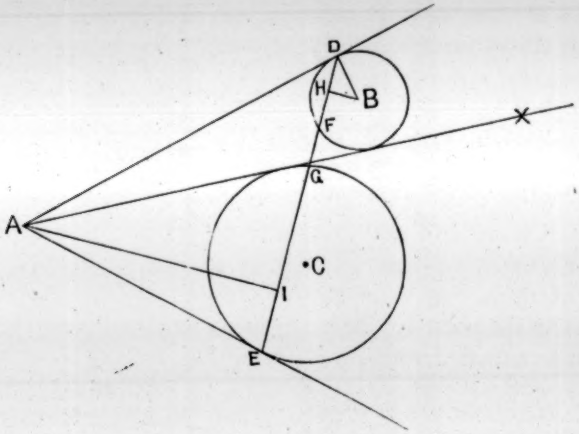

Fig. 7.

Demonstration.—Join BD, and draw the perpendicular BH, dividing the chord DF into two equal parts. Draw AI perpendicular to DE.

Then, since the triangles ADI, BDH are similar,

$$AD : AI :: BD : DH$$

$$\therefore DH = AI \cdot \frac{BD}{AD} = AI \cdot \tan \tfrac{1}{2}\theta$$

$$\therefore \text{chord } DF = 2\,AI \cdot \tan \tfrac{1}{2}\theta$$

By similar reasoning it may be shown that chord $GE = 2AI \tan \frac{1}{2}\phi$

$$\therefore DF : GE :: \tan \tfrac{1}{2}\theta : \tan \tfrac{1}{2}\phi$$

Corollary 1.—If $\theta = \phi$, $DF = GE$. That is:—"If the circles subtend equal angles at A, the intercepted chords are equal."

Corollary 2.—If θ is greater than ϕ, then DF is greater than GE.

Lemma 11.—If any angle DAE is bisected by the line AX, and two circles are inscribed anywhere in the semi-angles DAX, EAX touching the sides at D and E; then the tangent DK is equal to the tangent EL.

Demonstration.—Join DE. We have shown in Lemma 10 that in this case the intercepted chords DF, GE are equal. Subtract them from the whole line DE, and the remainders DG, EF will be equal. Therefore

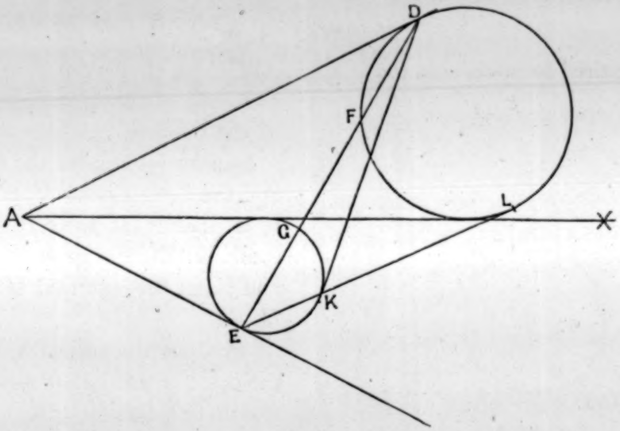

Fig. 8.

$$DG \,.\, DE = EF \,.\, ED \quad \therefore \; DK^2 = EL^2$$

and

$$\therefore \; DK = EL$$

Corollary.—Conversely, if DK = EL it follows that the angle DAX = angle EAX.

For, if those angles are not equal, let DAX be the greater. Then because the angle DAX is greater than the angle EAX, the chord DF is greater than the chord EG (by Lemma 10, corollary 2). Subtract them successively from the line DE, and the remainder EF will be smaller than the remainder DG, therefore EF . ED is less than GE . DE; therefore EL^2 is less than DK^2; and EL is less than DK. But on the contrary EL = DK by hypothesis. Consequently it is not true that the angle DAX is greater than the angle EAX. In the same way it is shown that it is not less; consequently it is equal to EAX. *Q.E.D.*

Lemmas 10 and 11 are particular cases of a much more general theorem which I propose to give on another occasion. In the first nine Lemmas I have chiefly followed Plücker, but have endeavoured to make his demonstrations more rigorous by going more into detail than he has done. But Lemmas 10 and 11, and the more general theorem of which they are particular cases, are original; at least I am not aware of their having been published elsewhere. They supply the link that was missing in PLÜCKER's investigation, and singularly facilitate the demonstration of STEINER's elegant construction.

By the help of the preceding Lemmas, we can show the truth of that construction in the following manner:—

Malfatti's Problem.

It is required to inscribe in the triangle ABC three circles touching each other, and each of them touching two sides of the triangle.

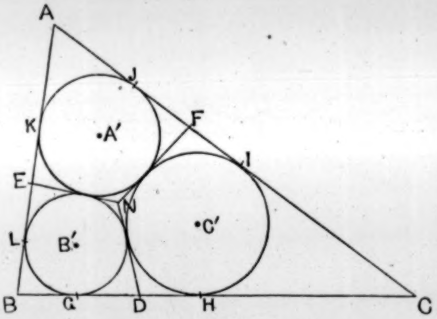

Fig. 9.

Solution.—Suppose the thing done, and the three circles A′, B′, C′, found, it is plain that their three internal common tangents meet in a point N, and bisect the external tangents GH, IJ, KL.

Produce the lines EN, FN beyond the point N until they meet the base BC in two points Q, R. Then if in the triangle QRN so formed, we inscribe a circle which may be[*] called α, it will touch the base BC in the point D (by Lemma 3).

By an exactly similar process we obtain a circle β, touching DN, EN produced, and the side AC in F; and a circle γ touching DN, FN produced, and the side AB in E.

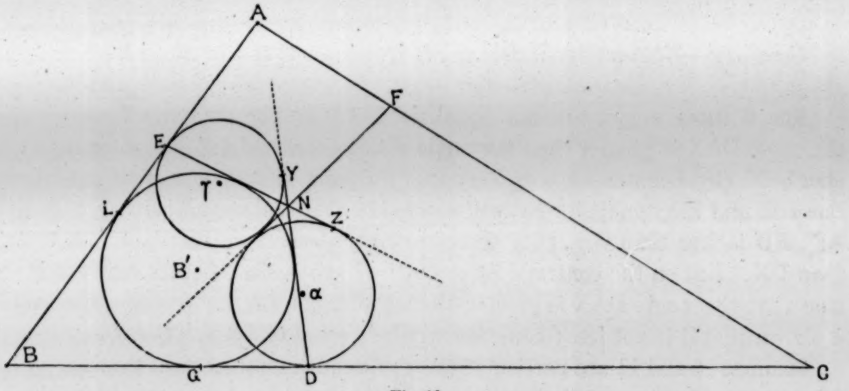

Fig. 10.

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[*] I have called it α, because it stands opposite to the angle A of the original triangle. Similarly for the names of β and γ. I have called α, β, γ the *secondary* circles.

This will be seen better by referring to fig. 10, in which, to avoid confusion, I have only represented one of the required circles B′, and the two secondary circles α, γ, which belong to it. I have represented the prolongations of the three tangents DN, EN, FN by dotted lines. The secondary circles touch these dotted lines, and also touch the sides of the triangle at D and E, where the tangents intersect them. A simple inspection of the figure suffices to show that the tangent DY drawn from D, which touches both the circles B′ and γ, is the sum of two parts, which equal the external tangents DG and EL respectively. And that the tangent EZ, drawn from E, which touches both the circles B′ and α, is the sum of two parts, which equal the external tangents EL and DG respectively. Therefore, these two tangents DY, EZ are equal to each other, since each of them equals DG + EL.

But the three lines DN, EN, FN meet in one point at N; that is, the three internal common tangents of the circles α, β, γ meet in one point. Therefore, by Lemma 8, their *other* three common tangents must also meet in one point, which point may be called O.

Moreover, the line DN produced is the external tangent of the circles B′ and γ; and EN produced is the external tangent of the same circle B′ and circle α; and (as we said before), FN produced is the *internal* tangent of the circles α and γ. But DN, EN, FN concur in a point; that is, two external and one internal common tangents of the circles α, γ, and B′ concur in a point. Therefore, by Lemma 9 the other three *corresponding* tangents of those circles meet in a point. And it is easy to see what that point is. For, the *second* external common tangent of the circles B′ and γ, is AB, one of the sides of the given triangle; and the *second* external common tangent of the circles B′ and α is BC, one of the sides of the triangle.

But we know the point of concourse of AB and BC, to be at B, one of the vertices of the triangle. Consequently, we attain this important result, that the second common internal tangent of the circles α and γ passes through the angular point B of the triangle. In a similar way, it may be shown that the second common internal tangent of the circles α and β passes through the angle C; and that the second common internal tangent of the circles β and γ passes through the angle A.

These three tangents are therefore three lines proceeding from A, B, C, the three angles of the given triangle, and meeting *in a single point* (which a few lines previously we named O). Now STEINER affirms that this point O is the centre of the inscribed circle of the given triangle ABC; and this is the theorem which PLÜCKER and other geometers have been unable to prove except by the use of analytical methods of investigation. But here we call to our assistance the Lemma 11, which we have demonstrated above; and we proceed as follows. Since we have shown that the line BO touches the circle α, and also the circle γ. And since we have also shown that the line DY drawn from the point of contact

D to the circle γ, is equal to the line EZ drawn from the point of contact E to touch the circle a. Therefore by the corollary to Lemma 11, the line BO necessarily bisects the angle B of the original triangle. Similarly it is shown that CO bisects the angle C, and that AO bisects the angle A. Therefore, O is the centre of the inscribed circle of the triangle ABC:—which is STEINER'S Theorem. The solution of MALFATTI'S Problem is therefore as follows:—

Bisect the angles of the triangle ABC, by the lines AO, BO, CO. In two of the smaller triangles thus made AOB, BOC, inscribe the circles γ and a. From D, the point of contact of a with the side BC, draw a line DY, touching the circle γ. Then DY will touch one of the required circles also; which circle also touches AB, BC, two sides of the triangle, and is therefore wholly determined.

XII.—*On the Law of Frequency of Error.* By Professor Tait.

(Read 3d January 1865.)

1. It has always appeared to me that the difficulties which present themselves in investigations concerning the Frequency of Error, and the deduction of the most probable result from a large number of observations by the *Method of Least Squares* (which is an immediate consequence of the ordinary "Law of Error"), are difficulties of reasoning, or logic, rather than of analysis. Hence I conceive that the elaborate analytical investigations of Laplace, Poisson, and others, do not in anywise present the question in its intrinsic simplicity. They seem to me to be necessitated by the unnatural point of view from which their authors have contemplated the question. It is, undoubtedly, a difficult one; but this is a strong reason for abstaining from the use of unnecessarily elaborate analysis, which, however beautiful in itself, does harm when it masks the real nature of the difficulty it is employed to overcome. I believe that, so far at least as mathematics is concerned, the subject ought to be found extremely simple, if we only approach it in a natural manner.

2. It occurred to me lately, while I was writing an elementary article on the Theory of Probabilities, that such a natural process might possibly be obtained by taking as a basis one of the common problems in probabilities, viz.:—*To find the relative probabilities of different combinations of mutually exclusive simple events in the course of a large number of trials.*

3. In fact, this is really the basis of Laplace's investigation, an elegant, but very troublesome piece of analysis. With the view, apparently, of attaining the utmost possible generality, he considers an error to be made up of an infinite number of contributions, each from a separate source. But he assumes at starting, that these separate contributions are as likely to be of one magnitude as another, which is, to say the least, questionable; as it seems to be inconsistent with the result finally arrived at. For instance, by far the larger part of the probability of a given finite error is thus made to depend upon a great number of infinite positive contributions, combined with a proper allowance of infinite negative ones. Now, though it is not a harsh assumption to suppose that finite effects should be, in certain cases, the results of additive and subtractive operations with infinite quantities, it does appear unlikely in the extreme, that finite effects should be due to such operations in a far greater measure than to operations with finite quantities. It is true that Laplace subsequently shows that the same law will be arrived at by assuming any law of probability for the contributions to

the error from each separate cause, provided positive and negative errors of equal amount are equally likely; but it is the complexity, not the sufficiency, of his processes, which I think requires attention.

4. Gauss' investigation is founded on the assumption, that *the arithmetical mean, of the results deduced from equally trustworthy observations, is the most probable value of the quantity sought.* So far as I can see, Ellis[*] has satisfactorily shown that this, however apparently natural, is not justifiable as an *à priori* assumption. In fact, it would seem that we have no right to assume that, because errors of equal magnitude and opposite signs are equally likely, their *sum* will vanish in a large number of trials, any more than that the sum of their third or fifth powers will vanish. Why the first powers should be chosen, appears to arise from the extreme simplicity of the requisite operations; yet, though complexity of calculations is undesirable, it must be submitted to, if necessary for the evolution of truth. The principle of the arithmetical mean has been adopted, among a multitude of others equally likely, just as we might suppose a calculator to insist on gravity varying as the direct distance instead of its inverse square, on the ground that the problem of Three Bodies would then become as simple and its solution as exact, as they are now complicated, and at best only approximate. "*La nature ne s'est pas embarrassée des difficultés d'analyse, elle n'a évité que la complication des moyens,*" in the words of Fresnel.

5. It is with some hesitation that I communicate the present paper to the Society; for I have not devoted much time to the study of the Theory of Probabilities; and I know well how easy it is to fall into the gravest errors of reasoning on such a subject, from the fact that D'Alembert, Ivory, and many others, have published investigations and proofs (sometimes in its most elementary parts), which are now seen to be entirely fallacious.

6. I proceed to show how I think the principle, above (§ 2) enunciated, may be applied. The most direct method would be, of course, to assume any one set of causes of error whatever, and to determine what will, in the long run, be the chance of each separate amount of error as due to their joint action. Supposing this to be determined, let us try to combine the probabilities of error from any indefinite number of sets of possible causes; and, if this process should lead to a definite law of error, such will be the law to which, by an inverse application of the Theory of Probabilities, we should expect each separate observation to be subject. But this process, which is analogous to that of Laplace, though not identical with it, cannot easily be carried out, for it essentially involves in its first steps the assumption of a law of error which it is the object of the investigation to determine. We must try a less direct method.

7. We shall, therefore, investigate what must be, in the long run, the chance of any combination whatever of independent events, and *consider the deviation of this combination from the most probable combination as the Error, and the ratio of its probability to that of the most probable combination, as the function which expresses the Law of Error.* If we find, as we proceed, that the law thus arrived at, is (in *form* at least) totally independent of the number, variety, &c., of the several simultaneously acting causes, we shall thus have a very strong argument in favour of the correctness of the process; whose real difficulty, be it remembered, is logical and not mathematical. The mathematical processes to be employed below are, of course, known, and will be found in most treatises on Algebra; but, for the present application, it will be convenient to put them in a form slightly different from the usual one.

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[*] Cambridge Phil. Trans. viii. p. 205.

8. Taking the simplest case, let us suppose a bag to contain white and black balls, whose numbers are as $p:q$, where $p+q=1$. The chance of drawing α white, and β black, balls in n $(=\alpha+\beta)$ drawings, replacing before each drawing, and disregarding the order in which they appear, is,—

$$\frac{|\underline{n}}{|\underline{\alpha}\;|\underline{\beta}}\, p^{\alpha} q^{\beta} \quad . \quad . \quad . \quad . \quad . \qquad (1)$$

This is a maximum, when $\alpha:\beta::p:q$; which, when n is indefinitely great, can always be exactly attained. This maximum value is,—

$$\frac{|\underline{n}}{|\underline{pn}\;|\underline{qn}}\, p^{pn} q^{qn} \quad . \quad . \quad . \quad . \quad . \quad . \qquad (2)$$

The ratio of these two numbers is,—

$$\frac{|\underline{pn}\;|\underline{qn}}{|\underline{\alpha}\;|\underline{\beta}}\, p^{\alpha-pn} q^{\beta-qn} \quad . \quad . \quad . \quad . \quad . \quad . \qquad (3)$$

Now, according to the principle above assumed, we must treat $\alpha-pn$, the deviation from the most probable result, as measuring the error in some observation, while the expression (3) measures the probability of it, as compared with that of the most probable result. To introduce the ordinary notation, let x be the error, and y the (indefinitely small) probability of that error; then, A and m being constants,—

$$\alpha-pn=mx, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (4)$$

while y may be expressed as the product of (3) into A, that is, by (4),

$$y=\mathrm{A}\,\frac{|\underline{pn}\;|\underline{qn}}{|\underline{pn+mx}\;|\underline{qn-mx}}\, p^{mx} q^{-mx} \quad . \quad . \quad . \qquad (5)$$

When n is a large number, the value of this is easily found from STIRLING's Theorem, viz.—

$$1\,.\,2\,.\,3\;\ldots\ldots\;n=|\underline{n}=\sqrt{2\pi}\;n^{n+\frac{1}{2}}\epsilon^{-n}\left(1+\frac{1}{12n}+\&c.\right)$$

where the inverse powers of n may be neglected if n is large. For (5) thus becomes,

$$y=\mathrm{A}\frac{(pn)^{pn+\frac{1}{2}+mx}(qn)^{qn+\frac{1}{2}-mx}}{(pn+mx)^{pn+mx+\frac{1}{2}}(qn-mx)^{qn-mx+\frac{1}{2}}}$$

$$=\mathrm{A}\frac{1}{\left(1+\frac{mx}{pn}\right)^{pn+mx+\frac{1}{2}}\left(1-\frac{mx}{qn}\right)^{qn-mx+\frac{1}{2}}}$$

Hence, $$\log y-\log \mathrm{A}=-(pn+mx+\tfrac{1}{2})\log\left(1+\frac{mx}{pn}\right)-(qn-mx+\tfrac{1}{2})\log\left(1-\frac{mx}{qn}\right)$$

$$=-(pn+mx+\tfrac{1}{2})\left\{\frac{mx}{pn}-\frac{m^2x^2}{2p^2n^2}+\frac{m^3x^3}{3p^3n^3}-\&c.\right\}-(qn-mx+\tfrac{1}{2})\left\{-\frac{mx}{qn}-\frac{m^2x^2}{2q^2n^2}-\frac{m^3x^3}{3q^3n^3}-\&c.\right\}$$

$$=-\frac{m^2x^2}{2n}\left(\frac{1}{p}+\frac{1}{q}\right)+\frac{m^3x^3}{6n^2}\left(\frac{1}{p^2}-\frac{1}{q^2}\right)-\frac{m^4x^4}{12n^3}\left(\frac{1}{p^3}+\frac{1}{q^3}\right)+\&c.$$

$$-\frac{mx}{2n}\left(\frac{1}{p}-\frac{1}{q}\right)+\frac{m^2x^2}{4n^2}\left(\frac{1}{p^2}+\frac{1}{q^2}\right)+\&c.$$

The first term of this expression is finite when mx is of the order $n^{\frac{1}{2}}$; and in this case the other terms in the first line are infinitely small, being of the orders $n^{-\frac{1}{2}}$, n^{-1}, &c. respectively. The latter remark applies to the second line of the expression, which depends upon the ½ in the exponents. When mx is of an order higher than $n^{\frac{1}{2}}$, it is obvious from the undeveloped form that the expression must be infinitely large, and negative. Hence, generally, we may neglect all but the first term, and we have therefore

$$y=\mathrm{A}\epsilon^{-\frac{m^2x^2}{2pqn}}$$

$$=\mathrm{A}\epsilon^{-\mu x^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (6),$$

which is the ordinary expression.

9. This shows that, as is well known, the chance of a result differing x from the most probable combination is, in this very simple case, represented by a number proportional to $\epsilon^{-\mu x^2}$ times that of the most probable event. But if we now consider, not *one* but, *any number of* causes conspiring to produce the observed result, we find that the law is still precisely the same in *form*, and this *whether the most probable event be the same as regards each cause or not.* And it is this fact which appears completely to justify the proposed method of regarding the question.

10. For, if the various causes all tend to produce the *same* most probable event, *its* probability will be, by (6),

$$\mathfrak{A}=\mathrm{A}_1\mathrm{A}_2\mathrm{A}_3 \ldots \mathrm{A}_\nu \quad . \quad . \quad . \quad . \quad . \qquad (7)$$

while that of a result, whose error is x, will be

$$y=y_1y_2y_3 \ldots y_\nu=\mathfrak{A}\epsilon^{-(\mu_1+\mu_2+\ldots+\mu_\nu)x^2}=\mathfrak{A}\epsilon^{-\mathrm{M}x^2} \quad . \quad . \qquad (8)$$

$$(\text{where } \mathrm{M}=\mu_1+\mu_2+\mu_3+\ldots+\mu_\nu)$$

which is the same form as (6).

If the most probable result, as depending on the several sets of causes, be different for each, the formula (6) becomes, for any one cause,

$$y = \mathrm{A}\epsilon^{-\mu(x-\gamma)^2} \quad . \quad . \quad . \quad . \quad . \qquad (9),$$

where A is the (small) chance of the most probable result, which is, of course, $x = \gamma$.

The chance of any particular value of x, as due to the simultaneous action of all the causes, is now

$$y = \mathrm{A}_1 \ . \ . \ . \ . \ \mathrm{A}_r \epsilon^{-\mu_1(x-\gamma_1)^2 - \ . \ . \ . \ . \ - \mu_r(x-\gamma_r)^2} \quad . \quad . \qquad (10),$$

which may, of course, be put in the form

$$y = \mathfrak{A}\epsilon^{-\mathrm{M}(x-\Gamma)^2} \quad . \quad . \quad . \quad . \quad . \qquad (11),$$

where the most probable result is now

$$x = \Gamma = \frac{\mu_1\gamma_1 + \mu_2\gamma_2 + \ . \ . \ . \ . \ + \mu_r\gamma_r}{\mu_1 + \mu_2 + \ . \ . \ . \ . \ + \mu_r},$$

while

$$\mathfrak{A} = \mathrm{A}_1 \ . \ . \ . \ . \ \mathrm{A}_r \epsilon^{-(\mu_1\gamma_1^2 + \ . \ . \ . \ . \ + \mu_r\gamma_r^2) + \mathrm{M}\Gamma^2}$$

(where, as before, $\mathrm{M} = \mu_1 + \mu_2 + \ . \ . \ . \ . \ + \mu_r$)

is its probability.

If we take this as our point of departure for the error x, we must write x for $x - \Gamma$, and we have

$$y = \mathfrak{A}\epsilon^{-\mathrm{M}x^2} \quad . \quad . \quad . \quad . \quad . \quad . \qquad (12),$$

for the form of the law of error, which is precisely that of (6) deduced from the simplest conceivable case.

11. Another remarkable confirmation of the validity of the process suggested above, is to be found in the fact that not only are the *curves* expressed by equations such as (6) and (9) compounded, by multiplication of corresponding ordinates, into another of the same class, whatever be the positions of their axes of symmetry, but that the same principle holds good in three, four, &c., dimensions also.

Thus, any number of hills on the plane of xy, represented by equations such as

$$z = \mathrm{A}\epsilon^{-\mu[(x-\alpha)^2 + (y-\beta)^2]} \quad . \quad . \quad . \quad . \quad . \quad . \qquad (13),$$

give, by multiplication of their corresponding ordinates, another hill of the same general form, the *values* only of the constants being changed.

[Many curious geometrical results may be derived from this construction. One of the most singular is the fact that the projection on xy of the line of intersection of any two surfaces whose equations are of the form (13) is a *circle*, and that another such surface (viz., that whose ordinates are mean proportionals between those of the former) can be described, passing through the curve of double curvature of which this circle is the projection. But, besides being foreign to our subject, these theorems follow at once from well-known properties of circles.]

12. Returning to equation (12), it is obvious that $\mathfrak{A}$ and M must be connected, since we have to satisfy the condition that the probability that the error lies between infinite positive and negative limits is certainty. Hence, as we may write

$$\mathfrak{A}\epsilon^{-\mathrm{M}x^2}\delta x \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14),$$

for the chance that the error lies between x and $x+\delta x$; we must have

$$\mathfrak{A}\int_{-\infty}^{+\infty}\epsilon^{-\mathrm{M}x^2}dx=1. \quad . \quad . \quad . \quad . \quad . \quad . \quad (15),$$

But we know that

$$\int_{-\infty}^{+\infty}\epsilon^{-y^2}dy=\sqrt{\pi},$$

which reduces (15) at once to the form

$$\mathfrak{A}\sqrt{\frac{\pi}{\mathrm{M}}}=1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16),$$

the required relation.

13. It is obvious from (12) that large errors have less probability when M is large; that is when h is small, if we put

$$\mathrm{M}=\frac{1}{h^2}.$$

Hence h becomes an indication of the comparative accuracy of the process whose errors we are testing, and it is thus desirable to retain it in the expression for the law of error.

By (16) we have

$$\mathfrak{A}=\frac{1}{h\sqrt{\pi}},$$

and therefore, by (14), we obtain

$$\frac{1}{h\sqrt{\pi}}\epsilon^{-\frac{x^2}{h^2}}\delta x$$

for the chance that the error lies between x and $x+\delta x$, the usual expression.

14. It only remains that we give an idea of the accuracy with which this law of error is approximated to, in cases such as we have assumed as the basis of our reasoning, even in a very small number of trials. For this purpose we take the case of 20 tosses of a coin. Here the most probable result is, of course, 10 heads and 10 tails, and the chances of the various possible combinations are the terms of the expansion of

$$\left(\frac{1}{2}+\frac{1}{2}\right)^{20}.$$

If we erect these as ordinates at successive distances, each equal to unit, along a line, we may graphically represent their relative values by a curve drawn, *liberâ manu*, through their extremities. The area of this curve will evidently approximate to unity, which is the exact value of the sum of the areas of the rectangles of unit breadth, each of which is bisected by one of the ordinates laid down from the expansion.

To find the corresponding curve of error, notice that the maximum ordinate is

$$\frac{20\,.\,19\,\ldots\ldots\,11}{1\,.\,\;2\,\ldots\ldots\,10}\cdot\frac{1}{2^{20}}=\frac{184756}{1048576}=0{\cdot}1762.$$

Taking this as the value of $\frac{1}{h\sqrt{\pi}}$ we have for (12) the expression

$$y=\frac{1}{5{\cdot}675}\epsilon^{-\frac{x^2}{10{\cdot}253}} \qquad \ldots\ldots \qquad (17).$$

The following table shows a few of the values of y from this formula, compared with the corresponding terms in the binomial: it is sufficient for our purpose, as it would not be worth while to take the trouble of calculating the *areas* of the curve of error corresponding respectively to the rectangles above mentioned.

x.	y from (17).	y from Binomial.	Difference.
0	0·1762	0·1762	0·0000
1	0·1598	0·1602	−0·0004
2	0·1193	0·1201	−0·0008
3	0·0733	0·0739	−0·0006
4	0·0370	0·0369	+0·0001
5	0·0154	0·0148	+0·0006
6	0·0053	0·0046	+0·0007

15. Nothing is better calculated to show the general soundness of the method we have adopted in this paper, than the fact of the excessive closeness of the above approximation: the case having been specially chosen as one in which we could hardly have expected more than a rude resemblance to the law of error.

XIII.—*On the Application of Hamilton's Characteristic Function to Special Cases of Constraint.* By Professor Tait.

(Read 20th March 1865.)

1. One of the grandest steps which has ever been made in Dynamical Science is contained in two papers, "*On a General Method in Dynamics,*" contributed to the Philosophical Transactions for 1834 and 1835 by Sir W. R. Hamilton. It is there shown that the complete solution of any kinetical problem, involving the action of a given conservative system of forces, and constraint depending upon the reaction of smooth guiding curves or surfaces, also given, is reducible to the determination of a single quantity called the *Characteristic Function* of the motion. This quantity is to be found from a partial differential equation of the first order, and second degree; and it has been shown that, from any *complete* integral of this equation, all the circumstances of the motion may be deduced by differentiation. So far as I can discover, this method has not been applied to inverse problems, of the nature of the Brachistochrone for instance, where the object aimed at is essentially the determination of the constraint requisite to produce a given result. It is easy to see, however, that a large class of such questions may be treated successfully by a process perfectly analogous to that of Hamilton; though the characteristic function in such cases is not the same function (of the quantities determining the motion) as that of the Method of *Varying Action.*

2. It is unnecessary to enter into any great detail with reference to the present subject; because any one who is familiar with Hamilton's beautiful investigations will have no difficulty in applying them, with the requisite slight modifications, to the subject of this paper. I shall therefore content myself with a brief explanation of the application of the method to the problem of the Brachistochrone, and a mere indication of some other curious problems which are easily solved in a similar manner.

3. The problem of the Brachistochrone for a single particle is, in its simplest form, as follows:—

Find the form of the (smooth) constraining curve along which a particle will pass, under the action of a given conservative system of forces, from one given point to another in the least possible time, the initial velocity being given.

The problem may easily be complicated by supposing, for instance, the terminal points not to be definitely assigned, but to lie each on a given surface:

still farther, by supposing the initial velocity to depend, according to some given law, upon the coördinates of the initial point, and so forth. But such complications introduce analytical difficulties of the quasi-arithmetical kind merely, not of a physical nature; and we leave them to those who are curious in such matters.

4. In symbols, if τ be the time of passing from x_0, y_0, z_0 to x, y, z, we must have

$$\tau = \int_{x_0, y_0, z_0}^{x, y, z} \frac{ds}{v}$$

a minimum: subject to the sole condition

$$v^2 = 2\,(\mathrm{H} - \mathrm{V})$$

where H is the whole energy, and V the potential of the system of forces on unit mass at the point x, y, z.

Hence, taking the variation,

$$\delta\tau = \int\left(\frac{d\delta s}{v} - \frac{ds\delta v}{v^2}\right).$$

But $$ds d\delta s = dx d\delta x + dy d\delta y + dz d\delta z;$$

and $$v\delta v = \delta(\mathrm{H} - \mathrm{V}) = \mathrm{X}\delta x + \mathrm{Y}\delta y + \mathrm{Z}\delta z + \delta\mathrm{H},$$

if X, Y, Z be the component forces on unit mass at x, y, z. Thus we have

$$\delta\tau = \left[\frac{1}{v^2}\left(\frac{dx}{dt}\,\delta x + \frac{dy}{dt}\,\delta y + \frac{dz}{dt}\,\delta z\right) - \delta\mathrm{H}\int\frac{ds}{v^3}\right]$$

$$-\int\left\{\delta x\left[d\left(\frac{\frac{dx}{dt}}{v^2}\right) + \frac{\mathrm{X}dt}{v^2}\right] + \&c.\right\};$$

where the whole, integrated or not, is to be taken between the given limits.

If the limits and the initial velocity be fixed, the first part of the expression for $\delta\tau$ disappears; and, that the integral may vanish, we must have

$$d\left(\frac{\frac{dx}{dt}}{v^2}\right) + \frac{\mathrm{X}dt}{v^2} = 0, \quad . \quad . \quad . \quad . \quad . \quad (\mathrm{A}).$$

with similar equations in y and z. This is simply the ordinary result given in treatises on kinetics.

But if we consider the effect of the alteration of the limits, or of the initial energy, we have

$$\left.\begin{array}{lcc} & \dfrac{\delta\tau}{\delta x}=\dfrac{1}{v^2}\dfrac{dx}{dt}, & \dfrac{\delta\tau}{\delta x_0}=-\left(\dfrac{1}{v^2}\dfrac{dx}{dt}\right)_0, \\ & \&\text{c.} & \&\text{c.} \\ \text{and} & \multicolumn{2}{c}{\dfrac{\delta\tau}{\delta\mathrm{H}}=-\displaystyle\int_{x_0,\,y_0,\,z_0}^{x,\,y,\,z}\dfrac{ds}{v^3}.} \end{array}\right\} \quad . \quad . \quad . \quad (1).$$

5. Hence, if τ could be found as a function of x, y, z, x_0, y_0, z_0, and H, it is obvious that its partial differential coefficients with respect to these quantities would give the motion completely.

But, neglecting altogether the initial limit, we see that

$$\left(\frac{d\tau}{dx}\right)^2+\left(\frac{d\tau}{dy}\right)^2+\left(\frac{d\tau}{dz}\right)^2=\frac{1}{v^4}\left(\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2+\left(\frac{dz}{dt}\right)^2\right)$$

$$=\frac{1}{v^2}=\frac{1}{2(\mathrm{H}-\mathrm{V})} \quad . \quad . \quad . \quad . \quad (2).$$

6. It can be easily shown, by a process similar to that employed for *Varying Action*,* that, if any integral of this equation can be found, its partial differential coefficients with respect to x, y, z are respectively equal to the corresponding components of the velocity, in a curve which is a brachistochrone for the given forces, *each divided by the square of the whole velocity.*

A *complete* integral of (2) must of course contain, besides H, two arbitrary constants α, β. If, then, τ be a complete integral, the equations of the brachistochrone are easily shown to be

$$\frac{d\tau}{d\alpha}=\mathfrak{A}, \qquad \frac{d\tau}{d\beta}=\mathfrak{B} \quad . \quad . \quad . \quad . \quad . \quad . \quad (3);$$

where $\mathfrak{A}$ and $\mathfrak{B}$ are two new arbitrary constants.

Also we have the relation

$$\frac{d\tau}{d\mathrm{H}}=-\int\frac{dt}{v^2}=-\int\frac{ds}{v^3} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4).$$

7. Before proceeding farther with the theory, we may apply the results already obtained to one or two well-known problems; commencing with the original case proposed by BERNOULLI.

8. *To find the brachistochrone, when gravity is the only impressed force, and the particle has the velocity due to a fall from a given horizontal plane.*

Taking the axis of y vertically downwards, we have

$$\mathrm{V}=-gy.$$

Also, we may write

$$\mathrm{H}=g\alpha.$$

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* THOMSON and TAIT's Natural Philosophy, § 323, or TAIT and STEELE's Dynamics of a Particle (2d edition), §§ 252, 253.

Hence

$$\left(\frac{d\tau}{dx}\right)^2+\left(\frac{d\tau}{dy}\right)^2+\left(\frac{d\tau}{dz}\right)^2=\frac{1}{2g(a+y)}.$$

This equation is obviously satisfied by

$$\left(\frac{d\tau}{dx}\right)=\mathrm{M},\qquad \left(\frac{d\tau}{dz}\right)=\mathrm{N},\qquad \left(\frac{d\tau}{dy}\right)^2=\frac{1}{2g(a+y)}-\mathrm{M}^2-\mathrm{N}^2.$$

But

$$\frac{\left(\frac{d\tau}{dx}\right)}{\left(\frac{d\tau}{dz}\right)}=\frac{\frac{dx}{dt}}{\frac{dz}{dt}}\ (\text{by § 6})\ =\frac{dx}{dz}.$$

Hence $\frac{dx}{dz}=\frac{\mathrm{M}}{\mathrm{N}}$, that is *the path is in a vertical plane.* We may take this as the plane of xy. Hence our equation becomes

$$\left(\frac{d\tau}{dx}\right)^2+\left(\frac{d\tau}{dy}\right)^2=\frac{1}{2g(a+y)}.$$

We may now write

$$\left.\begin{aligned}\frac{d\tau}{dx}&=\frac{1}{\sqrt{2gb}}\\ \left(\frac{d\tau}{dy}\right)^2&=\frac{1}{2g}\left(\frac{1}{a+y}-\frac{1}{b}\right),\end{aligned}\right\}\quad .\quad .\quad .\quad .\quad .\quad (5).$$

where b is an arbitrary constant.

By (5) we have, at once,

$$\sqrt{2g}\,\tau=\frac{x}{\sqrt{b}}+\int dy\sqrt{\frac{1}{a+y}-\frac{1}{b}}\quad .\quad .\quad .\quad .\quad .\quad (6).$$

Hence the equation of the brachistochrone is (by § 6)

$$\frac{d\tau}{db}=\text{const.}$$

or

$$\mathrm{C}=-\frac{x}{b^{\frac{3}{2}}}+\frac{1}{b^2}\int\frac{dy}{\sqrt{\frac{1}{a+y}-\frac{1}{b}}};$$

that is, changing the constant, and effecting the integration,

$$\mathrm{C}_1=-x-\sqrt{(b-\overline{a+y})\,(a+y)}+\frac{b}{2}\,\text{vers.}^{-1}\,\frac{2(a+y)}{b}\quad .\quad .\quad .\quad .\quad (7).$$

the common equation of the *Cycloid*, the velocity at any point being that due to a fall from the base.

In this case we have evidently

$$\frac{d\tau}{dH} = -\int\frac{ds}{v^3} = \frac{1}{g}\,\frac{d\tau}{da} = -\,\frac{1}{2\sqrt{2g^3}}\int\frac{dy}{(a+y)^2\sqrt{\frac{1}{a+y}-\frac{1}{b}}}$$

$$= \frac{1}{\sqrt{2g^3}}\sqrt{\frac{1}{a+y}-\frac{1}{b}} + C_2 .$$

The above (at first sight apparently too limited) assumptions

$$\frac{d\tau}{dx} = M, \quad \frac{d\tau}{dz} = N,$$

and the consequent reduction of the question to a *plane* problem, may seem to require some justification. This is easily supplied, thus: In the equation

$$\left(\frac{d\tau}{dx}\right)^2 + \left(\frac{d\tau}{dy}\right)^2 + \left(\frac{d\tau}{dz}\right)^2 = F^2,$$

the direction-cosines of the tangent to the brachistochrone, at the point x, y, z, are, by (1.),

$$l = \frac{1}{F}\frac{d\tau}{dx}, \quad m = \frac{1}{F}\frac{d\tau}{dy}, \quad n = \frac{1}{F}\frac{d\tau}{dz} .$$

At the adjacent point $x+\delta x$, $y+\delta y$, $z+\delta z$, where we have, of course,

$$\frac{\delta x}{l} = \frac{\delta y}{m} = \frac{\delta z}{n} = \delta s,$$

the value of l becomes

$$l' = \frac{\frac{d\tau}{dx} + \frac{d^2\tau}{dx^2}\delta x + \frac{d^2\tau}{dxdy}\delta y + \frac{d^2\tau}{dxdz}\delta z}{F + \delta F}$$

$$= \frac{\frac{d\tau}{dx} + \frac{\delta s}{F}\left(\frac{d\tau}{dx}\frac{d^2\tau}{dx^2} + \frac{d\tau}{dy}\frac{d^2\tau}{dxdy} + \frac{d\tau}{dz}\frac{d^2\tau}{dxdz}\right)}{F + \delta F}$$

$$= \frac{\frac{d\tau}{dx} + \left(\frac{dF}{dx}\right)\delta s}{F + \delta F} .$$

But in the above problem F *is a function of* y *only*, and we must therefore have

$$\frac{l'}{n'} = \frac{l}{n},$$

which shows that the curve is in a plane parallel to the axis of y.

9. *To find the Brachistochrone when the force is central, and proportional to a power of the distance; the velocity being also proportional to a power of the distance, that is, being the velocity from infinity if the force is attractive, from the centre if it is repulsive.*

Here
$$v^2 = 2(\mathrm{H}-\mathrm{V}) = \frac{\mu}{r^n},$$

and the central force at distance r is evidently

$$-\frac{d\mathrm{V}}{dr} = -\frac{n\mu}{2r^{n+1}}.$$

Thus (2) becomes

$$\left(\frac{d\tau}{dx}\right)^2 + \left(\frac{d\tau}{dy}\right)^2 + \left(\frac{d\tau}{dz}\right)^2 = \frac{r^n}{\mu}$$

or, changing to polar co-ordinates,

$$\left(\frac{d\tau}{dr}\right)^2 + \frac{1}{r^2}\left(\frac{d\tau}{d\theta}\right)^2 + \frac{1}{r^2\sin^2\theta}\left(\frac{d\tau}{d\phi}\right)^2 = \frac{r^n}{\mu}.$$

It is obvious that we must take

$$\frac{d\tau}{d\phi} = 0,$$

which shows that the path is in a plane passing through the centre of force. The above equation will then be satisfied by

$$\frac{d\tau}{d\theta} = a, \qquad \frac{d\tau}{dr} = \sqrt{\frac{r^n}{\mu} - \frac{a^2}{r^2}}.$$

Hence we have

$$\tau = a\theta + \int dr\, \sqrt{\frac{r^n}{\mu} - \frac{a^2}{r^2}},$$

$$= a\theta + \frac{2a}{n+2}\left\{\sqrt{\frac{r^{n+2}}{\mu a^2} - 1} - \cos^{-1}\frac{\sqrt{\mu}a}{r^{\frac{n+2}{2}}}\right\} + \mathrm{C}.$$

And the equation of the brachistochrone, which is evidently a plane curve, is

$$\mathfrak{A} = \theta + \frac{2}{n+2}\left\{\sqrt{\frac{r^{n+2}}{\mu a^2} - 1} - \cos^{-1}\frac{\sqrt{\mu}a}{r^{\frac{n+2}{2}}}\right\}$$

$$+ \frac{2a}{n+2}\left\{-\frac{\frac{r^{n+2}}{\mu a^3}}{\sqrt{\frac{r^{n+2}}{\mu a^2} - 1}} + \frac{\sqrt{\mu}}{r^{\frac{n+2}{2}}}\frac{1}{\sqrt{1 - \frac{\mu a^2}{r^{n+2}}}}\right\}$$

$$= \theta - \frac{2}{n+2}\cos^{-1}\frac{\sqrt{\mu}a}{r^{\frac{n+2}{2}}}:$$

or
$$r^{\frac{n+2}{2}} = \sqrt{\mu}a \sec\frac{n+2}{2}(\theta - \mathfrak{A}),$$

while the equation of the *free* path is

$$\left(\frac{r}{a}\right)^{\frac{n-2}{2}} = \cos\frac{n-2}{2}(\theta+\beta).$$

The above integration fails in the case of $n = -2$; that is, when the force is repulsive and directly as the distance, the velocity vanishing at the centre of force. But in this case

$$\tau = a\theta + \sqrt{\frac{1}{\mu} - a^2}\,\log Cr,$$

and the equation of the brachistochrone is

$$\mathfrak{A} = \theta - \frac{a}{\sqrt{\frac{1}{\mu} - a^2}}\log Cr,$$

the logarithmic spiral. Eliminating r between these equations, we see that the time is proportional to the polar angle.

Since a definite form has been assigned to the expression for the velocity in this problem, it is obvious that H is given, and therefore that there is no $\frac{d\tau}{dH}$.

The assumption

$$\frac{d\tau}{d\phi} = 0$$

is easily justified, in the case of any equation of the form

$$\left(\frac{d\tau}{dr}\right)^2 + \frac{1}{r^2}\left(\frac{d\tau}{d\theta}\right)^2 + \frac{1}{r^2\sin^2\theta}\left(\frac{d\tau}{d\phi}\right)^2 = F^2,$$

if F be a function of r only. For

$$\delta\left(\frac{d\tau}{d\phi}\right) = \frac{d^2\tau}{drd\phi}\delta r + \frac{d^2\tau}{d\theta d\phi}\delta\theta + \frac{d^2\tau}{\delta\phi^2}\delta\phi.$$

But

$$\frac{d\tau}{dr} = F^2\frac{dr}{dt}, \qquad \frac{d\tau}{rd\theta} = F^2\frac{rd\theta}{dt}, \qquad \frac{d\tau}{r\sin\theta d\phi} = F^2\frac{r\sin\theta d\phi}{dt}.$$

Hence

$$\delta\left(\frac{d\tau}{d\phi}\right) = \frac{\delta t}{F^2}\left\{\frac{d\tau}{dr}\frac{d^2\tau}{drd\phi} + \frac{1}{r^2}\frac{d\tau}{d\theta}\frac{d^2\tau}{d\theta d\phi} + \frac{1}{r^2\sin^2\theta}\frac{d\tau}{d\phi}\frac{d^2\tau}{d\phi^2}\right\} = \frac{\delta t}{F}\left(\frac{dF}{d\phi}\right) = 0.$$

That is, unless F contains ϕ, $\frac{d\tau}{d\phi}$ is necessarily a constant, β suppose.

But, in the present case, *if we give this constant any value but zero, we introduce a problem much more general than that proposed*, for the expression for the reciprocal of the square of the velocity becomes

$$\frac{r^n}{\mu} - \frac{\beta^2}{r^2\sin^2\theta}.$$

10. As an example of a tortuous curve we take the following:

Determine the form of the brachistochrone when the velocity at any point of space is proportional to the distance from a given line.

Taking the line as the axis of z, our equation obviously becomes

$$\left(\frac{d\tau}{dx}\right)^2+\left(\frac{d\tau}{dy}\right)^2+\left(\frac{d\tau}{dz}\right)^2=\frac{a^2}{x^2+y^2}.$$

Hence

$$\frac{d\tau}{dz}=a,$$

and, substituting this, and changing to polar co-ordinates in a plane parallel to xy,

$$\left(\frac{d\tau}{dr}\right)^2+\frac{1}{r^2}\left(\frac{d\tau}{d\theta}\right)^2=\frac{a^2}{r^2}-a^2.$$

Hence we may take

$$\frac{d\tau}{d\theta}=\beta,$$

and there remains

$$\frac{d\tau}{dr}=\frac{1}{r}\sqrt{a^2-\beta^2-a^2r^2}.$$

Integrating, we have

$$\tau=az+\beta\theta-\sqrt{a^2-\beta^2}\log.\left[\frac{\sqrt{a^2-\beta^2}}{r}+\sqrt{\frac{a^2-\beta^2}{r^2}-a^2}\right]+\sqrt{a^2-\beta^2-a^2r^2}.$$

By equating to constants the partial differential coefficients of τ with respect to a and β, we obtain the two equations of the brachistochrone

$$\mathfrak{A}=z-\frac{ar^2}{\sqrt{a^2-\beta^2}+\sqrt{a^2-\beta^2-a^2r^2}},$$

and

$$\mathfrak{B}=\theta+\frac{\beta}{\sqrt{a^2-\beta^2}}\log.\left[\frac{\sqrt{a^2-\beta^2}}{r}+\sqrt{\frac{a^2-\beta^2}{r^2}-a^2}\right].$$

The former of these is the equation of a sphere, as may be seen at once by putting it in the form

$$a\,(z-\mathfrak{A})=\sqrt{a^2-\beta^2}-\sqrt{a^2-\beta^2-a^2r^2}.$$

The remaining equation, by altering the value of $\mathfrak{B}$, may be reduced to the form

$$2\,\frac{\sqrt{a^2-\beta^2}}{a}=r\left(\epsilon^{\frac{\sqrt{a^2-\beta^2}}{\beta}(\theta-\mathfrak{B})}+\epsilon^{-\frac{\sqrt{a^2-\beta^2}}{\beta}(\theta-\mathfrak{B})}\right)$$

which is at once recognised as a cylinder, whose base is one of COTES' Spirals.

Also, if we remark that, by (1),

$$r\frac{d\theta}{dt}=v^2\frac{d\tau}{rd\theta}=\frac{r^2}{a^2}\cdot\frac{\beta}{r}=\frac{\beta v}{a}$$

we see that

$$\cos\psi = \frac{r\dfrac{d\theta}{dt}}{v} = \frac{\beta}{a} = \text{const}$$

where ψ is the inclination of the element $r\delta\theta$ to the corresponding element δs of the brachistochrone. That is, the brachistochrone cuts all circles on the above sphere, whose planes are parallel to xy, at a constant angle. (*Loxodrome.*)

11. It is easily seen that

$$\tau = \mathrm{C}$$

is the equation of an *Isochronous* surface.

Also, since

$$\frac{\left(\dfrac{d\tau}{dx}\right)}{\dfrac{dx}{dt}} = \frac{\left(\dfrac{d\tau}{dy}\right)}{\dfrac{dy}{dt}} = \frac{\left(\dfrac{d\tau}{dz}\right)}{\dfrac{dz}{dt}}$$

the brachistochrone cuts all such surfaces at right angles.

And the normal distance between two consecutive isochronous surfaces is proportional to the velocity in the brachistochrone of which it forms an element. For, of course,

$$\delta s = v\delta\tau.$$

12. Generally, putting

$$\mathfrak{T} = \left(\frac{d\tau}{dx}\right)^2 + \left(\frac{d\tau}{dy}\right)^2 + \left(\frac{d\tau}{dz}\right)^2 = \frac{1}{2(\mathrm{H}-\mathrm{V})} \quad . \quad . \quad . \quad (7),$$

we have

$$2(\mathrm{H}-\mathrm{V}) = \frac{1}{\mathfrak{T}},$$

and

$$\mathrm{X} = -\left(\frac{d\mathrm{V}}{dx}\right) = -\frac{1}{2\mathfrak{T}^2}\frac{d\mathfrak{T}}{dx} \quad . \quad . \quad . \quad . \quad . \quad . \quad (8),$$

with similar expressions for Y and Z.

Also, by (1), we have

$$\left.\begin{aligned} \frac{d\tau}{dx} &= \mathfrak{T}\frac{dx}{dt}, \text{ \&c.} \\ \text{and} \quad \frac{d\tau}{d\mathrm{H}} &= -\int \mathfrak{T}dt \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (9),$$

Hence

$$\begin{aligned} \frac{d^2x}{dt^2} &= \frac{d}{dt}\left(\frac{1}{\mathfrak{T}}\frac{d\tau}{dx}\right) \\ &= \frac{1}{\mathfrak{T}}\frac{d}{dt}\left(\frac{d\tau}{dx}\right) - \frac{1}{\mathfrak{T}^2}\frac{d\tau}{dx}\frac{d\mathfrak{T}}{dt} \quad . \quad . \quad . \quad . \quad . \quad . \quad (10). \end{aligned}$$

But
$$\frac{d}{dt}\left(\frac{d\tau}{dx}\right) = \frac{d^2\tau}{dx^2}\frac{dx}{dt} + \frac{d^2\tau}{dydx}\frac{dy}{dt} + \frac{d^2\tau}{dzdx}\frac{dz}{dt}$$

$$= \frac{1}{\mathfrak{T}}\left\{\frac{d^2\tau}{dx^2}\frac{d\tau}{dx} + \frac{d^2\tau}{dydx}\frac{d\tau}{dy} + \frac{d^2\tau}{dzdx}\frac{d\tau}{dz}\right\} = \frac{1}{2\mathfrak{T}}\frac{d\mathfrak{T}}{dx}, \qquad (11),$$

which is the ordinary form of the equation of the brachistochrone, (A) in § 4.

Also,
$$\frac{d\mathfrak{T}}{dt} = 2\left\{\frac{d\tau}{dx}\frac{d}{dt}\left(\frac{d\tau}{dx}\right) + \frac{d\tau}{dy}\frac{d}{dt}\left(\frac{d\tau}{dy}\right) + \frac{d\tau}{dz}\frac{d}{dt}\left(\frac{d\tau}{dz}\right)\right\}$$

$$= \frac{1}{\mathfrak{T}}\left\{\frac{d\tau}{dx}\frac{d\mathfrak{T}}{dx} + \frac{d\tau}{dy}\frac{d\mathfrak{T}}{dy} + \frac{d\tau}{dz}\frac{d\mathfrak{T}}{dz}\right\} \quad . \quad . \quad . \quad . \quad . \qquad (12).$$

The above value of $\frac{d^2x}{dt^2}$ becomes therefore

$$\frac{d^2x}{dt^2} = \frac{1}{2\mathfrak{T}^2}\frac{d\mathfrak{T}}{dx} - \frac{\frac{d\tau}{dx}}{\mathfrak{T}^3}\left\{\frac{d\tau}{dx}\frac{d\mathfrak{T}}{dx} + \frac{d\tau}{dy}\frac{d\mathfrak{T}}{dy} + \frac{d\tau}{dz}\frac{d\mathfrak{T}}{dz}\right\} \quad . \quad . \quad . \qquad (13),$$

which (8) reduces to the form

$$\frac{d^2x}{dt^2} = -\mathrm{X} + 2\frac{\frac{d\tau}{dx}}{\mathfrak{T}}\left\{\mathrm{X}\frac{d\tau}{dx} + \mathrm{Y}\frac{d\tau}{dy} + \mathrm{Z}\frac{d\tau}{dz}\right\} \quad . \quad . \quad . \qquad (14).$$

And we have, of course, similar expressions for $\frac{d^2y}{dt^2}$ and $\frac{d^2z}{dt^2}$.

13. We may thus easily prove the fundamental property of brachistochrones given in most treatises on dynamics.

The pressure on the curve, due to the motion, is equal to that due to the impressed forces.

For (14) may be written

$$\frac{d^2x}{dt^2} = -\mathrm{X} + 2\frac{dx}{dt}\mathfrak{T}\left\{\mathrm{X}\frac{dx}{dt} + \mathrm{Y}\frac{dy}{dt} + \mathrm{Z}\frac{dz}{dt}\right\}$$

$$= -\mathrm{X} + 2\frac{dx}{ds}\left\{\mathrm{X}\frac{dx}{ds} + \mathrm{Y}\frac{dy}{ds} + \mathrm{Z}\frac{dz}{ds}\right\}$$

$$= \mathrm{X} - 2\left\{\mathrm{X} - \frac{dx}{ds}\left(\mathrm{X}\frac{dx}{ds} + \mathrm{Y}\frac{dy}{ds} + \mathrm{Z}\frac{dz}{ds}\right)\right\}.$$

Now $\mathrm{X}\frac{dx}{ds} + \mathrm{Y}\frac{dy}{ds} + \mathrm{Z}\frac{dz}{ds}$ is the component of the impressed forces along ds. Hence

$$\mathrm{X} - \frac{dx}{ds}\left(\mathrm{X}\frac{dx}{ds} + \mathrm{Y}\frac{dy}{ds} + \mathrm{Z}\frac{dz}{ds}\right),$$

$$\mathrm{Y} - \frac{dy}{ds}\left(\mathrm{X}\frac{dx}{ds} + \mathrm{Y}\frac{dy}{ds} + \mathrm{Z}\frac{dz}{ds}\right), \qquad \mathrm{Z} - \frac{dz}{ds}\left(\mathrm{X}\frac{dx}{ds} + \mathrm{Y}\frac{dy}{ds} + \mathrm{Z}\frac{dz}{ds}\right),$$

are the rectangular components of the component of the impressed force perpendicular to the path.

But, if R be the force of constraint, λ, μ, ν, its direction-cosines, we have by ordinary kinetics

$$\frac{d^2x}{dt^2} = \mathrm{X} - \mathrm{R}\lambda, \quad \&c.$$

Hence
$$\mathrm{R}\lambda = 2\left(\mathrm{X} - \frac{dx}{ds}\left(\mathrm{X}\frac{dx}{ds} + \mathrm{Y}\frac{dy}{ds} + \mathrm{Z}\frac{dz}{ds}\right)\right), \quad \&c., \&c.,$$

and therefore the whole pressure is *double* that due to the impressed forces.

From the above follows also the well-known theorem, that *the osculating plane of the brachistochrone contains, at each point, the resultant of the impressed forces.* For it has been shown that this resultant coincides in direction with the centrifugal force, and the latter of course lies in the osculating plane.

14. Another, and perhaps simpler proof of the theorem above is furnished directly by (10). Thus, squaring and adding the three equations of that form, after substituting in them from (11), we have

$$\left(\frac{d^2x}{dt^2}\right)^2 + \left(\frac{d^2y}{dt^2}\right)^2 + \left(\frac{d^2z}{dt^2}\right)^2 = \frac{1}{4\mathfrak{T}^4}\left\{\left(\frac{d\mathfrak{T}}{dx}\right)^2 + \left(\frac{d\mathfrak{T}}{dy}\right)^2 + \left(\frac{d\mathfrak{T}}{dz}\right)^2\right\}$$
$$- \frac{1}{\mathfrak{T}^4}\frac{d\mathfrak{T}}{dt}\left\{\frac{d\tau}{dx}\frac{d\mathfrak{T}}{dx} + \frac{d\tau}{dy}\frac{d\mathfrak{T}}{dy} + \frac{d\tau}{dz}\frac{d\mathfrak{T}}{dz}\right\}$$
$$+ \frac{1}{\mathfrak{T}^4}\left(\frac{d\mathfrak{T}}{dt}\right)^2\left\{\left(\frac{d\tau}{dx}\right)^2 + \left(\frac{d\tau}{dy}\right)^2 + \left(\frac{d\tau}{dz}\right)^2\right\}$$
$$= \frac{1}{4\mathfrak{T}^4}\left\{\left(\frac{d\mathfrak{T}}{dx}\right)^2 + \left(\frac{d\mathfrak{T}}{dy}\right)^2 + \left(\frac{d\mathfrak{T}}{dz}\right)^2\right\} - \frac{1}{\mathfrak{T}^4}\frac{d\mathfrak{T}}{dt}\left(\mathfrak{T}\frac{d\mathfrak{T}}{dt}\right) + \frac{1}{\mathfrak{T}^4}\left(\frac{d\mathfrak{T}}{dt}\right)^2 (\mathfrak{T})$$

[by (12) and (7)]

$$= \frac{1}{4\mathfrak{T}^4}\left\{\left(\frac{d\mathfrak{T}}{dx}\right)^2 + \left(\frac{d\mathfrak{T}}{dy}\right)^2 + \left(\frac{d\mathfrak{T}}{dz}\right)^2\right\} = \mathrm{X}^2 + \mathrm{Y}^2 + \mathrm{Z}^2, \quad . \quad . \quad \text{by (8).}$$

Hence *the whole acceleration is equal to the resultant of the impressed forces;* and therefore the component of the acceleration, normal to the curve, must be equal to that of the resultant of the impressed forces; from which the theorem follows at once if we can show independently that the resultant of the impressed forces lies in the osculating plane. This is easily done as follows. We have

$$\delta x = \frac{\delta t}{\mathfrak{T}}\frac{d\tau}{dx}, \quad \&c., \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{by (9).}$$

Hence
$$\delta^2 x = \frac{\delta t}{\mathfrak{T}}\delta\left(\frac{d\tau}{dx}\right) - \frac{\delta x}{\mathfrak{T}}\delta\mathfrak{T}, \quad \&c.$$

Now, by (8) and (11), $\delta\left(\frac{d\tau}{dx}\right)$ &c., are proportional to the direction-cosines of the resultant force, which therefore lies in the common plane of two consecutive elements of the curve.

15. The equation of the surfaces which are orthogonal to the path is

$$\tau = \mathrm{C} ;$$

and that of equipotential surfaces

$$\mathrm{V} = \mathrm{C}_1 .$$

That these may coincide we must have

$$\tau = \phi(\mathrm{V}) ,$$

where ϕ is any function whatever.

Hence

$$\left\{\phi'(\mathrm{V})\right\}^2 \left(\left(\frac{d\mathrm{V}}{dx}\right)^2 + \left(\frac{d\mathrm{V}}{dy}\right)^2 + \left(\frac{d\mathrm{V}}{dz}\right)^2\right) = \frac{1}{2(\mathrm{H} - \mathrm{V})} .$$

If we write

$$\mathcal{V} = \int \sqrt{2(\mathrm{H} - \mathrm{V})}\, \phi'(\mathrm{V})\, d\mathrm{V} = \psi(\mathrm{V}) , \quad . \quad . \quad . \quad (15).$$

this becomes

$$\left(\frac{d\mathcal{V}}{dx}\right)^2 + \left(\frac{d\mathcal{V}}{dy}\right)^2 + \left(\frac{d\mathcal{V}}{dz}\right)^2 = 1 , \quad . \quad . \quad . \quad . \quad (16).$$

A complete primitive of this equation is, of course,

$$\mathcal{V} = lx + my + nz - p ,$$

where p is any function of l, m, n, and

$$l^2 + m^2 + n^2 = 1 .$$

The general primitive, equated to a constant, is therefore obviously the equation of a series of surfaces such that the normal distance between any two consecutive members of the series is everywhere the same. It is evident from (15) that the surfaces thus found are identical with the isochronous and equipotential surfaces, when these coincide. The equations of their orthogonal trajectory, that is, of the free path which is also a brachistochrone, are therefore,

$$\frac{\delta x}{\left(\frac{d\mathcal{V}}{dx}\right)} = \frac{\delta y}{\left(\frac{d\mathcal{V}}{dy}\right)} = \frac{\delta z}{\left(\frac{d\mathcal{V}}{dz}\right)} = \frac{\left(\frac{d\mathcal{V}}{dx}\right)\delta x + \left(\frac{d\mathcal{V}}{dy}\right)\delta y + \left(\frac{d\mathcal{V}}{dz}\right)\delta z}{\left(\frac{d\mathcal{V}}{dx}\right)^2 + \left(\frac{d\mathcal{V}}{dy}\right)^2 + \left(\frac{d\mathcal{V}}{dz}\right)^2} = \delta\mathcal{V} = \delta\mathrm{C}, \quad (17).$$

Hence,

$$\delta x = \delta\mathrm{C}\left(\frac{d\mathcal{V}}{dx}\right) , \text{ \&c.,}$$

and, therefore,

$$\delta^2 x = \delta C \left\{ \left(\frac{d^2V}{dx^2}\right) \delta x + \left(\frac{d^2V}{dxdy}\right) \delta y + \left(\frac{d^2V}{dxdz}\right) \delta z \right\} + \delta^2 C \left(\frac{dV}{dx}\right).$$

But, substituting the values of δx, &c., from (17), this becomes

$$\delta^2 x = (\delta C)^2 \left\{ \left(\frac{dV}{dx}\right)\left(\frac{d^2V}{dx^2}\right) + \left(\frac{dV}{dy}\right)\left(\frac{d^2V}{dxdy}\right) + \left(\frac{dV}{dz}\right)\left(\frac{d^2V}{dxdz}\right) \right\} + \delta^2 C \left(\frac{dV}{dx}\right),$$

and the first part vanishes, by (16).

Hence

$$\frac{\delta^2 x}{\delta x} = \frac{\delta^2 y}{\delta y} = \frac{\delta^2 z}{\delta z} = \frac{\delta^2 C}{\delta C},$$

which show that when the path is simultaneously a free path and a brachistochrone, it is necessarily rectilinear.

This might have been inferred at once, from the theorem of § 13, which shows that if the free path be a brachistochrone, there can be no pressure due to the motion, *i.e.*, no curvature. But the above investigation is given as containing curious additional information. It shows, for instance, that if the force be the same at all points of each of a series of equipotential surfaces, the lines of force are rectilinear. Also, that if the flux of heat be constant per unit of area over each one of a series of isothermal surfaces, though not necessarily the same for all, the propagation of heat takes place in straight lines. And, as particular cases of these theorems, if the force or the flux of heat be the same throughout a given space, the attraction, or the flux, therein takes place in parallel lines.

16. HAMILTON's equation for the determination of the Characteristic Function (A) in the case of the free motion of a single particle is

$$\left(\frac{dA}{dx}\right)^2 + \left(\frac{dA}{dy}\right)^2 + \left(\frac{dA}{dz}\right)^2 = 2(H - V) \quad . \quad . \quad . \quad . \quad (18).$$

The comparison of this with (2) suggests a useful transformation. Introducing in that equation a factor θ^2, an undetermined function of x, y, z, we have

$$\left(\theta\frac{d\tau}{dx}\right)^2 + \left(\theta\frac{d\tau}{dy}\right)^2 + \left(\theta\frac{d\tau}{dz}\right)^2 = \frac{\theta^2}{2(H - V)} \quad . \quad . \quad . \quad . \quad (19).$$

If we make

$$\theta = \phi'(\tau) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (20),$$

and

$$\frac{\theta^2}{2(H - V)} = 2(H_1 - V_1) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (21),$$

(19) becomes

$$\left(\frac{d\phi(\tau)}{dx}\right)^2 + \left(\frac{d\phi(\tau)}{dy}\right)^2 + \left(\frac{d\phi(\tau)}{dz}\right)^2 = 2(H_1 - V_1) \quad . \quad . \quad . \quad (22).$$

Here it is obvious, by (18), that $\phi(\tau)$ is the action in a *free* path coinciding with the brachistochrone, and that $2(H_1-V_1)$ is the square of the velocity in this path.

Hence the curious result that, *if* τ *be the time through any arc of a given brachistochrone, the same path will be described freely under the action of forces whose potential is* V_1, *where*

$$2(H_1 - V_1) = \frac{(\phi'(\tau))^2}{2(H - V)},$$

ϕ' *being any function whatever; and* $\phi(\tau)$ *representing the action in the free path.*

17. The simplest supposition we can make is that $\phi'(\tau)$ is constant. In this case the velocity in the free path is inversely proportional to that in the brachistochrone at the same point; and the action in the one is proportional to the time in the other. In fact, as Professor W. THOMSON has pointed out to me, in this case the investigation may be made with extreme simplicity, thus—

In the brachistochrone we have

$$\int \frac{ds}{v} \text{ a minimum.}$$

Putting $\nu = \frac{1}{v}$, and considering ν as the velocity in the same path due to another (easily determinable) potential; we must have

$$\int \nu ds \text{ a minimum.}$$

This is the ordinary condition of *Least Action*, and belongs, therefore, to a free path.

Hence, since the cycloid is the brachistochrone for gravity, and since in it $v^2 = 2gy$, it will be a free path if $\nu^2 = \frac{1}{2gy}$, that is for a system of force where the potential is found from

$$H_1 - V_1 = \frac{1}{4gy}.$$

This gives

$$-\frac{dV_1}{dx} = 0, \quad -\frac{dV_1}{dy} = -\frac{1}{4gy^2}.$$

In other words, a cycloid may be described freely under the action of a force towards, and inversely as the square of the distance from, the base; and the velocity at any point will be the reciprocal of that in the same cycloid when it is the common brachistochrone.

This result is easily verified by a direct process.

18. But we have, by § 16, an infinite number of other systems of forces under which this cycloid will be described freely.

For by § 8 we have, putting $a=0$, since the base is now the axis of x,

$$\sqrt{2g}\,\tau = \frac{x}{\sqrt{b}} + \int dy \sqrt{\frac{1}{y} - \frac{1}{b}}$$

$$= \frac{x}{\sqrt{b}} - \sqrt{b}\,\overset{-1}{\cos}\sqrt{\frac{y}{b}} + \sqrt{\frac{y}{b}}\sqrt{b-y} + \mathrm{C}.$$

Hence, whatever be ϕ', the cycloid is a free path for the system

$$v^2 = 2(\mathrm{H}_1 - \mathrm{V}_1) = \frac{\left\{\phi'\left(\frac{x}{\sqrt{b}} - \sqrt{b}\,\overset{-1}{\cos}\sqrt{\frac{y}{b}} + \sqrt{\frac{y}{b}}\sqrt{b-y} + \mathrm{C}\right)\right\}^2}{2gy}.$$

19. The converse of the proposition in § 16 is also curious. Taking HAMILTON's equation (18), we have,

$$(\phi'(\mathrm{A}))^2\left\{\left(\frac{d\mathrm{A}}{dx}\right)^2 + \left(\frac{d\mathrm{A}}{dy}\right)^2 + \left(\frac{d\mathrm{A}}{dz}\right)^2\right\} = 2(\mathrm{H}-\mathrm{V})(\phi'(\mathrm{A}))^2 \quad . \quad . \quad (23).$$

Comparing this with (2), we see that $\tau = \phi(\mathrm{A})$ is the brachistochronic expression for the time in a path which is a free path for potential V. The requisite potential is now found from

$$\frac{1}{2(\mathrm{H}_1 - \mathrm{V}_1)} = 2(\mathrm{H}-\mathrm{V})(\phi'(\mathrm{A}))^2 \quad . \quad . \quad . \quad . \quad . \quad (24).$$

Hence, *if* A *be the action in a given free path, the same path will be a brachistochrone for forces whose potential is* V_1, *determined by* (24), V *being the potential in the free path.*

Thus, the parabola

$$(x - \mathfrak{A})^2 = 4a(y - a)$$

is the free path for $v^2=2gy$. And the action is given by

$$\frac{1}{\sqrt{2g}}\mathrm{A} = x\sqrt{a} + \frac{2}{3}(y-a)^{\frac{3}{2}}.$$

Hence this parabola is the brachistochrone for

$$2(\mathrm{H}_1 - \mathrm{V}_1) = \frac{1}{2gy(\phi'(\mathrm{A}))^2}.$$

In the simplest case $\phi'(\mathrm{A}) = 1$, and we have

$$-\frac{d\mathrm{V}_1}{dx} = 0, \qquad -\frac{d\mathrm{V}_1}{dy} = -\frac{1}{4gy^2}.$$

Hence, by § 17, the parabola is a brachistochrone when a cycloid is the free path.

20. Again, if

$$v^2 = 2\left(\frac{\mu}{r} - \mathrm{H}\right), \quad . \quad . \quad . \quad . \quad . \quad (25),$$

where H and μ are essentially positive, the free path is an ellipse of which the origin (the centre of force) is a focus.

This ellipse is the brachistochrone for the potential V_1, and whole energy H_1, where

$$\frac{C}{2(H_1 - V_1)} = 2\left(\frac{\mu}{r} - H\right),$$

or

$$V_1 = H_1 - \frac{Cr}{4(\mu - Hr)}.$$

This corresponds to a central force

$$-\frac{dV_1}{dr} = \frac{C}{4(\mu - Hr)} + \frac{CHr}{4(\mu - Hr)^2}$$

$$= \frac{C\mu}{4(\mu - Hr)^2}.$$

The velocity at any point is

$$\sqrt{\frac{Cr}{2(\mu - Hr)}}.$$

In the ellipse, we know by ordinary kinetics that

$$v^2 = \mu\left(\frac{2}{r} - \frac{1}{a}\right).$$

Comparing this with the above formula (25) we have

$$\frac{\mu}{H} = 2a.$$

Hence the velocity in the free ellipse is

$$v = \sqrt{\frac{\mu}{a}}\sqrt{\frac{2a - r}{r}}. \quad . \quad . \quad . \quad . \quad (26).$$

That in the same ellipse, when it is a brachistochrone, is, as above,

$$v_1 = \sqrt{\frac{Cr}{2(\mu - Hr)}} = \sqrt{\frac{Ca}{\mu}}\sqrt{\frac{r}{2a - r}}.$$

But if we refer it to the other focus of the ellipse we have

$$r_1 = 2a - r.$$

Hence

$$v_1 = \sqrt{\frac{Ca}{\mu}}\sqrt{\frac{2a - r_1}{r_1}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (27).$$

Comparing (26) and (27), we have the singular result that *a planet moving freely about a centre of force in the focus of its elliptic orbit is describing a brachistochrone* (*for the same law of velocity as regards position*) *about the other focus.* The reason of this remarkable property, as well as of the connected one that *while the time in an elliptic orbit is (of course) measured by the area described about one focus, the action is measured by that described about the other*,* is easily traced to the fact that the rectangle under the perpendiculars from the foci on any tangent is constant.

21. It follows from HAMILTON's investigations, that in the free ellipse we have

$$A = \int \frac{2\left(\frac{\mu}{r} - H\right) dr}{\sqrt{2\left(\frac{\mu}{r} - H\right) - \frac{a^2}{r^2}}},$$

where a depends upon the excentricity of the ellipse by the formula

$$a^2 = \frac{\mu^2}{2H}(1 - e^2).$$

The theorem may therefore be generalized as follows:—The free ellipse will be a brachistochrone, if the velocity be given by

$$v^2 = 2(H_1 - V_1) = \frac{1}{2\left(\frac{\mu}{r} - H\right)\left\{\phi'(A)\right\}^2}$$

where ϕ' is any function, and A is the integral last written. By differentiation with respect to r, we get the law of central force requisite.

But results of this nature may be deduced to any desired extent, without more trouble than the requisite integrations involve.

22. The examples immediately preceding are but particular cases of the following general theorem, which is easily seen to be involved in the results of §§ 16, 19. *If we have two curves, P and Q, of which P is a free path, and Q a brachistochrone, for a given conservative system of forces; P will be a brachistochrone for a system of forces for which Q is a free path—and the action and time in any arc of either, when it is described freely, are functions of the time and action respectively, in the same arc, when it is a brachistochrone.*

23. It is easy to see, that there exists a very singular analogy between the processes we have just given, and those suggested by certain problems in optics.

Assuming, for an instant, the exploded corpuscular theory of Light, Varying Action is at once applicable to the determination of the path of a corpuscle. On the other hand, if we assume, as our fundamental hypothesis, that light takes the least possible time to pass from one point of its path to another, the foregoing investigations would be directly applicable to find the path in a medium whose refractive index (on which the velocity depends), at any point, is a given function of the co-ordinates; in other words, in a heterogeneous singly refracting medium.

In the beautiful investigations of HAMILTON, on the *Theory of Systems of Rays*

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* Proc. R.S.E. March 1865, or TAIT and STEELE's Dynamics of a Particle (2d edition) § 258.

(Trans. R.I.A., 1824–32), the path of a ray is assumed to be a straight line in any one medium. Here the velocity depends only upon the *direction* of the ray, as in homogeneous doubly refracting media, and the problem has no analogy with the conservative case which is treated above.

24. As an instance of an optical problem I take the following, due I believe to MAXWELL.* *If the refractive index of a medium be such a function of the distance from a given point that the path of any one ray is a circle, the path of every other ray is a circle; and all rays diverging from any one point converge accurately in another.* Or, in another form, find the relation between the velocity and the distance from the centre of force that the brachistochrone may always be a circle.

The symmetry shows that our investigations need only involve two dimensions. Taking the centre of force as pole, the equation of a circle is

$$r^2 - 2ar\cos(\theta - \mathfrak{A}) = \varrho^2 - a^2, = b^2 \text{ suppose.}$$

Hence

$$\mathfrak{A} = \theta - \cos^{-1}\frac{b^2 - r^2}{2ar}.$$

This is obviously the equation before written (3) in the form

$$\frac{d\tau}{da} = \mathfrak{A}.$$

Hence

$$\tau = a\theta - \int da \cos^{-1}\frac{b^2 - r^2}{2ar}.$$

But, if v be the velocity (the reciprocal of the refractive index in the optical problem),

$$\left(\frac{d\tau}{dr}\right)^2 + \frac{1}{r^2}\left(\frac{d\tau}{d\theta}\right)^2 = \frac{1}{v^2}.$$

Hence

$$\frac{d\tau}{dr} = \sqrt{\frac{1}{v^2} - \frac{a^2}{r^2}} = -\frac{d}{dr}\int da \cos^{-1}\frac{b^2 - r^2}{2ar} = -\int da \frac{b^2 + r^2}{r\sqrt{\left(4a^2r^2 - (b^2 - r^2)^2\right)}}.$$

But v is not a function of a, so that we get by differentiation with respect to that quantity

$$\frac{\dfrac{a}{r^2}}{\sqrt{\dfrac{1}{v^2} - \dfrac{a^2}{r^2}}} = \frac{b^2 + r^2}{r\sqrt{\left(4a^2r^2 - (b^2 - r^2)^2\right)}}.$$

This is easily reduced to

$$v^2a^2 = \frac{(b^2 + r^2)^2}{4(a^2 + b^2)} = \frac{(b^2 + r^2)^2}{4\varrho^2}.$$

The condition, that v is a function of r and absolute constants only, thus leads at once to two conclusions: b is an absolute constant; and so is $2\rho a$, for which we may write c. a is therefore inversely as the diameter of the circle; and

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* Cambridge and Dublin Math. Journal, IX., p. 9.

$$v = \frac{b^2 + r^2}{c}.$$

From the form of the equation of the path it is obvious that $-b^2$ is the rectangle under the segments of any chord drawn through the centre of force.

Hence, in the optical problem, if a ray leave, in any direction, a point distant r from the origin, it will pass through another point in the prolongation of r, distant $\frac{b^2}{r}$ from the origin; and, in the kinetic problem, there is an infinite number of brachistochrones (circles all, and the time being the same for all) when two points thus related are taken as the initial and final points.

25. Such examples might be multiplied indefinitely. For instance, if the refractive index of a medium be inversely proportional to the square root of the distance from a given point, the path is a parabola about the point as focus; that every ray may be a cardioid whose cusp is at the point, the square of the refractive index must be inversely as the cube of the distance: and so on.

26. The processes of § 4 may of course be applied to innumerable problems besides the determination of the form and properties of brachistochrones, but I shall content myself with an example or two. Thus, if we take

$$\Phi = \int f(v)\,ds$$

as the characteristic function, we have

$$\frac{d\Phi}{dx} = \frac{f(v)}{v}\frac{dx}{dt}, \text{ \&c., and } \frac{d\Phi}{dH} = \int f'(v)\,dt.$$

Of this, besides the cases $f(v)=v$, and $f(v)=\frac{1}{v}$, which we have already considered, the most curious is that where

$$f(v) = \frac{v^2}{2};$$

that is, when *the space average of the kinetic energy is a minimum.* In this case,

$$\left(\frac{d\Phi}{dx}\right)^2 + \left(\frac{d\Phi}{dy}\right)^2 + \left(\frac{d\Phi}{dz}\right)^2 = \frac{v^4}{4} = (H-V)^2,$$

and

$$\frac{d\Phi}{dH} = s.$$

Again, if we take

$$\Phi = \int F(x,y,z)\,f(v)\,ds$$

$$\frac{d\Phi}{dx} = \frac{Ff}{v}\frac{dx}{dt}, \text{ \&c., and } \frac{d\Phi}{dH} = \int F f'(v)\,dt.$$

Hence, if
$$F(x, y, z) = \frac{\text{Constant}}{f'(v)},$$
we have
$$\frac{d\Phi}{dH} = Ct,$$
so that there is an infinite number of values of the characteristic function, besides that of HAMILTON, which give the time through any arc of the orbit by their differential coefficients with respect to H.

27. Enough of this; I conclude with the remark that various investigations in Statics supply us with excellent examples in our subject.* Take the common catenary, for instance, its equation is found by the conditions
$$\int y ds = \text{minimum}, \quad \text{and} \int ds = \text{constant},$$
the axis of y being directed vertically upwards.

This gives
$$\delta \int (y + a)\, ds = 0.$$

Hence the catenary is the free path of a particle whose velocity is given by
$$v = C(y + a);$$
that is, if the force be in the direction of, and proportional to, the ordinate, and repulsive from the axis of x. In the same way we see that the catenary is the brachistochrone if the velocity be inversely as the distance from the axis; that is, if the force be attractive, and inversely as the cube of the distance from the axis.

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* Compare THOMSON and TAIT's Natural Philosophy, §§ 581, 582.

XIV.—*On the Tertiary Coals of New Zealand.* By W. LAUDER LINDSAY, M.D., F.L.S., &c., Honorary Fellow of the Philosophical Institute of Canterbury, New Zealand.

(Read 20th February 1865.)

Coal deposits of Tertiary age have now been found more or less throughout the two great islands (north and south) of New Zealand. They are best known, and they have been chiefly worked, however, in the provinces of Otago, Nelson, Canterbury, and Auckland. Their apparent more meagre development in other provinces is probably simply due to the circumstance that the latter have not, as yet, been so thoroughly explored or so extensively colonised and peopled as the others. The explorations, however, of provincial and geological surveyors, of prospecting gold-miners and sheep-owners, and of other pioneers of civilisation, are daily adding to the number of the known coal-fields of New Zealand; and it is probably not going too far to assert, in general terms, that the whole area of its two great islands is studded over with coal-basins of various extent, depth, age, and quality of coal.

In *Otago* the following are the districts which possess coal-fields or basins of Tertiary age:—

1. District between Dunedin (the capital) and the Taeri plains; including especially what I may designate the *Saddlehill* or *Greenisland Basin;* and the Silverstream valley.
2. Tokomairiro valley; *a.* Upper (Woolshed), and *b.* Lower (Tokomairiro gorge).
3. Great valley of the Clutha river—
    - *a. Upper* (Dunstan, Kawarau, and Manuherikia districts; Cromwell, Clyde, and Alexandra townships).
    - *b. Middle* (Teviot, Tuapeka, and Waitahuna districts; Laurence and Wetherstone's townships).
    - *c. Lower* (Kaitangata and Coal Point).
4. Valley of the Upper Taeri and Shag river—
    - *a. Upper* (Mount Ida or Highlay district).
    - *b. Lower* (Shag or Vulcan Point).
5. Valleys or beds of the Waitaki and Waikawa rivers; Otepopo &c.; all mostly in the central and eastern districts.

These localities include, fortunately for the gold-mining interest, the great gold-fields of Tuapeka, Dunstan, and Mount Ida or Highlay; and, as a general rule, there are few of the gold-fields destitute of local deposits of Tertiary or brown coals. The discovery and rapid development of the rich and extensive gold-fields of Otago are the main causes why in this province the coals in question have been more largely worked than in any other part of New Zealand.*

Throughout the province of *Canterbury* similar coal-deposits occur, and more especially in—

1. The valleys or beds of the Selwyn, Upper Waikamariri, Rakaia, Rangitata, Ashburton, Northern Hinds, Potts, Tenawai, and other rivers.
2. The Malvern Hills, Mount Somers, Big Ben Range, Thirteen Mile Bush, &c.; all mostly in the central or eastern districts.

In the province of *Nelson*, Tertiary coal-deposits stretch along the—

1. *West coast*, from Cape Farewell with little interruption to the Grey river on the Canterbury boundary line—overlying unconformably the secondary Coal-measures of the Buller and Grey rivers.
2. On the *northern* and *north-western coasts*, they occur at Motupipi in Massacre Bay ;Ennerglyn, near the town of Nelson, &c.

In the province of *Auckland* they stretch along the—

1. *West coast* with little interruption from Kawhia to Hokianga (including Aotea, Raglan, Manukau, and Kaipara).
2. On the *east coast* they show themselves at Mongonui in the north, and further south at Wangeroa and Matakana, &c.
3. In the *interior* they abound in the Upper and Lower Waikato ; Waipa ; and Drury (or Hunua) districts.

Towards the close of 1861 I lived for three months in the centre of the *Saddlehill Coal Basin* in the immediate vicinity of Dunedin. Within a couple of miles of my residence (ten miles from Dunedin) an excellent artificial section of the brown coal strata was exposed in the Saddlehill Colliery, on the flank of the conical basaltic mass of Saddlehill (height 1565 feet). There is here a regular adit of considerable length, with relative tramways and other works. This colliery had been in successful, though limited, operation for several years; but at the period of my visit the superior attractiveness of the recently discovered gold field of Tuapeka had absorbed all available labour, and the works were consequently temporarily stopped. A semi-artificial section I also found in the immediate neighbourhood of my headquarters (the farm-house of Fairfield),—viz., in Abbott's Creek, where brown coal had also been worked for a very short time, and on a very limited scale. Natural outcrops or sections of similar strata I found on Scrogg's Hill—a continuation southwards of Saddlehill—where they were included in a Government coal-reserve; in M'Coll's Creek, on the seaward base of Saddlehill; and at other points on the flanks or base of this basaltic nucleus. I was led, while on the spot, to the conclusion that the Trappean mass of Saddlehill, with adjacent minor eruptive Trappean masses, had burst through what had at one time been a continuous coal-bed or basin subsequent to the deposition of the latter; or, in other words, in newer Tertiary times. That the coal-bed in question is more or less continuous is so far proved by the fact, that since I left Otago no less than three large collieries have been established in the immediate vicinity of my former residence. The first of these—the Fairfield Colliery—was opened on the lands of Fairfield itself, very near the old workings of Abbott's Creek, which I frequently examined. The proprietor, my friend Mr MARTIN, late Member of the Provincial Council of Otago, tells me* there are here two main seams, the upper six and the lower four feet thick, with a "dirt-bed" between them. The other pits, all within a few hundred yards of each other, are the Shand and the Walton Park Collieries. The strata associated with the brown coal are mostly various shales; clays, bituminous, and arenaceous, some of them plastic, white, and micaceous; and sands or sandstones of various degrees of coarseness. The whole are overlaid by the newer Tertiaries so abundant in the district, consisting of variously coloured clays, sands, and conglomerates.

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* In all probability it will yet be found that the Tertiary coals of New Zealand are referrible to groups of *three distinct ages*,—corresponding so far to our Eocene, Miocene, and Pliocene subdivisions. Dr HECTOR already regards the Otago brown coals as of *three* distinct ages, and Dr HAAST those of Canterbury as of at least *two*. Those which are mined as fuel are—for the most part at least—apparently referrible to the older or lower groups, or subdivisions, of the New Zealand Tertiaries.

Daily pedestrian excursions during my residence in the Saddlehill district gave me frequent opportunities of studying all the natural and artificial sections of the brown coal strata of this basin, and of collecting hand-specimens of every quality of the coals so exposed. The latter were brought home, and a suite thereof submitted in 1862 to careful chemical analysis by Professor MURRAY THOMSON, a Fellow of this Society, then an analytical chemist in Edinburgh, and now Professor of Experimental Science in the Thomason College, Roorkee, Bengal. The results of his analysis are embodied in Table II., and fully bear out the opinions to which, when in Otago,† I gave public expression, regarding the economical value of the Otago brown coals.

While in Otago I also visited the Kaitangata coal-field at the mouth of the Clutha, about 60 miles southward of Dunedin. Here there are works which were originated, and are now carried on, under the patronage of the Otago government. There are not only regular adits at the pit; but the pit is connected by means of a rail or tramway two miles in length, with a jetty on the Clutha for the ready delivery of the fuel into small coasting traders for the Dunedin market.

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* Letters of January 1862 and September 1863.

† In a lecture on "The Place and Power of Natural History in Colonisation, with special reference to Otago (New Zealand)," prepared for and printed by the "Young Men's Christian Association of Dunedin," Dunedin, January 1862. Extracts therefrom reprinted in the "Edinburgh New Philosophical Journal" for April and July 1863.

Vide chapter on the "Geology of the Otago *Lignites*."

At Coal Point, in the coast cliffs, two or three miles northwest of the mouth of the Clutha river, occur the best natural sections of the brown coal strata I saw in Otago. Here these strata consist of various seams of coal of different characters or qualities, separated by or associated with laminæ or beds of conglomerates, quartzose gravels, grits, and sandstones; clays, including fire-clay, pipe-clay, fine coloured clays, and carbonaceous and arenaceous clays, sometimes laminated; and carbonaceous and other shales. They contain various fossils, in the form chiefly of dicotyledonous leaves, fragments of lignite, stems of trees, and other plant impressions; as well as ironstone concretions and iron pyrites. Overlying the brown coal strata is a series of conglomerates, gravels, sands, and clays, generally more or less ferruginous; of newer Tertiary age; essentially identical with those overlying the brown coals of the Saddlehill district. These beds, too, contain nodules of clay ironstone, which are further scattered in all directions on the beach at the foot of the cliffs. Their appearance and position reminded me strongly of those of the carboniferous shales of Wardie. It does not appear that the stratigraphical relations of the Tertiary coals in other parts of Otago and New Zealand differ—save in minor and local details—from those of the brown coals of the Saddlehill and Kaitangata basins, as here roughly sketched.

Before leaving Otago I visited the Tuapeka gold-field, where I had an opportunity of seeing the relations of the brown coal, which is now being worked at Laurence, Wetherstone's, and Waitahuna. From Otago I passed northwards to the provinces of Nelson and Auckland. In neither of these, however, had I any opportunity of inspecting the Tertiary coal strata *in situ*. My examinations were confined to the suites of coal specimens contained in the Provincial Museums of these provinces, or in the hands of coal proprietors or amateur geological collectors. Unfortunately, the value of the Museum series—which, in the Auckland Museum at least, is somewhat extensive—is seriously detracted from by their careless nomenclature and classification, and by the improper or defective method of exhibition. The plan adopted in the Museum of Science and Art, Edinburgh, and in other of our own national Museums, should forthwith be copied in these, as well as other, colonial museums,—viz., to accompany each specimen with a full descriptive label, setting forth not only its locality, and date and circumstances of collection, but its chemical composition; and to classify it after some uniform plan, geological or chemical. Were this done, such series of specimens could not fail to acquire a great local as well as general value, instead of being, as at present, little more than a mass of lumber.

In the Great Exhibition at London in 1862 (New Zealand department), I was further enabled to examine a pretty complete series of all the New Zealand coals known up to that date,—especially those of the North Island, and northern portions of the South Island. Desirous of comparing the New Zealand *Tertiary* coals with local coals of greater age and superior quality, as well as with coals of all ages from every part of the world, I availed myself of the opportunities presented by the Exhibition in question; the Australian Museum, Sydney; the Museum of Economic Geology and British Museum, London; the Museums of Economic Botany at Kew and Edinburgh; the Museum of Science and Art, Edinburgh, and other minor museums, British or colonial.

I have selected the *Brown Coals* of *Otago*, as representative of the Tertiary coals of New Zealand, for three reasons,—viz., that I am more familiar with them; that they are the best known and most extensively worked in the colony; and that their stratigraphical relations and chemical constitution appear *essentially* those of all other New Zealand tertiary coals.

The general results of my inquiries as to the geology and chemistry of the Tertiary coals of New Zealand I have given in the "Abstract" [published in the Society's "Proceedings,"] of the paper which I had the honour of presenting to the Royal Society on 20th February last. The only section of the said paper which it seems desirable here to print in detail, is the tabular exhibition of the physical characters and chemical constitution of the brown coals of Otago; and as standards of comparison of certain other or older coals of other provinces of New Zealand or of Australia.

Table I. refers exclusively to specimens collected by the Geological Survey of Otago, or submitted to analysis by the chemist attached to that survey.* The majority of the Otago specimens are from the same collieries or localities from which my own collections were made. But inasmuch as the collections of the Geological Survey were made subsequently to mine, and at a period when the various works were in full operation, the survey specimens are likely to be of a quality superior to mine, which were necessarily, to a great extent, surface specimens. Moreover, the mode and scope of the analyses differ somewhat in the two tables; wherefore, and on other accounts, Table I. is to be regarded as the natural complement of Table II.†

Table II. refers exclusively to specimens collected by myself in Otago in the course of my various excursions in 1861. The chemical analyses were made, as before stated, by Professor MURRAY THOMSON.

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* It is constructed chiefly from materials contained in a Report by the Government geologist of Otago, Dr HECTOR—a Fellow of this Society—of date 13th April 1864.

† The natural order or sequence of these Tables has been reversed to suit the requirements of the printer.

TABLE I.

I. TERTIARY (CAINOZOIC) COALS OF OTAGO.

Locality.	Physical Characters of Coal-Specimen Analysed.	Character of			Specific Gravity.[1]	Percentage[1] of					
		Powder.	Ash.	Coke.[2]		Coke.	Fixed Carbon.	Volatile Hydro-carbons.	Ash.	Sulphur.	Water.[3]
I. CLUTHA—COAL POINT.	1. COMMON BROWN COAL.—*Fracture* moderately bright—conchoidal; becomes dull on exposure; *streak* deep brown; *burns* freely with a very slight *odour;* contains laminæ of *Jet*.	Deep brown.	White or light grey.	Dull.	1280	43·28	38·99	37·28	5·46	...	16·10
Do. *Top* of Main Seam.	2. BROWN COAL, apparently of very superior quality.—Distinctly laminated; traversed irregularly by thin plates of *Jet;* contains abundance of *Mineral Resin*, scattered through its substance in small specks, or occurring in large nodulated masses; also *Iron Pyrites* in its cracks.	Dull brown.	White.	Do.	1258	42·79	43·23	33·87	3·36	3·63	12·94
Do. *Middle* of Main Seam.	3. Quality and characters as in No. 2.	Do.	Do.	Do.	1267	41·10	39·12	35·67	4·43	...	16·35
Do. *Bottom* of Main Seam.	4. Blacker; more compact and massive; *Fracture* conchoidal in all directions; *streak* chocolate brown; possesses higher illuminating power than Nos. 2 and 3; and contains fewer impurities, with exception of nodules of *Iron Pyrites*.	Light chocolate brown.	Do.	Do.	1275	40·46	39·71	32·24	7·28	3·85	16·96
Do. Associated Sandstones.	5. *Jet;* bright; lustrous; compact; streak black; *burns* with very great difficulty, *caking* slightly.	Black.	Light red.	Iridescent.	...	50·35	39·04	27·61	11·31	...	16·43
II. GREENISLAND.	6. COMMON BROWN COAL.—Dull; earthy; very moist when first extracted from the mine; absorbs water readily, with a crackling noise; laminated; *fracture* subconchoidal; no trace of woody structure; *streak* dull, blackish-brown; contains thin plates of *Jet;* ignites slowly, and rather smoulders than burns; forms a useful fuel, when *mixed with a better class of coals*.	Dark brown.	Very light; almost like that of wood.—Colour light buff.	Slightly metallic or iridescent.	1256	39·72	36·83	33·83	4·32	2·24	19·33
Do. from another Pit.	7. Quality and characters as in No. 6.	Dull brown.	Light buff.	Dull.	...	...	43·12	34·37	2·25	...	20·26
III. SADDLEHILL.	8. EARTHY BROWN COAL.—Soft; dull; very moist; structure homogeneous; no trace of vegetable tissue; *streak* dark brown; in burning gives off a fœtid smell.	Do.	Very light.—Colour pure white.	Do.	1294	38·97	39·07	35·93	2·78	2·22	15·95
IV. SHAG POINT.	9. PITCH COAL.—Brownish black; very compact; *fracture* conchoidal and splintery; *lustre* fatty or resinous; *streak* lustrous; does not absorb water; does not soil the fingers; *burns* with a rich oily flame and a slight odour; on distillation at high temperatures yields a rich *gas*, and at lower ones a large amount of *oils*. Probably one of the oldest tertiary coals of Otago. The freest burning quality of Brown Coal yet found (1864) in that province. Its characters correspond with Dr Percy's definition of "*Pitch Coal:*" and it holds the same relative position among the Brown Coals of Otago that Cannel does among the household coals of Britain.	Dull brownish black.	Bulky; light; argillaceous—Chocolate brown.	Light; dull or semi-metallic.	1258	50·68	41·83	37·31	9·25	4·78	10·91
V. TOKOMAIRIRO.	10. DULL BROWN COAL.—Physical characters resemble those of Clutha and Greenisland Coal. *Streak* brown; contains occasional *quartz* pebbles.	Dull brown.	Light grey.	Dull or semi-metallic.—Does not cake.	1299	41·38	39·93	37·55	5·91	...	11·40
VI. WAITAHUNA.—FLAT OR JUNCTION.	11. LIGNITE, or WOODY BROWN COAL.—Dull; friable; slightly laminated; contains a considerable amount of *ligneous tissue;* also *mineral resin* in small masses; *burns* slowly, like turf; with a heavy, fœtid odour. Perhaps of more recent geological age than the Brown Coals proper.	Light brown.	Light buff.	Imperfect; lustrous; slightly iridescent.	...	43·74	35·20	37·63	11·15	6·21	11·06
	Mean of the foregoing 11 Classes, which include varieties of *Jet*, *Lignite*, *Brown Coal*, and *Pitch Coal*.	...	...	...	1273	43·24	39·64	34·84	6·13	3·82	15·24

	heavy, fœtid odour. Perhaps of more recent geological age than the Brown Coals proper.			iridescent.						
	Mean of the foregoing 11 Classes, which include varieties of *Jet*, *Lignite*, *Brown Coal*, and *Pitch Coal*,	...	...	...	1273	43·24	39·64	34·84	6·13	8·82	15·24

I. OTAGO. Preservation Inlet; South-West Coast.	12. SEMI-BITUMINOUS COAL.—Hard; black; compact; *fracture* splintery, cubical, rarely conchoidal; *streak* black, shining; approaches in character the coals of the West Coast of Nelson and Canterbury, and of Newcastle, New South Wales; has a greater resemblance to British palæozoic coal than any other hitherto found in Otago.	Black; with occasional faint brownish tinge.	Light buff.	Dull; unchanged; does not cake.	1305	64·43	59·42	27·50	5·14	...	7·15
II. NELSON. 1. Pakawau (or Seaford); Golden (or Massacre) Bay; North-West Coast.	13. PITCH COAL.—Pitch black; compact; heavy; *burns* with a large amount of flame and heat, and without disagreeable odour (Hochstetter). Lithological characters those of true coal. Quality of specimens exhibited to me, while at Nelson, by Mr Lewthwaite, proprietor of the colliery, seemed inferior to that of Grey River coal; but specimens in the Great Exhibition (London, 1862), labelled *Gas Coal*, appeared of superior quality.	Brownish black.	Light red (Hector); Clean and white (Hochstetter)	Semi-metallic.	1330	...	50·10	38·08	8·26	1·04	3·56
2. Buller River; West Coast.	14. PITCH COAL.—The only specimens I have seen were some water-worn fragments from the river's bed in the Exhibition of 1862 (London; Nelson section of the New Zealand department). They had the external characters of No. 13.	Do.	White.	Do.	1255	...	60·72	33·76	4·62	1·85	0·87
3. Grey River; West Coast, at or near River's mouth.	15. PITCH COAL.—By far the best quality of coal I saw in New Zealand; has all the lithological attributes of a true coal, and is quite comparable, *quoad* external characters, with the best English Newcastle qualities.	Black.	Light brown.	Cakes.	1330	...	62·37	29·44	6·20	...	1·99
	Mean of the foregoing 4 Classes of *Semi-Bituminous* or *Bituminous Coals*,	...	...	...	1305	64·43	58·15	32·19	6·05	1·44	3·37

III.—PRIMARY (PALÆOZOIC) COALS OF AUSTRALIA.

I. NEW SOUTH WALES. Newcastle; Hunter River.	16. PITCH COAL: with all the lithological characters of British Palæozoic Coals. Of true carboniferous age (according to Clarke, Keene, Daintree, and others).	Dull black.	Light buff.	Semi-metallic: cakes.	1385	...	56·28	31·68	9·09	...	2·94

ABSTRACT.

I. TERTIARY (CAINOZOIC) COALS—OTAGO,	Maximum,	1308	50·98	46·77	46·77	12·13	6·21	20·26
	Minimum,	1229	37·45	33·13	27·61	1·92	2·22	10·91
	Average,	1273	43·24	39·64	34·84	6·13	3·82	15·24
II. SECONDARY (MESOZOIC) COALS—NELSON AND OTAGO,	Maximum,	1330	68·67	64·43	38·08	11·20	1·85	15·60
	Minimum,	1250	58·85	50·10	25·55	1·98	1·04	0·70
	Average,	1305	64·43	58·15	32·19	6·05	1·44	3·39
III. PRIMARY (PALÆOZOIC) COALS—NEW SOUTH WALES,	Maximum,	...	...	57·20	31·78	10·05	...	3·00
	Minimum,	...	...	55·36	31·59	8·14	...	2·88
	Average,	1385	...	56·28	31·68	9·09	1·60	2·94

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[1] The *mean* of several examinations of, or experiments on, a variety of specimens. Up to October 1864, the Geological Survey of Otago had made fifty analyses of its *Brown Coals* alone.

[2] *None* of the Tertiary Coals of Otago, as here analysed, *cake*.

[3] Water *of constitution* as contradistinguished from *accidental water* (that given off at ordinary temperatures). Dr Hector informs me that he regards the Otago Brown Coals as of three distinct ages, the age being in each case indicated by the relative percentage of water of constitution. (*Letter*, October 1864).

[4] *Precise age* for the most part as yet undetermined.

TABLE II.

Locality.	Physical Characters of Coal—Specimen analysed.	Quantity[1] per Ton.						Percentage.[1]			
		Coke.	Carbon in Coke.	Ash.	Volatile Matter.	Gas.	Oil.	Coke.	Carbon in Coke.	Volatile Matter.	Ash.
		lbs.	lbs.	lbs.	lbs.	cub. ft.	gals.				
	I. TERTIARY COALS OF OTAGO.										
I. SADDLEHILL COLLIERY—on the property of Mr Jaffrey of Saddlehill Park, about 10 miles southward from DUNEDIN, on the Great South or Invercargill Road.	1. From Main Adit. COMMON BROWN COAL.—Brown; compact; massive; earthy; comparatively heavy;[2] brittle; *lustre* dull; *fracture* uneven; no vegetable structure apparent to naked eye; cracks on exposure as if baked, apparently from desiccation[2] and shrinkage; subsequently splinters into fragments or "small."	1021	659	362	1217	3175	...	45	29	54	16
	2. BROWN COAL.—Blackish-brown; contains occasional *quartz* pebbles or grains; physical characters otherwise as in No. 1.	972	887	85	1267	2954	17·	43	39	56	3
	3. From Roof of Main Adit. Transition state between BROWN COAL and LIGNITE.—Obscurely fibrous or ligneous; laminated; associated with arenaceous shales.	1262	681	586	977	3024	7·4	56	30	43	26
	4. LIGNITE.—Consists of, or contains, much vegetable débris; very friable; bears a close resemblance, save in colour, to the *Mineral Charcoal* of the Scotch coal measures.	1046	964	82	1194	3175	15·5	46	43	53	3
	Mean of four foregoing classes of Coal, which include *Brown Coal, Lignite,* and their transition states, ...	1075	797	278	1163	3082	13·3	47	35	51	12
	5. From Roof of Main Adit. CARBONACEOUS SHALE.[3]—Largely intermixed with arenaceous clays; very impure; not used as fuel.	1099	733	366	1140	1889	...	48	32	50	16
II. SADDLEHILL—M'Coll's Creek, on its seaward base: near the Coast, and about 10 miles southward of Dunedin.	6. BROWN COAL.—Laminated; earthy; ochrey; friable; of light specific gravity; resembles more than do most of the Otago Tertiary Coals, the *Brown Coals of Bohemia.*	943	632	303	1296	3931	...	42	28	53	13
III. ABBOT'S CREEK[4]—on the Kaikorai Stream, Chain Hills, Greenisland, about 6 miles southward from Dunedin, on the Great South Road.	7. Transition state between BROWN COAL, LIGNITE, and JET: partaking of the lithological characters of all. Black; *texture* fibrous, partly woody; of light specific gravity; contains occasional *quartz* pebbles, and *iron pyrites;* associated with *pyritous shales,* which yield on decomposition with water considerable quantities of *sulphate of iron.*	1068	737	331	1170	3024	...	47	32	52	14
IV. MOUTH OF THE CLUTHA.—KAITANGATA COAL-FIELD—Lewis' Colliery, Coast Cliffs, south of Coal Point, about 60 miles southward of Dunedin.	8. Transition state between LIGNITE, BROWN COAL, and PITCH COAL.—Black; compact; *texture* sub-fibrous; *lustre* dull; *fracture* sub-conchoidal; of light specific gravity; graduates into a *Pitch Coal,* resembling in external characters the *Scotch Cannels* or the Mesozoic coals of Nelson and Canterbury.	861	787	73	1378	3250	...	38	35	61	3
	Mean of the last three groups of specimens, which include *Brown Coals* and *Pitch Coals,* and their transitions into *Lignite* and *Jet,*	957	718	235	1281	3401	...	42	31	55	10
	Mean of seven varieties of Otago Tertiary Coals, including the transitions between *Jet* and *Pitch Coal,*	1016	758	257	1222	3241	...	44	33	52	11
	ABSTRACT.										
	Maximum, ...	1262	964	586	1378	3931	17·0	56	43	61	26
	Minimum, ...	861	632	73	977	2954	7·4	38	28	43	3
	Average, ...	1016	758	257	1222	3241[5]	13·3[6]	44	33	52	11
	II. TERTIARY COALS OF NELSON.										
V. ENNERGLYN—up the Waimea Valley, on a hill about 4 miles south of the town of NELSON; Jenkins' Colliery.	9. PITCH COAL: having the external characters of the "*Glanzkohle,*" of the Germans and of the English "Newcastle." Black; *lustre* vitreous or splendent; *fracture* conchoidal. Much used in the town of Nelson as a domestic coal; but not used for steam purposes, avowedly on account of its large proportion of *ash.*	973	911	62	1266	4400	...	43	40	56	2

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(1). In round numbers (without decimals).

(2). The Otago Brown Coals generally contain, when first extracted from their beds, from 20 to 30 per cent. of *water,* of which sometimes as much as one-half is given off in the process of natural desiccation.

(3). Among these carbonaceous sandy shales I found *Lignite,* closely resembling the Icelandic "*Surturbrand.*" It could only be distinguished from recent or fresh wood by its greater darkness, density, and weight. In other localities, *e. g.,* near Coal Point, I have found the same form of Lignite, always in strata *overlying* the Brown Coal proper, and referrible, I suspect, to the *Newer or Upper Tertiaries.*

(4). In the immediate vicinity of the Fairfield, Shand's, and Walton Park Collieries, now in successful operation, and probably part of the bed on which these works are situated.

(5). As a standard of comparison, I append the following Analyses attached to certain specimens of familiar Scotch Palæozoic (or Primary) Coals or allied Minerals, contained in the Museum of Science and

Description of Coal.	Quantity per Ton.	
	Gas, in cubic feet.	Crude Paraffin Oil, in gallons.
1. Torbanehill Mineral, Boghead, Linlithgowshire,	15,000	120
2. Lesmahagow, Duke of Hamilton's Pit, . .	12,000	...
3. Brown Cannel, Methil, Fifeshire, . . .	10,000	90
4. Cappeldbrae, do. . . .	...	80
5. Cannel, Rochsoles, Lanarkshire, . . .	...	72
6. Overton, do. . . .	...	64
7. Black Cannel, Methil, Fifeshire, . . .	...	50
8. Wellwood, do. . . .	...	48

XV.—*On Variability in Human Structure, with Illustrations, from the Flexor Muscles of the Fingers and Toes.*

By Wm. Turner, M.B. (Lond.), F.R.S.E., Senior Demonstrator of Anatomy in the University of Edinburgh.

(Read 19th December 1864.)

Deviations from the usually described arrangements of the parts, of which the human body is composed, have from time to time attracted the attention of the anthropotomist. In many anatomical text-books, as well as in sundry memoirs specially devoted to the subject, numerous examples of such variations have now been recorded. To the scientific anatomist these have always had a certain value, but of late years this department of anatomical inquiry, more especially in connection with variations in the muscular system, has had additional importance and interest attached to it, on account of the attention which has been directed to the correspondence, or want of correspondence, in the muscular arrangements in man and the other mammalia, more particularly the apes.

Into this aspect of the question it is not my intention to enter in this communication. My object on this occasion is rather to compare certain structures in one human body with corresponding structures in others, and to point out the extent of variability which may occur in similar parts in different individuals.

Every one is conscious that of the multitude of individuals he may meet with in the course of a day's experience, no two are alike. Leaving altogether out of consideration all mental differences, each possesses some peculiarity of form and gait which enables him at once to be distinguished from those around him, and that these external manifestations of variability are in their prominent features correlated with internal structural differences will, I suppose, be generally admitted. That diversities in the shape of the skull, for example, occasion corresponding diversities in the form of the head and face, so as to impart to them characters diagnostic not only of the race, but of the individual, have been recognised from the time of Blumenbach and Camper. But the osseous is not the only organic system in which distinct evidence of structural variability may be traced; the muscular, vascular, nervous, and visceral systems all exhibit it. In some cases, undoubtedly, the departure from what may be termed the standard method of arrangement, as set forth by descriptive writers, is greater than in others, but evidence of its existence to a greater or less degree in every individual may be obtained not only by the examination of the systems taken as a whole, but of the separate structures of which they are composed. But though some of the best marked examples of internal structural variations are correlated with corresponding variations in the external configuration of the body, yet there are a large number which, either from their minuteness, or from being situated in the deeper seated parts, give no sign externally, and to distinguish them requires close and careful dissection. For many years I have been in the habit of preserving a record of the most remarkable "irregularities," as they are often called, which have come under my notice; and I could cite many cases from my note-book in which, in the course of dissection, variations in the different organic systems were noted. During the present winter session, for example, four arms have been met with in which that very curious process of bone, known as the supra-condyloid process, projected from the inner part of the shaft of the humerus. In all, this process was connected to the inner condyle by a ligament. The process, the ligament, and the shaft of the humerus, covered by the brachialis anticus muscle, formed collectively the boundaries of a supra-condyloid foramen. In all the median nerve went through the foramen. So far these limbs, though varying greatly from the usual arrangement of parts in the human upper arm, corresponded closely with each other, but in other respects they differed considerably amongst themselves. In three specimens the pronator radii teres muscle arose from the process and the ligament connecting it to the condyle; in the fourth the pronator muscle did not arise from these structures, but the ligament gave origin to some of the fibres of the brachialis anticus muscle. In one specimen the brachial artery, after giving off an accessory radial artery high up in the limb, accompanied the median nerve through the supra-condyloid foramen; in another the brachial artery, after giving off its ulnar branch of bifurcation high up in the limb, also passed along with the median nerve behind the process; in the third, the brachial artery pursued its usual course along the inner margin of the biceps to the bend of the elbow previous to its bifurcation, and sent simply a small branch through the foramen along with the median nerve; in the fourth not only was the brachial artery not deflected from its customary course, but it did not even send a small branch through the foramen, through which, consequently, the median nerve proceeded unaccompanied by any vessel.* In three of the specimens, also, a muscular slip arose along with the flexor sublimis digitorum from the coronoid process of the ulna. This slip terminated on a tendon, which in one specimen became blended with the tendon of the flexor profundus passing to the middle finger; in another with the tendon of the same muscle going to the ring finger; and in the third with the tendon going to the little finger. In the fourth specimen the flexor sublimis was not connected to the flexor profundus by any intercommunicating structures. In one specimen the middle, ring, and little finger tendons of the flexor profundus were connected together by a network of intertendinous bands; in another such bands only connected the middle and ring finger tendons of that muscle; in the remainder these structures were absent. In one of the specimens the abductor pollicis received a distinct slip of origin from the styloid process of the radius, whilst in another the abductor minimi digiti received a slender muscular slip, which arose in the lower part of the forearm from an accessory palmaris longus tendon.

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* The four cases described in the text of the occurrence of a supra-condyloid foramen in the human upper arm are not the only specimens which have come under my notice in the dissecting-room. In former years I had observed five specimens, in three of which both brachial artery and median nerve passed through the foramen, in the remaining two I had unfortunately not preserved a note of the arrangement. But by far the most complete account of the anatomy of the supra-condyloid foramen which has yet appeared has been drawn up by Professor Wenzel Gruber in an elaborate memoir presented to the Imperial Academy of St Petersburg. Vol. viii. 1859. This anatomist has collected from the works of previous writers, as well as from material which has come under his own observation, sixty-two cases in which this foramen was noticed in the human body, and in which there was at the same time a greater or less amount of variation in the arrangement of the pronator teres, the median nerve, the brachial artery or some of its branches. One of the chief features of interest connected with the supra-condyloid foramen is the circumstance that it furnishes, as an occasional occurrence in human structure, an approximation to an arrangement

It would be quite possible for me to multiply examples to serve as additional illustrations of variations occurring in several of the most important organic systems in the same body,—variations so well marked, indeed, that though, as in the cases above cited, it is probable no outward evidence of their existence was manifested, yet they furnished the individuals in whom they occurred with characters as distinctive as any peculiarities of external configuration. Hence we may conclude that in the development of each individual a morphological

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which is very frequently met with in some of the Mammalia. For there has now been recorded a considerable number of instances in which a distinct canal, generally with bony walls, existed in this locality in various Quadrumana, in Galeopithecus, in the Edentata and Monotremata, in many Carnivora, Marsupialia, Rodentia, and in some of the Pinnepedia; whilst it would appear to be absent in the Ruminantia, Solidungula, Multungula, and Cetacea. But though the canal would seem to occur almost constantly in all the genera of some orders and families of the Mammalia, yet it by no means follows that in other orders and families, though it may occur in one genus, that it exists in all, or even though it may occur in one species of a genus, that all the species of the same genus should possess it. Thus, as Professor OWEN has shown (Article *Marsupialia* in "Cyclopædia of Anatomy and Physiology"), whilst it exists generally amongst the Marsupialia, yet it is absent in Dasyurus and Thylacinus; and though most of the species of Phalangista possess it, yet it does not exist in *Phalangista Cookii;* and whilst GRUBER saw it in *Erinaceus auritus*, he did not find it, and I have not seen it, in *Erinaceus europæus.* In the Pinnepedia also it has been described by various anatomists as present in the *Phoca vitulina*, and GRUBER has seen it in other species of the same genus. In a common seal which I dissected, I found that it only transmitted the median nerve, neither the brachial artery, nor any of its branches passed through it; in the Walrus (Trichechus), however, anatomists agree in stating that it does not occur, a fact which I have observed in three skeletons of that animal which have come under my observation. Again, in some of the Mammalia variations in its occurrence take place in individuals of the same species, a circumstance which has been noticed not unfrequently amongst the Quadrumana, though it has not as yet been seen, I believe, in the humeri of any of the Anthropoid Apes. Thus, whilst TIEDEMANN describes it as present in *Cercopithecus sabæus* and *Cercocebus fuliginosus*, MECKEL and OTTO state that it is wanting in those species; a discrepancy of statement which may probably be explained by regarding the arrangement as a variety present in one individual but absent in another. Of the skeletons of the Quadrumana personally examined, I have found the foramen absent in two specimens of the Orang, in a Chimpanzee, in two specimens of the Gibbon, in *Cercocebus fuliginosus* (agreeing thus with MECKEL and OTTO), in *Macacus cynomolgus*, in *Cynocephalus maimon*, *Hapale jacchus*, and *Ateles paniscus;* whilst I have found it present in a species of Cebus, and in the prosimian *Stenops tardigradus.*

specialisation occurs both in internal structure and external form, by which distinctive characters are conferred, so that each man's structural individuality is an expression of the sum of the individual variations of all the constituent parts of his frame.

But it is not essential that, for the demonstration of this specialisation of structure in the individual, we should extend our inquiries over all the organic systems. Any one, if carefully examined, will afford us sufficient evidence of its existence. The muscular system is the one I have especially selected for illustration. There are some parts of this system in which, from the mode of arrangement of single muscles, and from the manner in which they are collected into groups, we are enabled to study more precisely than in other localities the extent of variation which is permitted, and the various forms which it assumes. None are better fitted for this purpose than the flexor muscles of the fingers and toes; for not only can we define with great exactness the arrangement of these muscles and their tendons, but we can employ in connection with them a method of description precise enough to convey a conception, not only of the stronger and best marked varieties, but of the more minute deviations from their usually recognised disposition. During the past twelve months, I have made a series of special dissections of these groups of muscles, and I shall now record the general results which I have arrived at in the examination of these parts. It must be understood that all the dissections were made on the bodies of the inhabitants of these islands, natives of either Great Britain or Ireland.

The flexor muscles in the forearm and hand, to which my attention has especially been directed are the flexor longus pollicis, the flexor sublimis digitorum, the flexor profundus digitorum, and the lumbricales. The long flexor muscles exhibited numerous variations in their bulk, in the extent of their attachment to the bones of the forearm (the extent of the radial origin of the superficial flexor was especially variable), and to the interosseous or other fibrous membranes from which they arose. The superficial and deep flexors of the fingers also varied as to the mode in which they divided into their terminal bundles; in some cases the division took place lower down in the forearm than in others. This was especially the case with the deep flexor, in which it was not unfrequent to see the separation between the more internal of its terminal tendons still incomplete at the carpal end of the forearm, or beneath the annular ligament. In one specimen in my possession, the muscle divided into five bundles, two of which afterwards united to form the tendon for the ring finger.* But, in addition, other variations were met with of a more remarkable character. A more clear conception of their nature may perhaps be formed if we conceive the long flexors of the digits as composed of muscles situated on two planes, a superficial and a deep, and then bear in mind that both sets of muscles are subdivided into bundles, each of which terminates in a tendon possessing a distinct attachment to its proper digit. Now, between the different subdivisions of the muscle or muscles, situated on the same plane, and between the muscles situated on different planes, tendinous or musculo-tendinous bands not unfrequently proceed so as to connect them together. Thus, whilst it is customary to consider the flexor sublimis as dividing into four distinct bundles, each ending in a tendon, I not unfrequently saw a tendinous or musculo-tendinous slip proceed between adjacent bundles, and keep up a lateral communication between the divisions of the muscular mass situated on the same superficial plane. In a similar manner, the divisions of the muscular mass situated on the deeper plane were not unfrequently connected together by lateral bands. In the flexor profundus digitorum these lateral connecting bands presented various arrangements in different individuals. Sometimes the three inner tendons were closely tied together in the forearm, either by simple bands passing from one to the other, or by a more complicated reticular structure. At others only the two inner tendons; at others again the tendons for the middle and ring fingers were intimately connected (fig. 1), whilst the little and index tendons were quite free. As a rule, indeed, the index tendon appeared to be less liable to form a connection with the tendon of the same muscle than was the case with the other subdivisions of the flexor profundus. But on the other hand, I saw several specimens in which the index division of the deep flexor was intimately connected to the flexor longus pollicis,* a junction of considerable interest, as it approximates in their arrangement these muscles in the forearm and hand with the flexor hallucis and flexor communis digitorum in the foot. The nature of this union varied considerably in different specimens. In some it consisted of a muscular bundle, passing obliquely downwards from the fleshy part of the flexor of the thumb to the fleshy part of the index division of the deep flexor; in others it consisted of a musculo-tendinous slip proceeding obliquely downwards from the muscular part of the former to the tendon of the latter (fig. 1); and in some of these

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* Multiplication of the bundles of this muscle has been recognised by ARNOLD, HENLE (Muskellehre, p. 196, 1858), and THEILE (Traité de Myologie), 1843, p. 246. THEILE also states that it sometimes receives a special head of origin from the inner condyle of the humerus; and THEILE, HALLETT (Ed. Med. and Surg. Journ. vol. lxxii. p. 12), and HENLE state that it sometimes receives fibres from the radius.

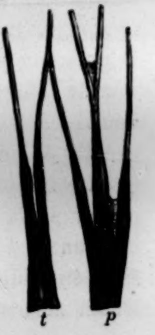

Fig. 1.†

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* Various anatomists have recognised the occasional connection of these tendons, without, however, specialising its different forms. See Theile, p. 246; M'Whinnie, Lond. Med. Gaz., vol. xxxvii. p. 191; Henle, p. 196; Wood, Proc. Roy. Soc. of London, p. 301, 1864. I am disposed to regard the connection in one or other of its forms as more common than is usually supposed.

† Fig. 1, *t*, flexor longus pollicis; *p*, flexor profundus digitorum. It shows the connection of the index tendon of the latter muscle with a strong musculo-tendinous band from the former, also the close union for some distance of the middle and ring-finger tendons of the deep flexor. This, and the other illustrative figures, have been drawn from the dried preparations of my dissections by my pupil, Mr Richard Caton.

cases the connecting slip was in great measure, though not altogether, formed of the fibres of the rounded head of the flexor longus pollicis, which arose from the coronoid process of the ulna. In one case the connecting slip received almost one-half the fibres of the long flexor muscle, a specimen which illustrates how large and important this intermuscular tendon may at times become. In one very remarkable specimen the bond of union passed in the opposite direction from those above described—viz., from the index tendon of the flexor profundus to the tendon of the flexor longus pollicis (fig. 2).

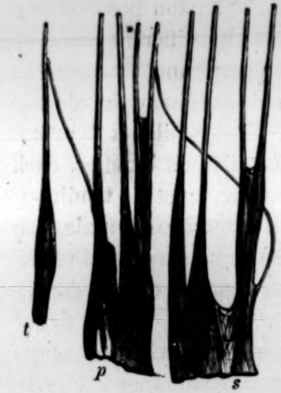

Fig. 2.*

Amongst the intermuscular structures which not unfrequently connect together the superficial and deep flexor muscles of the forearm, I am disposed to place that rounded musculo-tendinous band, which is so often met with, as a second head of origin of the flexor longus pollicis, for it arises along with the flexor sublimis from the coronoid process of the ulna, and ends inferiorly in the inner part of the long flexor of the thumb. But the superficial is also not unfrequently connected to the deep flexor of the fingers by intermuscular bands. For I have frequently seen a slip of muscle arise from the coronoid process, along with, and apparently forming a part of, the flexor sublimis, which, after it became tendinous, blended in five specimens with the tendon of the flexor profundus going to the little finger (*e.g.* fig. 2), in one with the tendon passing to the ring-finger, in four with the tendon of the same muscle going to the middle finger, and in one it divided into three slips which joined the deep tendons for the middle, ring, and little fingers. The blending usually occurred opposite, or slightly below, the carpal articulations.†

The lumbricales muscles exhibit many forms of variation in size, number, extent, surface of origin, and mode of insertion; but as both THEILE and HENLE have entered fully into these varieties, I need do no more than state that I have seen, in addition to most of the forms which they have described, a variety in which an accessory first lumbricalis arose tendinous from the flexor sublimis.

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* Fig. 2, *t*, flexor longus pollicis; *p*, flexor profundus digitorum; *s*, flexor sublimis digitorum. The connection of the first and second muscles by a tendinous band passing from the index tendon of the latter to the long flexor of the thumb, is shown; also a tendon connecting the ulnar side of the superficial with the tendon of the deep flexor for the little finger; also a close connection low down the limb between the ring and little finger tendons of both the superficial and deep flexor muscles.

† The presence of slips proceeding between the flexor sublimis and F. profundus, though without precise statement as to their connections, has been recognised by COWPER (Myotomia reformata), THEILE, and WOOD.

I may in this place also refer to an arrangement which I saw on one occasion in the left forearm. A slender fasciculus of muscular fibres proceeded from the flexor sublimis immediately to the inner side of the palmaris longus. It ended on a tendon which passed beneath the palmaris, and joined the tendon of the supinator radii longus at the lower end of the forearm.

After a course of about two inches it became muscular, and then ran parallel to the first lumbricalis, and was inserted along with it.

The flexor muscles of the toes, the arrangements of which I have more especially studied, were the flexor brevis digitorum, the flexor communis digitorum, the flexor longus hallucis, the flexor accessorius, and the lumbricales,—an important group of muscles, the different members of which are more or less intimately related to each other in the sole of the foot. In order to form as precise a conception as possible of their mode of arrangement in the foot I carefully dissected fifty specimens, taking them without selection from the subjects which came in my way in the ordinary course of my anatomical work, so that the variations described must not be regarded as unusual or abnormal forms. The results I have arrived at differ in many respects from the descriptions of these muscles usually given in treatises on anatomy. Of these fifty specimens no two were exactly alike, so that it would be necessary, in order properly to bring out the extent of individual variation which they presented, that each should have a separate description; but as this would be tedious both to writer and reader, it may suffice if I adopt some method of arrangement which may exhibit their most important variations.

In all the specimens the tendon of the flexor longus hallucis gave off, in the sole of the foot, a slip or band which connected that tendon either to one or more of the subdivisions of the flexor communis digitorum, or in part to that tendon, and in part to the flexor accessorius. In its size this connecting slip varied somewhat, and though at times flattened and membrane-like, yet was mostly in the form of a rounded band. In every specimen it took a more or less important part in the formation of the deep flexor tendons for one or more of the four outer toes. In eleven specimens it ended solely in the deep flexor tendon for the second toe; in twenty specimens it bifurcated and ended in the deep flexor tendons for the second and third toes; in eighteen specimens it trifurcated, and ended in the deep flexor tendons for the second, third, and fourth toes; in one specimen it divided into four parts, and ended in the deep flexor tendons for the four outer toes.

Of the eleven specimens in which the connecting band went solely to the second toe, it formed about one-half the deep flexor tendon for that toe in four cases (fig. 9), the remaining half being formed partly by the flexor communis and partly by the flexor accessorius. A much larger proportion than one-half in six cases (fig. 11); and in one case it and the flexor accessorius together formed the whole of the deep flexor tendon for the second toe, in the construction of which the flexor communis did not consequently enter (fig. 3).

Of the twenty specimens in which the connecting slip went to the second and third toes, it contributed a larger share to the second than the third toes in twelve specimens, in one of which it formed, with the addition of a few fibres from the flexor accessorius, the whole of the deep flexor tendon for the second toe;* it was divided almost equally between the two in six specimens (fig. 4); and in two specimens it and the flexor accessorius together formed almost the whole of the deep tendons for these toes, the share taken in their construction by the common flexor being limited to a few fibres (fig. 5).

Of the eighteen specimens in which the connecting slip went to the second, third, and fourth toes, it contributed a larger share to the second than to either the third or fourth in seven specimens (fig. 6), in one of which the process for the deep flexor tendon of the second toe was much larger than that supplied by the flexor communis; a larger share to the second and third than to the fourth in five specimens; a larger share to the second and fourth than to the third in one specimen; and about equally to these three toes in the remainder.

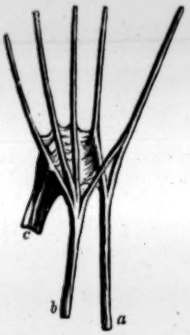

Fig. 3.†

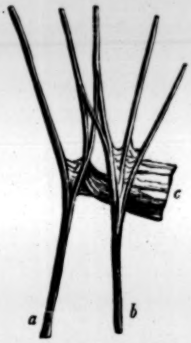

Fig. 4.

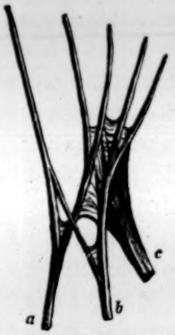

Fig. 5.

In the solitary specimen in which the connecting slip went to the four outer toes, the subdivisions for the second and third toes were larger than those for the fourth and fifth, that for the fifth being a comparatively slender thread (fig. 7).

In none of the fifty specimens did the connecting band join the tendon of the common flexor previous to the subdivision of the latter tendon. In every instance it proceeded either single, bifurcated, trifurcated, or in four subdivisions, to its appropriate toe or toes, and in its course joined the divisions of the flexor communis, or the portions of the flexor accessorius passing to the same toe or toes. To the deep tendons for the second and third toes, more especially, it not unfrequently contributed quite as much as the flexor communis, and occasionally it and the flexor accessorius together, entirely or almost entirely, were substituted for the flexor communis in the construction of the deep flexor tendons for those toes.*

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* The flexor communis in this case trifurcated for the third, fourth, and fifth toes.

† Fig. 3. In this and the succeeding figures, *a*, is the flexor hallucis longus tendon; *b*, the flexor communis digitorum tendon; *c*, the flexor accessorius. In figure 3 the flexor communis forms no portion of the deep tendon for the second toe, and after giving a slip to the flexor hallucis tendon, trifurcates for the three outer toes.

In nine specimens a band, sometimes of considerable size, proceeded from the common flexor tendon previous to its subdivision, which joined the tendon of the flexor hallucis longus beyond the origin of the connecting slip for the common

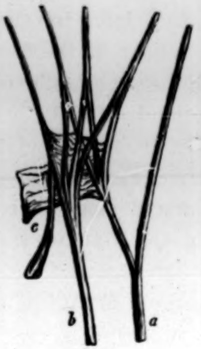

Fig. 6.

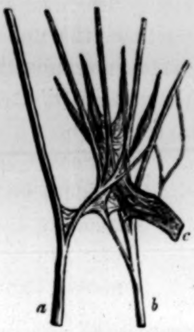

Fig. 7.

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* That the tendon of the long flexor of the great toe gives off a band more or less strong to the common flexor of the toes in the sole of the foot has been almost universally recognised by anatomists, but the exact nature of the connection between them has not at all times been clearly expressed. Amongst the older anatomists, Vesalius describes this band as passing from the tendon of the great toe to the tendon proceeding to the second toe, and sometimes in an equal degree to the tendon of the common flexor for the middle toe. Diemerbroeck again states that sometimes the long flexor of the great toe is divided in the sole into two parts, one of which goes to the great, the other to the second toe, and then the common flexor sends but three tendons to the other toes. Cowper and Bidloo simply describe a connecting band passing from the proper to the common flexor, without specialising its mode of termination, and this method of description has been followed by most systematic writers in the latter part of the last century, and in the present, as Innes, Monro, Sabatier, Bichat, Boyer, John Bell, Fyfe, Cloquet, Cruveilhier, Dodd, Quain, Harrison, Hyrtl, Ledwich, Ellis, Knox, Holden, Heath, and Gray. Meckel employs, in his description of the long flexor of the great toe almost the same method as Diemerbroeck, but, in addition, states that the long flexor tendon for the second toe is for the most part formed by the connecting band and the flexor accessorius. Theile follows very closely the latter statement of Meckel, but, under the head of anomalies, he describes the connecting band as dividing for the second and third toes. Arnold gives the connecting band as strengthening the tendon for the second toe, though it often goes also to the third toe. Henle states that the strong process from the proper to the common tendon is for the most part, and at times altogether, continued into the tendon destined for the second toe. Mr Church, in a recent monograph on the myology of the Orang (*Natural History Review*, 1862), has also directed attention to the connection of the band from the flexor hallucis with the second and third toes. Professor Rolleston has advanced evidence to the same effect. Last of all, Mr Huxley (*Reader*, 13th February 1864) states, as the results of his dissections, that the tendon of the flexor hallucis longus, besides giving off the tendon to the great toe, furnishes distinct slips to the two or three succeeding digits, uniting with the tendons of the flexor digitorum and flexor accessorius. That considerable variability occurs in the mode of termination of the connecting band might almost be inferred from the different descriptions given of it by the numerous anatomists just quoted, each apparently, of those at least who go into details, basing his description on the specimen or specimens he may more particularly have examined. A more exact conception, however, of the extent of this variability may be gathered from the analysis of the fifty specimens recorded in the text.

flexor tendon from it (figs. 3, 5, 8, 10). In these cases, therefore, the tendons were doubly connected by intertendinous bands.[*]

The flexor accessorius varied greatly in its mode of termination on the flexor tendons. In but a few instances (fig. 5, for example) could it be said to end in the manner usually described in the text-books, by joining the outer border and upper, and sometimes the under surface of the tendon of the flexor communis. In many cases it had no connection whatever with the outer border of that tendon; in several of these it contributed no fibres to the tendon for the little toe, and in a few it had no connection with the tendons for the fourth and fifth toes. In other cases, however, it gave off a distinct tendinous or musculo-tendinous bundle, sometimes of considerable size, to the deep tendon for the little toe (figs. 3, 6, 7). In a few cases the deep flexor tendon for that toe was almost entirely (fig. 10), and in one case (fig. 8) apparently, entirely formed of a tendon proceeding from the flexor accessorius, the common flexor tendon sparingly in the former (fig. 10), and not at all in the latter case (fig. 8), entering into its construction. In most cases the accessory flexor ended partly on the flexor communis, and partly on the connecting slip from the flexor hallucis, and through one or both of these contributed materially to the formation of the deep tendons for the second, third, and fourth toes. In one case it sent, in addition, a few fibres to the primary tendon of the flexor hallucis, and in another all its fibres terminated on the connecting slip, and through it were transmitted to the deep flexor tendons of the second and third toes. In one case it gave off a distinct slip, which, separating into two parts, gave one to each process of bifurcation of the tendon of the flexor brevis digitorum for the third toe.

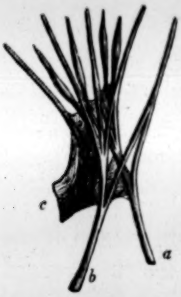

Fig. 8.†

In two cases the flexor accessorius had an accessory muscle connected to it, which arose from the deep fascia of the back of the leg in its lower third, concealing at its origin the posterior tibial vessels and nerve. It passed downwards, and ended in a rounded tendon, which extended through the inner ankle beneath the abductor pollicis, and joined the inner margin of the flexor accessorius (fig. 9).[‡]

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[*] It would almost appear as if some of the systematic writers of the last century had recognised the band proceeding from the flexor communis to the flexor hallucis, but not the one passing in the opposite direction. *Vide* ALBINUS, WINSLOW, TARIN, SANDIFORT, and DOUGLAS. Several of the more recent writers have described an arrangement similar to the one recorded in the text.—*Vide* SABATIER, ARNOLD, and THEILE.

[†] Fig. 8 shows the deep flexor tendon for the little toe entirely formed of the flexor accessorius. The flexor communis, after sending off a connecting band to the flexor hallucis, trifurcates for the second, third, and fourth toes, to which the connecting band from the flexor hallucis also proceeds.

[‡] The two specimens described in the text were found amongst the fifty specimens specially analysed, but I have in former years, and in other subjects, met with additional instances of an accessory muscle in this locality. The region of the inner ankle appears, indeed, to be frequently the seat of such accessory muscular structures, *e.g.*

The lumbricales muscles presented many variations, some of the leading forms of which it may be advisable to particularise, more especially that Froment (with whom Henle seems to agree) states that variations in the arrangement of these muscles are extremely rare. The first lumbricalis, in the specimens under analysis, arose sometimes from the tibial side of the deep tendon for the second toe, after the junction of the connecting slip from the flexor hallucis with the division of the common flexor tendon for that toe. Sometimes only from the tibial side of the connecting slip; sometimes only from the division of the common flexor to the second toe; in one case from the expanded part of the common flexor before it divided into its terminal tendons; in other cases by a continuous origin both from the tibial side of the second toe tendon, and from the expanded part of the common tendon; and in two cases by two distinct heads,—one from the tibial side of the connecting slip from the flexor hallucis, the other from the expanded part of the common flexor before its division into the terminal tendons. In one case no lumbricalis was present in the first metatarsal space, but two were situated in the second space (fig. 8). In another case the second lumbricalis was absent. In several cases, not only did the second, third, and fourth muscles arise from adjacent sides of the tendons between which they were situated, but also from special slips derived from those tendons. Sometimes their fibres of origin extended for some distance backwards over the general expansion of the common flexor tendon. In other instances the fibres of the fourth lumbricalis, or of the third and fourth lumbricales, were continuous with (or received fasciculi from) those of the flexor accessorius; in others they

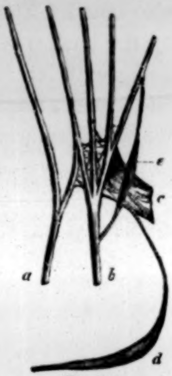

Fig. 9.[*]

1*st*, A large slip springing from the inner side of the soleus, and passing quite distinct from the tendo Achillis, to be inserted into the inner concave surface of the os calcis.

2*d*, A muscle arising from the deep fascia of the back of the leg, and inserted into the inner side of the os calcis, close to the inner head of the flexor accessorius; this apparently constitutes the accessorius ad calcaneum of Gantzer.

3*d*, Two muscular bundles connected to the deep fascia of the back of the leg, one as high as the middle of the tibia, the other close to the origin of the flexor hallucis longus from the fibula; these bundles united to form a muscle which passed beneath the internal annular ligament to the sole where its tendon bifurcated, one slip joining the tendon of the flexor hallucis longus, the other the tendon of the flexor communis digitorum. A corresponding arrangement was found in both limbs.

4*th*, A well-marked muscle arose from the deep surface of the soleus tendon. It concealed the tendons of the deep muscles, and the posterior tibial vessels and nerves in the lower third of the leg, and was inserted into the deeper surface of the tendo Achillis, immediately above the os calcis. A similar case to this has been described by R. Quain.

Other irregularities in this locality have been recorded by Mayer, Rosenmüller, Gantzer, Meckel, Hallett, Theile, Henle, and John Wood.

[*] Fig. 9. *d*, the accessory muscle to the flexor accessorius. It has been bent out of its proper direction so as to occupy less space in the wood block. *e*, the displaced fasciculus of the flexor brevis for the little toe, which simply blends, without bifurcating with the deep flexor tendon for that toe.

received a special fasciculus of fibres, arising from the process sent by the connecting slip of the flexor hallucis to the third or fourth toes; in one case the fourth lumbricalis was absent.

Variations in the mode of arrangement of the flexor brevis digitorum were also noted. The tendon passing to the little toe was sometimes not perforated by the tendon of the common flexor. In one case it was blended and inserted along with it; in others it was so thin as to be lost in the fascia of the foot; in one it was altogether absent. In five cases the short flexor tendon for the little toe was displaced at its origin, and arose from the common flexor tendon previous to the subdivision of that structure (fig. 10).* At its origin it either consisted partly of fibres continuous with those of the common flexor tendon, and partly of distinct muscular fibres attached to and springing from that tendon, or it arose tendinous, and then muscular fibres appeared in it, which again terminated on the tendon of insertion. In three of these cases the tendon bifurcated, to allow the common flexor tendon for the little toe to pass through and beyond (fig. 10); in the other two it blended with the common flexor tendon for that toe, and was inserted along with it (fig. 9). In one specimen the tendon for the

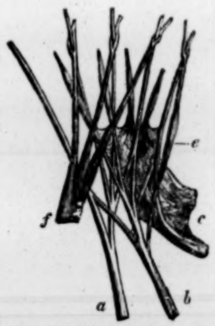

Fig. 10.*

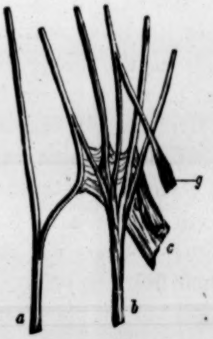

Fig. 11.†

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* Fig. 10. In this drawing, *f* is the flexor brevis digitorum, which divides into fasciculi for the second, third, and fourth toes, the fasciculus for the fifth toe, *e* arises from the tendon of the flexor communis. In this figure the deep flexor tendon for the little toe is formed almost entirely from the flexor accessorius, the flexor communis contributing but a few fibres. The connecting slip from the flexor hallucis trifurcates for the second, third, and fourth toes, and the flexor communis gives off a connecting band to the flexor hallucis. Brugnone, Meckel, Theile, Hyrtl, Henle, Church, and Huxley, have all recognised the occasional origin of the short flexor tendon for the little toe from the flexor communis. In another subject I saw the fasciculus forming the short flexor tendon for the little toe arise in part from the external inter-muscular septum, and in part through fibres continuous with the muscular part of the flexor accessorius.

† Fig. 11, *g*, the tendon of the flexor brevis for the third toe, its junction with a slip from the expanded part of the flexor communis is represented. In both feet of a subject not included in the above analysis, I saw an arrangement similar to that represented in fig. 11, except that the slip from the common flexor tendon bifurcated before joining the two branches of bifurcation of the flexor brevis.

third toe received a strong tendinous slip from the expanded part of the common flexor tendon, and the two were blended and inserted together (fig. 11). In another, the short flexor tendon for the third toe received an additional slip from the flexor accessorius.

In the foot, therefore, as in the forearm and hand, intermuscular structures not unfrequently connected together not only the flexor muscles situated on the same plane, but those situated on the superficial and deeper planes.

The fifty specimens of the flexor muscles of the foot, the special analysis of the mode of arrangement of which I have now recorded, will be sufficient, I think, to show that in the construction of this group of muscles an amount of variation existed much greater than might at first sight have been supposed. In a portion of one muscle alone, viz., the connecting band from the tendon of the flexor hallucis longus, a considerable number of modifications occurred. In the flexor accessorius also great variability was displayed, and in some proportional relation, apparently, to the extent of variation in it and the connecting band, did the flexor communis digitorum undergo certain modifications in its arrangement, so much so, indeed, in certain cases, as to permit those structures to be to a great extent substituted for it in the formation of the deep flexor tendons for some of the toes.

Variability in the construction of parts, however, was not manifested merely in different individuals, but in the same individual the corresponding structures on opposite sides of the body were by no means symmetrically disposed. Thus, in two of the four examples recorded in the earlier part of the paper in which a supra-condyloid process existed, it occurred only in one arm of each subject in two cases; whils in a third subject, though both humeri exhibited the process, yet the relations of the brachial artery to it on the two sides were by no means symmetrical. The tendons in the left foot, in many of the individuals in whom both feet were examined, varied also more or less from those in the right foot, so that in the construction of the limbs, as well as in the form and arrangement of the organs contained in the great cavities of the body, an asymmetrical disposition of parts is to be looked for. Thus, we arrive at the conclusion that the plan on which the human body is constructed, although constant in all its essential characters, yet admits of variations (within itself as it were) in certain directions and within certain limits. Neither form nor structure is absolutely stereotyped, but modifications occur which, when regarded singly, may be considered, perhaps, as slight and of comparatively little importance, yet when viewed collectively, are sufficient to give to the individual well-marked distinctive characters.

Much has been said and written of late years on the existence of structural differences between the fair and coloured races of mankind, more especially between the white man and the negro,—differences which, according to some writers, are so great as to constitute an actual specific distinction between them. But those who have advanced and supported this view seem to me to have ignored, or at least not to have taken sufficiently into consideration the fact, that in the white races themselves, nay, as we have shown in this paper, in a limited section of them even, variations occur in the arrangement of certain of the soft parts so great as to permit of the office usually performed by one muscle to be, in a great measure, or even altogether, exercised by another. But the extent of variation which the white races may exhibit is by no means exhausted by what we have detailed in this communication. Numerous isolated examples of variations, both in the muscular and other systems, have been recorded elsewhere by myself and other anatomists, and additional observations in the same profitable field of inquiry will, I have no doubt, add many other forms to our already extensive list. Until, however, the deviations from the usually described arrangements in the fair races are more systematically inquired into than has hitherto been the case, we cannot hope to reach an accurate conception of the latitude which may be allowed them.

Of the extent of the structural variability which may exist in the soft parts of the dark races, we as yet know but little. It is seldom that their bodies have been critically examined; and of many of the coloured races, indeed, the number of dissections has been too small to permit of any satisfactory conclusions to be arrived at, for opportunities of making the necessary observations seldom fall in the way of the European anatomist. His dissections are made and his descriptions are based on the examination of the inhabitants of his own continent. Our knowledge of the comparative anatomy of the soft parts of the races of men is still in its infancy. To make good, indeed, the proposition that the negro is specifically distinct from the white man, it would be necessary to show that any peculiarities of arrangement which may be exhibited in the construction of his body, are either constant, or, if variable, that the variations are not in accordance with those which have been or may be met with in similar parts in the bodies of men of the fair races. For until we have determined not only the amount of structural variability in the different races, but the comparative frequency of occurrence of its principal forms, we shall not be in a position even to discuss the question of specific difference on anything like positive scientific data, still less to pronounce dogmatically on the subject.

From the special difficulties which surround the study of human anatomy, it will not be an easy matter to determine with precision the laws which regulate the development of these structural variations. In some cases, indeed, it would appear that a variation in one structure is not unfrequently correlated with variations in adjacent parts. Thus in the four specimens described in the early part of this communication in which, in conjunction with a supra-condyloid process, a foramen existed above the inner condyle of the humerus, the median nerve passed through that foramen. The deflection of the nerve from its course and the existence of the process were, it is evident, not only from these but from many similar cases, correlated events. Again, in the flexor tendons of the foot, a deficiency in the size of the flexor communis digitorum was not unfrequently correlated with an increase in the size of either the connecting-band from the flexor hallucis, or of the flexor accessorius, or it might be of both; and an absence of a tendon from the flexor brevis digitorum for the little toe was not unfrequently correlated with the presence of a fifth tendon from the flexor communis digitorum.

To how great an extent the conditions of life of the individual, in whom these and other varieties present themselves, may be concerned in their production, or how far they may be transmitted from parent to offspring, and thus be considered as family peculiarities, are questions which for the present at least must be left undetermined. But in regard to the variations in the muscular arrangements which have been specially illustrated in this communication, it may safely be stated that the power of performing the appropriate movements of the part must be modified in accordance with the modifications in its structure.

XVI.—*Examination of the Storms of Wind which occurred in Europe during October, November, and December* 1863. By Alexander Buchan, M.A., Secretary to the Scottish Meteorological Society. (Plates XIII. to XXI.)

(Read 3d April 1865.)

A brief account of the weather of this period as regards temperature was read before the Royal Society last year. It was drawn up at the request of Professor Balfour, to accompany his paper "On the Remarkable State of Vegetation in the Edinburgh Botanic Garden in December 1863."

From the 26th of October to the end of December the weather was in every way remarkable. Though frost occurred in the end of October and beginning of November it was not severe, and the temperature continued on the whole seasonable till the 12th of November. From this date till the end of the month it ranged unprecedently high, being 9° above the average temperature of the season. It then fell for the next ten days, but on no occasion below the average; and again rose considerably above the average during the week ending with the 18th of December. Under this genial weather vegetation in the open air advanced rapidly to a state of forwardness not usually seen till the month of March. In December 245 plants were in flower in the Gardens in the open air, and of these 35 were spring flowers. The frost which had occurred was insufficient to damage, to any material extent, 210 autumn-flowering plants; and the high temperature of November, which was as high as what ordinarily occurs in the beginning of May, brought the spring flowers prematurely into bloom, so that there was to be seen the rare spectacle of sweet peas and hepaticas flowering together.

Thus, then, the atmosphere during this time was in a most abnormal condition in respect of temperature, which, of all the elements concerned, plays the most conspicuous part in destroying its equilibrium. It is not surprising, therefore, that the weather was equally remarkable, or even more so, for storms. From the 27th October to the 18th December, eleven well-marked storms passed over Europe in succession.

Since the space embraced by storms frequently includes the greater part of Europe, it is only recently, owing to the extension and growing popularity of meteorology, and the countenance now happily given to it by most European governments, that sufficient data could be obtained for a satisfactory treatment of the subject. For the observations of the principal observatories of Europe are too few in number, and at too great distances apart, to enable any one to lay down the isobarometric lines and general course of the winds, without drawing largely on conjecture and imagination. The recent multiplication of meteorological observatories is a great step in advance toward the discovery of the law of storms.

Observations have been received from 135 places scattered over Europe, from the Mediterranean to Archangel in the north of Russia, and from the extreme west to the Ural Mountains. All parts of Europe are pretty well represented except Central Russia, the south-east of Austria, and Turkey. The following are the sources from which the observations have been obtained:—The places in Scotland have been selected from the stations of the Scottish Meteorological Society; and most of the places in England and Ireland from Admiral FITZROY's Tables, published daily in "The Times,"—the omitted observations on Sundays having been, to some extent, supplied by the observers themselves. Most of the continental stations have been taken from the lists given in LE VERRIER's bulletins of the weather, published daily in Paris, and from the "Meteorologische Jaarboek" of Dr BUYS BALLOT of Utrecht. I am further indebted to Dr BALLOT for his valued assistance in supplying me with additional observations to those printed in the Jaarboek. The observations from Russia were kindly furnished by M. FERDINAND MÜLLER, assistant in the Physical Central Observatory of Russia; those from Sweden, by M. BONNIER, Stockholm; those from Norway, by M. C. FIARNLEY, director of the Observatory, Christiania; and those from Denmark and Greenland, by Professor HOLTEN, Copenhagen. Rev. FRANCIS REDFORD supplied the observations from Silloth; Mr E. J. LOWE, those from Nottingham; Mr W. C. BURDER, those from Clifton; Mr HENRY DENNY, those from Leeds; Mr WILLIAM JOHNSTON, those from Banbury; Captain WILLIAMSON, those from Dublin; and Mr A. DICKEY, Queen's College, those from Belfast. The importance of observations from Iceland and Faroe was not overlooked, but we regret to say that no observations were made in those places during the period. I beg also to return my most grateful thanks to the MARQUIS OF TWEEDDALE and BARON BRUNOW for the interest they took in this inquiry in procuring some of the most valuable of the observations, especially those from the north of Europe.

Construction of the Tables and Maps.

The observations at the different places were made at 8 A.M. At the few places where they were made at a different hour, such as 9½ A.M. at Dublin, a slight correction was adopted to bring them into accordance with the others. The amount of this correction was deduced from the preceding and succeeding observations at the place, modified by the apparent course and rate of motion of the storm, as suggested by the observations of neighbouring stations. It not being necessary for this inquiry to descend to the thousandth of an inch of barometric pressure, or parts of a degree of temperature, the tables and maps may be accepted as representing the pressure and temperature of the air over Europe, with scarcely any deviation from the truth

The *barometric* observations (Table I.) were brought to English inches, and then reduced to 32° and sea-level. Each observation, so reduced, was entered in its place on the map, and lines were then drawn through all those places where the pressure was equal. These isobarometric lines are given for every two-tenths in the difference of the pressure,—for 30·5, 30·3, 30·1, 29·9, &c., inches.

The lines of *temperature* have been laid down on a different principle. For lines exhibiting the actual temperature would fail to show, in a sufficiently clear manner, the real bearing of this important element, since the isothermals of October, November, and December run in a very irregular manner over the continent of Europe. Hence not the actual temperature, but the difference between the actual temperature and the mean temperature of each day at the several stations is traced on the maps.

Dr Buys Ballot has calculated the mean temperature of many places in Europe for every alternate day of the year, and for a few other places ten-day means. The results were published in 1861, in "*La Marche Annuelle du Thermomètre et du Baromètre en Neerlande et en Divers Lieux de l'Europe.*" In the observations of temperature in the "Jaarboek," 1863, not the actual temperature, but the deviations from the mean temperature of each day, are alone given. These I have adopted *simpliciter*. Of the other stations, I have calculated the mean temperature of each day, using for this purpose Dr Ballot's tables, Professor Dove's mean temperatures, as given in "*Darstellung der Wärmeerscheinungen durch Fünftägige Mittel,*" 1863; the same author's "*Monats-und-Jahresisothermen,*" 1864; and the data in the Scottish Meteorological Society's "Proceedings" bearing on the subject. The differences between these daily means and the daily observed temperatures are entered in Table III., in which the minus sign shows that the temperature was under the mean; and if no sign is used it was above the mean. The stations are pretty well distributed over Europe, and are sufficiently numerous to show the changes of temperature which occurred near the earth's surface before, during, and after the successive storms. They were entered on the maps, and then, as in the case of the barometer, lines were drawn through those places where the differences were equal. They show where the temperature was the average 0°, and then in succession where it was 4°, 8°, 12°, &c., above the average or below it.

The temperatures as actually observed are given in Table II.

The state of the sky with respect to *rain*, *cloud*, and *fog* is indicated in Table II. by means of letters—R showing that it was raining at the time of observation; C, that at least three-fourths of the sky was covered with clouds; B, that the sky was either quite clear, or not so much as three-fourths covered; and F, that fog prevailed. When *r* is attached to any of the above letters, rain fell at that station sometime during the previous twenty-four hours, but was not falling at the time of observation. The state of the sky at several places was not known. These places are marked with an asterisk, and R in such cases means that it rained sometime during the previous twenty-four hours; but whether it was raining at the time of observation or not cannot be learned from the returns.

The direction of the wind (Table V.) is indicated on the maps by arrows represented flying with the wind. The force of the wind is shown (1.) by plain arrows →, which represent light air to a moderate breeze; (2.) by arrows feathered on one side only ↣, which represent a fresh breeze to a fresh gale; and (3.) by arrows feathered on both sides ➼, which represent a strong gale, storm, tempest, or hurricane. A calm is shown by ⊙. In the tables the force of the wind is shown by the different types employed, as there explained.

The observations comprehend four periods, viz., (1.) from the 26th October to the 12th November; (2.) from the 20th to the 26th November; (3.) from the 30th November to the 5th December; and (4.) from the 14th to the 18th December —in all thirty six days. Maps were constructed, as described above, for each of these days. A selection from these accompanies this paper Plates XIII. to XVIII. give the barometric pressure, the temperature, the state of the sky, and the winds, as observed on the mornings of the 30th and 31st October, and the 1st, 2d, 10th, and 11th November. Plates XIX. to XXI. give only the observations of the barometer and winds on the mornings of the 28th and 29th October, the 3d, 4th, and 12th November, and the 1st, 2d, 3d, 4th, 5th, 16th, and 17th December.

Observations of the Barometer.

The observations of the barometer are the most important of all the observations, since it is within the area where the barometer falls to some extent below the average that storms occur. Speaking roughly, the mean atmospheric pressure for these months is 29·9 inches. Therefore, the space comprehended within the isobarometric line 29·7, and the other lines showing a less pressure, may be called, for convenience' sake, *the area of low barometer*. Hence, while we trace the progress of these low pressures over Europe from day to day, we trace at the same time the progress of the storms.

A brief account of these lines is desirable, to give some idea of the extent and course of the storms. An area of low barometer occupied the greater portion of the northern half of Europe, from the 28th October to the 9th November, during which time its eastern limit advanced slowly and steadily eastwards from Norway to the Ural Mountains; while its southern limit, having first oscillated backwards and forwards over the space lying between Spain and Ireland, ultimately moved northward, and left Europe by the North Cape. During this time four storms passed across this disturbed area, which was generally about 1900 miles in length by 1400 in breadth.

Storm I. on the 28th October (Plate XIX.), embraced the British Islands and the west of Norway, having its centre at Elgin, where the pressure was 29·41. On the 29th (Plate XIX.), the area of low barometer now included the north-west of France, the north of Germany, and the whole of Denmark and Scandinavia. The centre of the storm was near Christiansund, in Norway, and on the following day had passed out of the map by the North Cape.

Storm II. had advanced so far by the morning of the 29th October (Plate XIX.) that its centre had all but approached the west of Ireland, where the pressure was 28·56, being nearly an inch lower than the depression which accompanied the previous storm. On the 30th (Plate XIII.), it had arrived at Shetland, the lowest pressure being 29·44. At that moment the whole atmospheric system of Europe, to use a familiar illustration, appeared to be swinging round Warsaw as a centre, in the direction of the motion of the hands of a watch, so that in the south and south-west barometers were everywhere rising, whilst in the north and north-east they were falling. On the 31st (Plate XIV.), it had advanced eastward to Christiania. At the same time the isobarometric 28·9 had greatly extended its area, and a new depression (II. *b*) had been formed in its western part, due west of the former, and contiguous to it. On the 1st November (Plate XV.), the isobarometric 28·9 had contracted to one-half of its former dimensions, the two depressions in the centre had united and advanced a considerable way to the north-east; and on the 2d (Plate XVI.), it may be observed further to the north-east, and to be now leaving Europe by way of Lapland.

Storm III. had its centre, on the 1st November (Plate XV.), apparently within a hundred miles of the west of Ireland, where the pressure was 28·9. On the 2d (Plate XVI.), it had travelled east to Liverpool. On the 3d (Plate XIX.), it had continued its eastern course, and was now on the west of Jutland. At the same time the area of the storm had contracted to a fourth part of its former diameter; and the lowest pressure of the centre, instead of 28·9, was only 29·3. It was thus giving unmistakable signs of wasting away, and next morning (Plate XIX.), it had quite disappeared—a wide space between the barometric lines at the entrance of the Gulf of Finland being all that remained to show where it had died out.

The general features of the other storms were similar to those already described. The storm of the 10th and 11th November, and the storm in the beginning of December, had, however, certain peculiar features of their own to which I shall briefly advert.

The chart for the 11th November (Plate XVIII.) is the most remarkable of the charts, and the more so if compared with that of the 10th (Plate XVII.). Though the barometer fell a little to the N.E. at the head of the Gulf of Bothnia, yet the great fall took place to the S.W. To so great an extent did this occur, that the whole atmospheric system of Western Europe must be considered as having retreated on its course, and to have travelled from the N.E. to the S.W. As the translation proceeded, the depression widened and deepened, and a new depression was formed. The depression near Shetland, circumscribed by 29·3 on the 10th, and measuring 410 by 250 miles, increased to 1100 by 550 miles on the 11th, and the pressure in the centre was two-tenths of an inch greater. The isobarometric 29·5 had changed its position in a most remarkable manner. Its distance from 29·3 was greatly increased in Great Britain, and a new storm (VI.) was formed in the interval, 300 miles in diameter, having its centre near Plymouth. A little to the north of this, a space of about the same extent was noted for its high temperature on the 10th. The isobarometric 29·7 had also changed to a position equally remarkable, leaving a large space between it and 29·5 from Sardinia northwards; and in the interval a depression was formed round Genoa. On the 12th the two northern depressions had coalesced, and the isobarometric 29·3 contracted to a fourth part of what it was on the 11th, and the whole driven backward toward the N.E. The southern depression had travelled to the N.W., tripled its area, and was one-tenth of an inch lower.

The storm in the beginning of December will be afterwards described under the head of the "Direction of Storms."

Form of Storm Areas.

The forms of forty-two different areas circumscribed by the isobarometric lines admitted of examination. Of these, thirty were either circular or slightly elliptical. In ten cases, the major axis of the ellipse was nearly double the length of the minor axis, and in one case it was three times the length. In two instances, 29·5 on the 11th November, and 29·3 on the 3d December, the outline of the areas was very irregular, owing to the occurrence of two central depressions in one case, and three in another, within it. It follows from this, that the storms most commonly assumed a circular or oval form, and that the ellipses were seldom much elongated.

The area over which the storms spread themselves was very variable in size, being seldom less than 600 miles across, but often two or three times that amount. This area was not constant, even as regarded the same storm from day to day, but varied in size, sometimes contracting and sometimes expanding. If it contracted, the central depression at the same time gave signs of filling up, and the storm of dying out. On the other hand, if the area widened, the central depression generally became deeper, and occasionally was broken up into two or more separate depressions, which appeared to become separate storms with the wind circling round each, as shown in the maps for 11th November and 3d and

17th December. These different depressions, however, soon reunited, and the storm proceeded as before.

Direction in which the Major Axis of the Storm Area lay.

The direction in which the major axis of the storm lay, could be determined on twenty-eight occasions. On seven occasions it pointed to the N.E.; on six to the E.; on five to the N.N.E.; on four to the S.E.; on three to the E.N.E.; and on three to the N. In most cases the major axis was coincident, or nearly coincident, with the direction in which the storm happened to be moving at the time. These two features of storms have important bearings on the prediction of storms, and on the direction and veering of the wind.

It has been sometimes affirmed of the European storms that they are constantly marked by a barometric depression stretching in a north and south direction over Europe; but the analyses of these storms given above show that this assertion, in these cases at least, receives no support from fact.

Direction in which the Storms advanced over Europe.

The direction in which the storms advanced from the position they occupied on one day to the position they occupied on the next day, could be ascertained in twenty-four cases. In eleven of these, the progressive movement was to the N.E.; in four to the E.; in four to the S.E.; in two to the E.S.E.; and one to the E.N.E., S.S.E., and S.W. Thus, twenty-two travelled towards some point in the quadrant from N.E. to S.E., and only one took a westerly direction. Hence, these storms travelled as often toward the N.E. as toward all other points of the compass put together, and almost every one toward some point between N.E. and S.E.

The storms seldom proceeded in the same uniform direction from day to day. Though generally the change was not great, yet occasionally it was so. Thus of the many interesting features which marked the storm of the beginning of December, none were more remarkable than the sudden changes of its progressive movement. I have added in the Appendix observations relative to this storm at shorter intervals of time than 24 hours. From these it appears that the centre of the storm on the 2d was near Liverpool at 9 A.M. (Plate XX.); Worcester at noon; Oxford at 3 P.M.; Cherbourg at 6 P.M.; and Oxford at 9 P.M. The greater number of observations at 9 P.M. show three depressions:—1*st*, at Shetland, 28·88; 2*d*, at Oxford, 28·89; and, 3*d*, in Holland, 29·22. It is very probable that the storm had separated into two parts near Liverpool, one of which took a north-easterly course toward Shetland, and the other a south-easterly course toward Cherbourg. At 9 A.M. of the following morning (Plate XX.), the first had advanced to the N.E. to Christiansund; the second had advanced northward to Shields; and the third had advanced eastward to Denmark. At 9 P.M. of the 3d, the first was no longer visible, having probably left Europe by the North Cape, while the second had advanced to the north of Holland. At 9 A.M. of the 4th (Plate XXI.), it had advanced to Copenhagen, had greatly diminished in area, and the central depression was five-tenths less than on the previous morning; and at 9 P.M., the observations at Christiania, Kiel, and Konigsberg show that the atmospheric equilibrium was restored, and the storm consequently had died out.

Most of the storms left Europe by the North Cape or the north-east of Russia; but two of them (Storms III. and X.) wasted away and died out before reaching Russia.

Rate at which the Storms travelled.

The distance between the points indicating the centre of the barometric depression, or the centre of the storm on two consecutive days, could be determined on twenty-one occasions, which are given in the following table:—

No. of Storm.	For Twenty-four Hours, ending 8 A.M.	Distance travelled by Storm in Eng. miles in one day.	Rate per Hour.
Storm I. . . .	Oct. 29	580	24
„ II. . . .	„ 30	600	24
„ II. . . .	„ 31	420	18
„ II. . . .	Nov. 1	400	17
„ II. . . .	„ 2	420	18
„ III. . . .	„ 2	400	17
„ III. . . .	„ 3	430	18
„ III. . . .	„ 4	510	21
„ IV. . . .	„ 5	700	29
„ IV. . . .	„ 6	470	20
„ IV. . . .	„ 7	360	15
„ IV. . . .	„ 8	470	20
„ V. . . .	„ 11	260	11
„ V. . . .	„ 12	390	16
„ VII. . . .	„ 22	460	19
„ VII. . . .	„ 23	560	23
„ IX. . . .	Dec. 2	500	21
„ IX. . . .	„ 3	390	16
„ IX. . . .	„ 4	425	18
„ XI. . . .	„ 17	485	20
„ XI. . . .	„ 18	460	19
Means, .	...	461	19

Hence, the mean distance the storms travelled each day was 460 miles, being at the rate of 19 miles an hour. The least distance was from the 10th to the 11th, being only 260 miles, or 11 miles an hour. It was on this occasion that Storm V. retrograded toward the S.W., and the distance given is in all likelihood too small, it being probable that it did not begin its retrograde motion till sometime *after* 8 A.M. of the 10th. Hence, 15 miles per hour may be accepted as the minimum rate per hour travelled by any of these storms. The greatest distance travelled on any day was from the 4th to the 5th, being 700 miles, or 29 miles per hour.

If Storm VI. really overtook Storm V. by the morning of the 12th, its progressive motion must have far exceeded any of those in the table, since it must have travelled, in twenty-four hours, at least 1050 miles, or 46 miles an hour. The following observations, in addition to those given in the table, bear on this interesting point:—

TABLE SHOWING THE BAROMETRIC PRESSURE AND DIRECTION OF THE WIND AT TWENTY-FOUR PLACES, AT 8–9 A.M., 2 P.M., AND 8–9 P.M. OF THE 11TH NOV. 1863.

11th November 1863.		E. Yell.	Sandwick.	Stornoway.	Portree.	Dunrobin.	New Pitsligo.	Greenock.	South Cairn.	Drumlanrig.	Barry.	Milne-Graden.	Liverpool.
8-9 A.M.	Barometer,	29·09	29·14	29·29	29·14	29·23	29·22	29·36	29·46	29·34	29·30	29·39	29·38
	Wind, . . .	WNW	W	W	W	W	W	NW	E	W	Calm	...	S by W
2 P.M.	Barometer,	...	...	...	...	...	...	...	...	...	...	...	29·44
	Wind, . .	...	...	...	...	...	...	...	...	...	...	...	NW
8-9 P.M.	Barometer,	29·30	29·40	29·53	29·47	29·38	29·36	29·53	29·55	29·52	29·46	29·51	29·59
	Wind, . .	NWW	N	W	W	NW	W	NW	N	N	Calm	W	WNW

11th November 1863.		Nottingham.	Oxford.	Greenwich.	Flushing.	Helder.	Groningen.	Utrecht.	Luxemburg.	Pederborn.	Kiel.	Christiania.	Königsberg.
8-9 A.M.	Barometer,	29·45	29·38	29·39	29·50	29·41	29·43	29·47	29·68	29·53	29·45	29·35	29·67
	Wind, . .	...	NNE	E	SW	W	S	SW	NW	SE	SW	SSW	SE
2 P.M.	Barometer,	...	29·39	29·38	29·42	29·43	29·43	29·39	...	...	...	...	...
	Wind, . .	...	N by W	NE	S	SW	S	SW	...	...	...	...	...
8-9 P.M.	Barometer,	29·61	29·60	29·56	29·48	29·46	29·47	29·46	29·43	...	...	29·24	29·59
	Wind, . .	NNW	WNW	W	S	SE	S	S	SW	SE	SW	SE	S

At 12 noon, the pressure at Paris was 29·39, and wind S.S.E.; and at Luxemburg 29·52, and wind S.W. From these observations, it is probable that this storm continued to advance southwards for an hour or two after the morning observation; it then turned to the N.E., and in the evening advanced over the North Sea nearly to the south of Norway, as shown by the pressure and direction of the winds at that time, and soon after became absorbed in Storm V.

As the position of the centres of Storms VIII. and X. could be ascertained only on one day, the rate of their motion cannot be determined.

Since storms generally travel to the N.E. at an average speed of about twenty miles an hour, and since the distance of the S.W. of Ireland from any British port does not exceed 500 miles, it follows that these storms might have been predicted at least twenty-four hours before their occurrence at the eastern seaports of Great Britain; and as their approach could have been foreseen some hours before they burst upon the west of Ireland, they might have been predicted from thirty-six to forty-eight hours beforehand.

Comparison of the Barometric and Thermometric Lines.

The observations of the thermometer do not equal in importance those of the barometer, for this among other reasons, that while the barometer measures the weight of the whole atmosphere pressing on it, the thermometer gives only the temperature of that portion of the air which is in immediate contact with the earth. There appears to be little apparent connection between these lines at first sight; for while the barometric lines approach more or less closely the curves of the circle or the ellipse, the lines of equal thermometric disturbance present the greatest possible irregularity of form. When, however, the attention is confined to the region of greatest barometric disturbance, a remarkable connection is at once observed. It will be seen that in all cases the temperature rose a few degrees over the space toward which and over which the front part of the storm was advancing, and fell at those places over which the front part of the storm had already passed. In other words, the temperature rose as the barometer fell, and fell as the barometer rose. Generally, the temperature in advance of the storm was above the average, and in the rear of the storm below it. But if it was considerably above the average in advance of the storm, it was still above the average when the storm had passed, though lower than it was before.

In one or two cases the temperature, after falling a little, rose in what appeared to be the wake of a storm; but in these cases the observations of the following day showed that another storm was advancing close upon the one already past. The high temperature thus indicated the approach of the second storm, and properly belonged to it.

Observations of Rain and Cloud.

We learn from the observations that as long as the barometer did not fall below the mean, there was no continuous rain anywhere, but blue sky prevailed, varied with partially clouded sky or with fog. But when the barometer fell, the sky began to be obscured, and rain to fall at intervals; and as the central depression advanced, the rain became more general, heavy, and continuous. After the centre of the storm had passed, or when the barometer had begun to rise, the rain generally became less heavy, falling more in showers than continuously; the clouds began to break up, and fine weather, ushered in with cold breezes, ultimately prevailed.

In order to show where the greatest amount of rain fell, Table IV. has been prepared, giving the rainfall at those places over which, or near which, the storms passed. It is necessary to explain that, as far as known, the rain given in the table fell during the twenty-four hours preceding the date of each entry. Thus, as the centre of Storm II. was west of Ireland on the 1st November, and in the centre of England on the 2d, the rainfall of the 2d took place as the storm travelled between these places. The rainfall at the Irish, English, and French stations, over which the front part of the storm had passed, was excessive—two inches having fallen at Brest; one inch at Liverpool and Dover; and about three-quarters of an inch at L'Orient, Galway, &c. On the same day the rainfall in Scotland was everywhere small, none falling at many places, and the largest fall being about one-sixth of an inch at Portree, in Skye. In Scotland, where the fall of rain was small, the wind was feeble, in no case blowing a gale; and the barometer, though low, had varied little during the twenty-four hours, and a pressure almost equally low prevailed for a considerable distance round. On the other hand, where the rainfall was in excess, violent gales prevailed, the fluctuations of the barometer had been great, and the isobarometric lines were much crowded together in the vicinity. On the following day the storm had passed eastward to Denmark. The rain over the west of England and of France had diminished, but increased over the *east* of England, and in Belgium and the Netherlands, over which the storm had travelled on its way to Denmark. The Irish rainfall was small, but not so small as would have been, but for the advance of Storm III., whose rainfall swelled the amount; and the same cause increased the Scottish rainfall.

All the other storms showed similar relations to the rainfall. The amount precipitated was greatest during the time the front part of the storm passed any place, and appeared to be in a great measure proportioned to the atmospheric disturbance experienced during the twenty-four hours, and the violence of the wind occasioned by that disturbance. In the wake of storms, though the atmospheric disturbance was equally great, and the violence of the wind as great, or even greater, the rainfall was very much less, except when the advance of another storm increased the amount.

Observations of the Wind.

Every one of the storms on each day presented the winds under the same conditions, viz., whirling round the area of low barometer in a circular manner, in a direction contrary to the motion of the hands of a watch, with a constant tendency to turn inwards towards the centre of lowest barometer. The wind in storms neither blows round the centre of least pressure in circles (or as tangents to the concentric barometric curves), nor does it blow directly towards that centre. It takes a direction nearly intermediate, approaching, however, more nearly the direction and course of the circular curves than of the radii to the centre. To this general rule none of the eleven storms any day offered an exception. When the centre of the storm was in a situation where observations were made all round it, the following were the general directions as observed:—

At places S. of the centre of least pressure, the wind was generally S.W.
„ S.E. „ „ „ S.
„ E. „ „ „ S.E.
„ N.E. „ „ „ E.
„ N. „ „ „ N.E.
„ N.W. „ „ „ N.
„ W. „ „ „ N.W.
„ S.W. „ „ „ W.

The greater the force of the wind at any place, the more nearly did it approximate to the directions here indicated. On those occasions when no observations were obtained from one or more of the sides of the storm, such observations as were obtained followed the same rule. Hence the atmosphere on every occasion rotated round the centre of the storm; and it should be kept in mind that this is no theoretical statement, but the result of observations faithfully put down on maps.

It will follow from this, that as the storms advanced to the eastward the general veering of the wind at places lying north of the central path of the storm would be from the N.E. by N. to W.; and at places to the south of the centre, from the N.E. by E. and S. to N.W.

On referring to the chart of the 2d November (Plate XVI.), it will be seen that the violence of the wind was greatest in the north of France, and south of England and Ireland, where there were great differences in the pressure, as shown by the crowding together of the isobarometric lines. On the other hand, it will be seen that the wind was nowhere blowing a gale in North Britain, where the pressure varied little for a great distance all round, as shown by the distance between the isobarometric lines, even though the pressure there was absolutely low. Again, on the 11th November (Plate XVIII.), the isobarometric lines were far apart in Storm V. in the north, and the wind was nowhere strong within that disturbed area; whereas the lines were much crowded in Storm VI. round Plymouth, and the wind was blowing strongly all round. This blowing of the wind from a high to a low barometer, and with a force generally proportioned to the differences of the pressure, would appear from these storms to be the most important law concerned in regulating the movement of the wind. As the wind approached the centre of least pressure, its violence gradually abated, till, on reaching the centre, a lull or calm prevailed.

Calms and light winds also prevailed along the ridge of highest barometer, or the region where the pressure was greatest, and on receding from which, on each side, the pressure diminished. It may not unaptly be compared to the watershed in physical geography, since from it the winds flowed away towards those places where the pressure was less. It sometimes extended over the Continent from N. to S., sometimes from E. to W., and sometimes in other directions; frequently it curved through Europe in a very irregular manner, forming the boundary line between a disturbed area in the north and another in the south; occasionally it was broken up into different parts; and more rarely it was concentrated in one locality, forming an area of high barometer approaching a circular form. In this last case, which happened on the 5th December in western Europe (Plate XXI.), and on several other occasions, the wind was always observed gently *whirling out of the area of high barometer*, in the direction of the motion of the hands of a watch—being the opposite direction to that assumed by the wind when it blows round and in towards an area of low pressure.

Storms of the Mediterranean.

The observations from Austria, Turkey, Greece, Russia, and Syria are too scanty to enable us to trace satisfactorily any of the storms which occurred there during the period. There is enough, however, to show that the conclusions which may be drawn from those storms which passed over northern and western Europe cannot safely be applied to the storms of the Mediterranean, as regards their form, the direction from which they come, and the course generally pursued by them.

App.

APPENDIX.

Additional observations showing the progress of Storm IX. from the 2d to the 4th December 1863, referred to at p. 197.

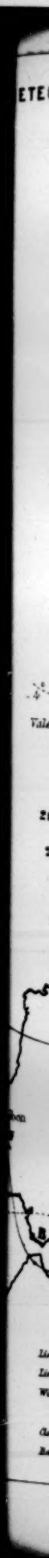
TABLE I.—SHOWING COMPARATIVE READINGS OF THE BAROMETER AT OXFORD AND GREENWICH, FROM DECEMBER 1^{d} 22^{h} TO DECEMBER 3^{d} 22^{h} 1863. (*From Proceedings of the British Meteorological Society, Vol. II. pp.* 48–59.)

1863. Month, Day, and Hour.	Barometric Readings.		1863. Month, Day, and Hour.	Barometric Readings.		1863. Month, Day, and Hour.	Barometric Readings.		1863. Month, Day, and Hour.	Barometric Readings.	
	Oxford.	Greenwich.		Oxford.	Greenwich.		Oxford.	Greenwich.		Oxford.	Greenwich.
d. h.	in.	in.	d. h.	in.	in.	d. h.	in.	in.	d. h.	in.	in.
Dec. 1 22	28·77	28·82	Dec. 2 10	29·31	29·38	Dec. 2 21	28·89	28·90	Dec. 3 10	29·76	29·73
23	78	81	11	30	38	22	97	97	11	84	80
Dec. 2 0	81	84	11½	28	37	23	29·04	29·03	12	89	84
1	84	84	12	27	36	Dec. 3 0	09	09	13	94	89
2	95	87	13	24	35	1	13	14	14	98	92
3	29·04	95	14	19	32	2	17	18	15	30·01	96
4	10	29·06	15	04	28	4	23	23	16	04	30·01
5	18	14	16	28·93	14	3	29	27	17	07	05
6	24	23	17	80	06	5	37	33	18	11	09
7	26	27	18	73	28·92	6	44	40	19	12	13
8	28	30	18¾	68	85	7	53	48	20	14	15
9	31	34	19	74	83	8	62	57	21	16	18
Dec. 2 9½	31	38	Dec. 2 20	80	79	Dec. 3 9	70	66	Dec. 3 22	18	20

Dec. 1^{d} 22^{h} minimum reading at Oxford; 1^{d} 23^{h} minimum reading at Greenwich.
2^{d} 8^{h} 40^{m} to 10^{h} 40^{m} stationary at maximum value at Oxford.
2 9 30 to 11 20 stationary at maximum value at Greenwich.
2 18 30 to 18 45 minimum value at Oxford.
2 19 30 to 20 12 minimum value at Greenwich.

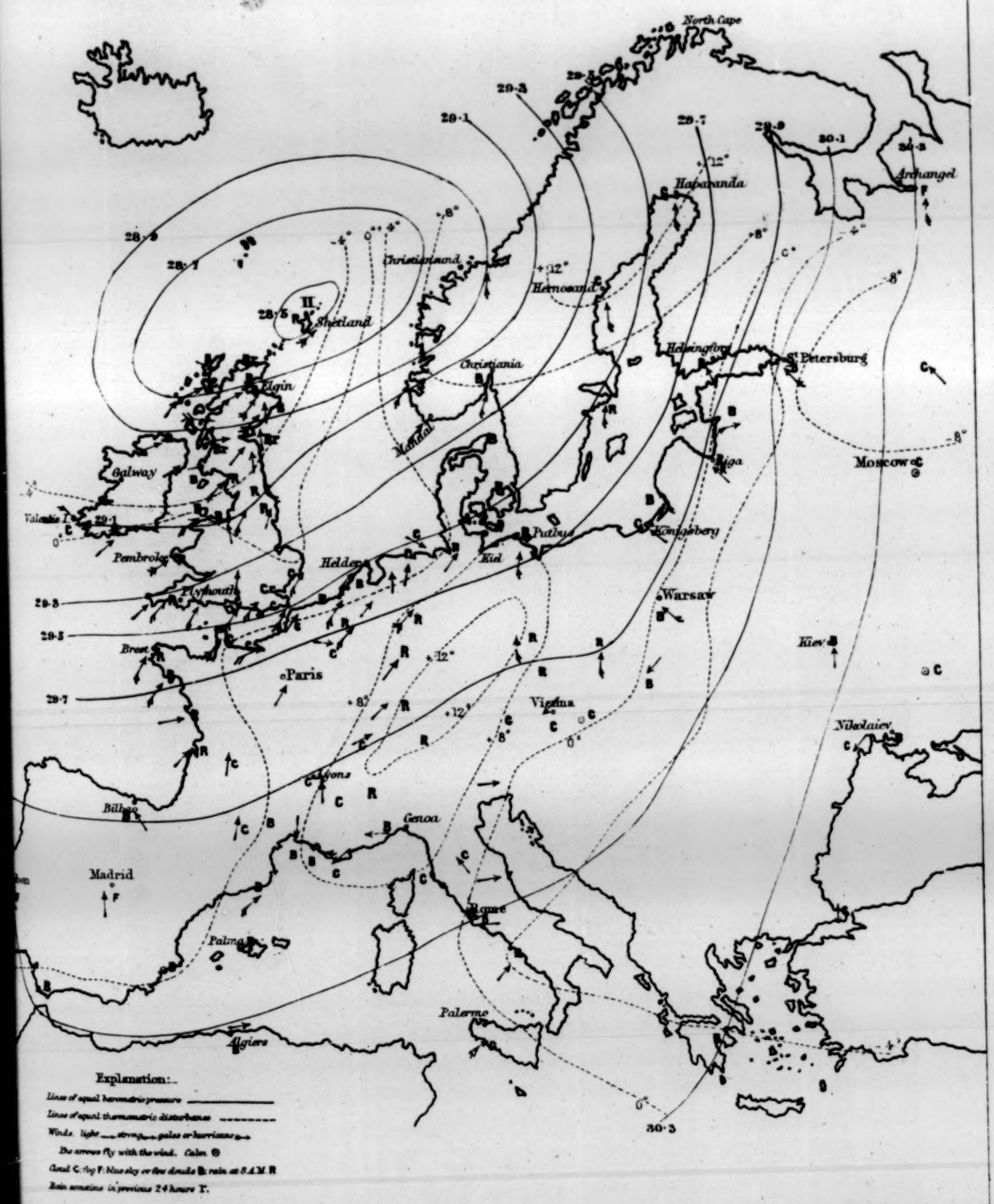
TABLE II.—OF BAROMETRIC OBSERVATIONS IN DIFFERENT PARTS OF EUROPE FROM 9 A.M. OF THE 2D TO 9 P.M. OF THE 4TH DECEMBER 1863.

	2d December 1863.						3d December 1863.						4th December 1863.				
	9 A.M.	Noon.	3 P.M.	6 P.M.	9 P.M.	Mid-night.	9 A.M.	Noon.	3 P.M.	6 P.M.	9 P.M.	Mid-night.	9 A.M.	Noon.	3 P.M.	6 P.M.	9 P.M.
East Yell, . . .	29·84	...	...	...	28·88	...	28·97	...	...	...	29·23	...	29·62	...	...	...	29·40
Stornoway, . . .	08	...	...	...	29·08	...	99	...	...	...	28	...	66	...	...	...	48
New Pitsligo, . .	17	...	...	...	05	...	94	...	...	...	52	...	82	...	...	...	64
Barry, ...	18	...	...	...	17	...	90	...	...	...	59	...	95	...	...	...	79
Cargen, ...	09	...	...	...	26	...	76	...	...	...	70	...	30·08	...	...	...	87
Liverpool, . . .	04	29·05	29·24	29·34	40	29·37	91	29·10	29·30	29·58	84	30·02	24	30·26	30·24	30·20	30·15
Nottingham, . .	29·00	28·94	10	41	...	41	83	00	20	48	...	29·90	25	30	25	22	...
Belvoir Castle, . .	...	...	06	...	...	...	...	...	12	...	...	...	...	...	20	...	...
Oxford, ...	31	29·27	04	28·73	28·89	09	29·70	89	30·01	30·11	30·16	...	...	...	...	...	...
Greenwich, . . .	34	36	28	92	90	09	66	84	29·96	09	18	...	...	...	...	...	...
Clifton, ...	28·97	...	43	...	...	...	25	...	63	...	...	...	38	...	45	...	...
Paris, ...	29·30	22	29	29·53	29·74	82	45	45	65	29·83	29·96	30·10	30·44	48	49	52	52
Luxemburg, . .	80	65	...	...	58	...	55	54	...	...	91	...	32	36	...	...	55
Flushing, . . .	34	...	13	...	33	...	25	...	25	...	42	...	26	...	40	...	38
Utrecht, ...	43	...	20	...	25	...	24	...	10	...	34	...	13	...	22	...	26
Leeuwarden, . .	54	...	26	...	24	...	34	...	28·85	...	28·77	...	29·96	...	15	...	20
Helvoetsluis, . .	44	...	13	...	23	...	28	...	29·09	...	...	...	30·22	...	34	...	34
Paderborn, . . .	77	...	...	...	33	...	54	...	...	...	29·15	...	29·73	...	...	...	30
Kiel, ...	80	...	...	...	37	...	30	...	...	...	17	...	18	...	...	...	10
Christiania, . . .	88	...	...	...	54	...	37	...	...	...	15	...	53	...	...	...	29·88
Konigsberg, . . .	30·20	...	...	...	96	...	85	...	...	...	65	...	71	...	...	...	30·13
Breslau, ...	08	...	...	...	74	...	83	...	...	...	55	...	69	...	...	...	09
Kremsmunster, .	10	...	...	...	82	...	30·03	...	...	...	84	...	91	...	...	...	39


ETE

ETEOROLOGICAL CHART OF EUROPE

For 8 A.M. October 30th 1863



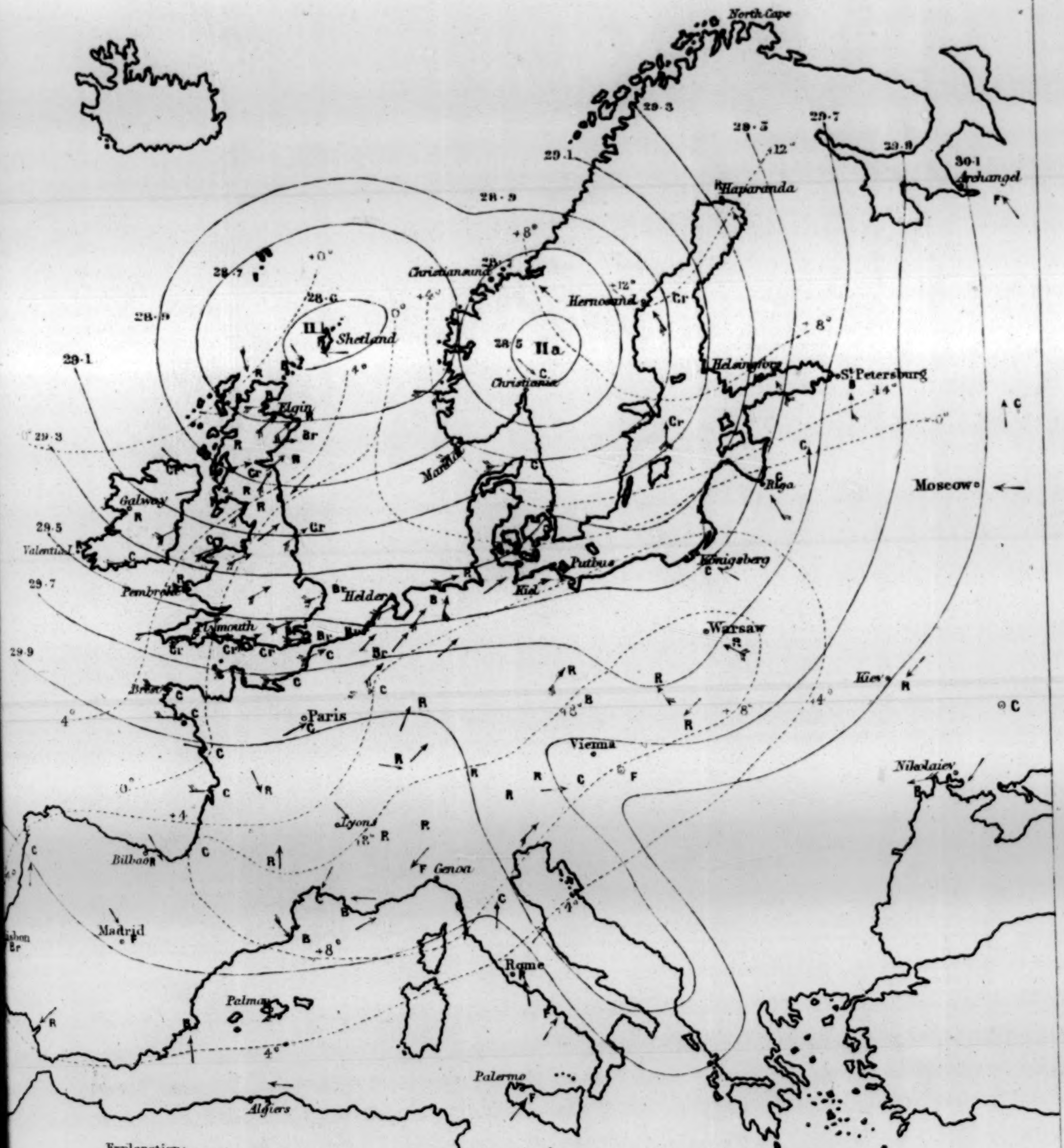
W. & A.K. Johnston, Edin.

29.9

PLATE XIV. Royal Soc. trans. Edin. Vol. XXIV.

ETEOROLOGICAL CHART OF EUROPE

For 8 A.M. October 31st 1863



Explanation:–

Lines of equal barometric pressure ————

Lines of equal thermometric disturbance - - - - - - -

Winds. light → *strong* → *gales or hurricane* →

The arrows fly with the wind. Calm. ⊙

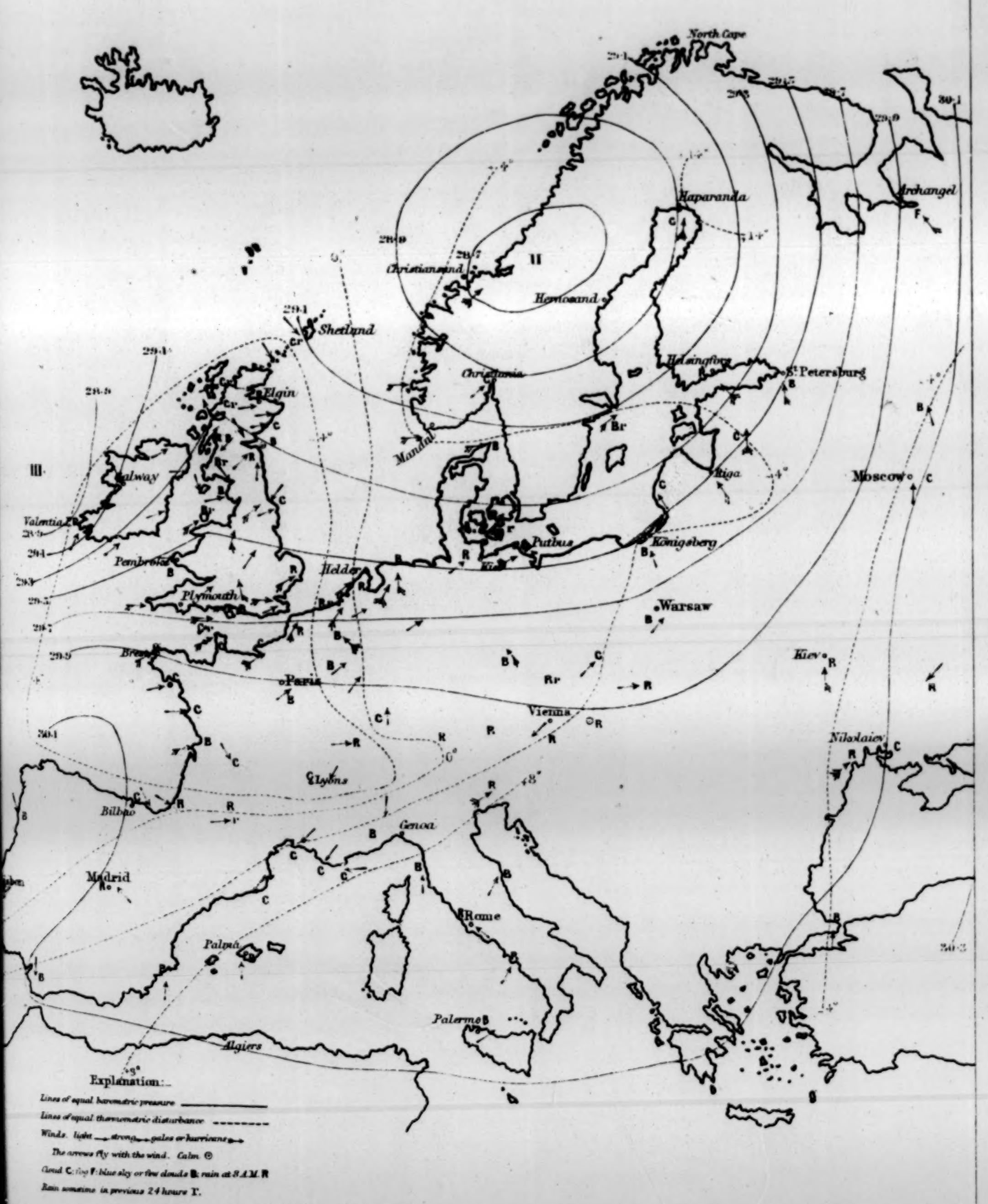
Cloud C. *fog* F. *blue sky or few clouds* B. *rain at 8 A.M.* R

Rain sometime in previous 24 hours Ṙ.

MET

METEOROLOGICAL CHART OF EUROPE

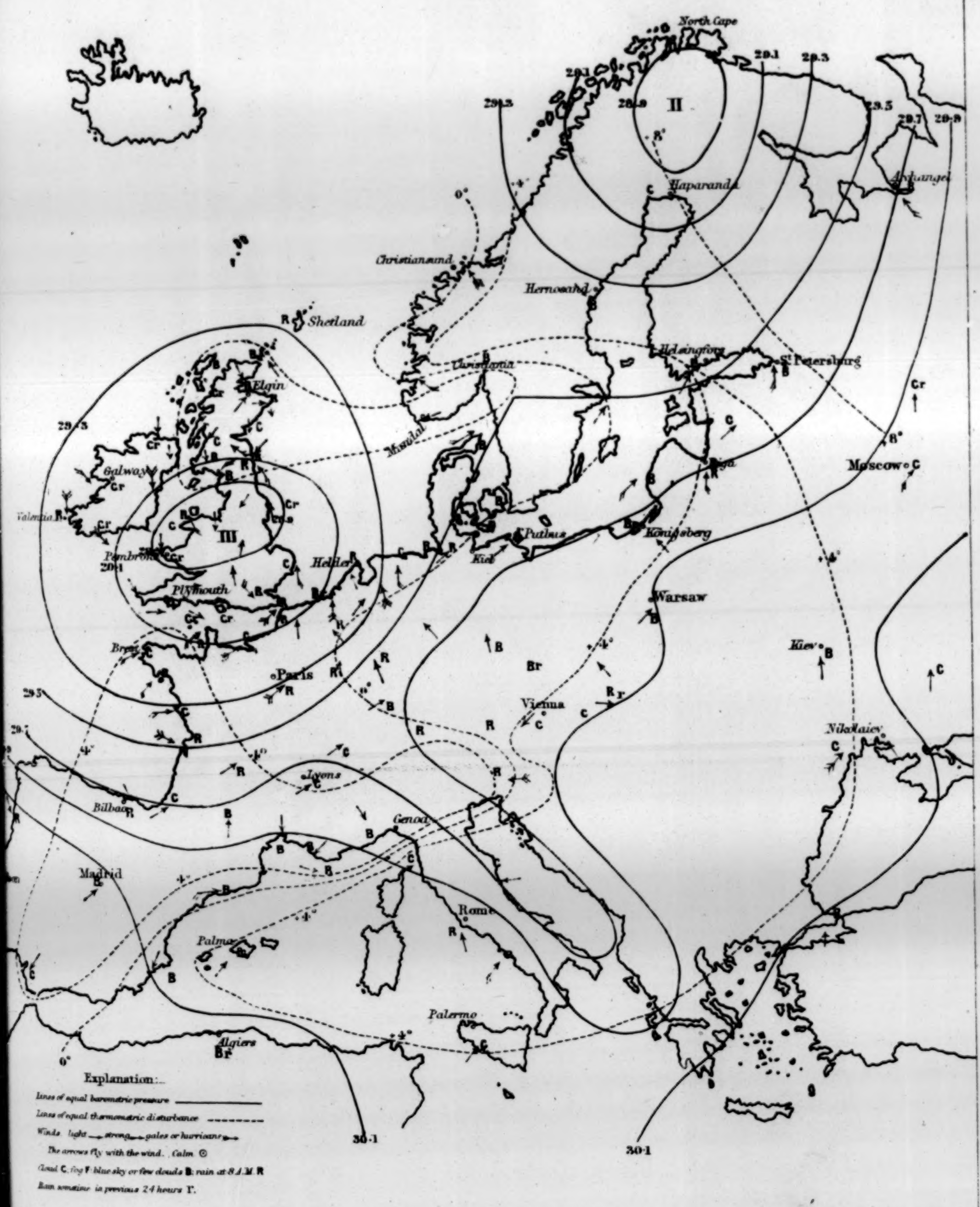
For 8 A.M. November 1st 1863



Vale
29.3
29.7

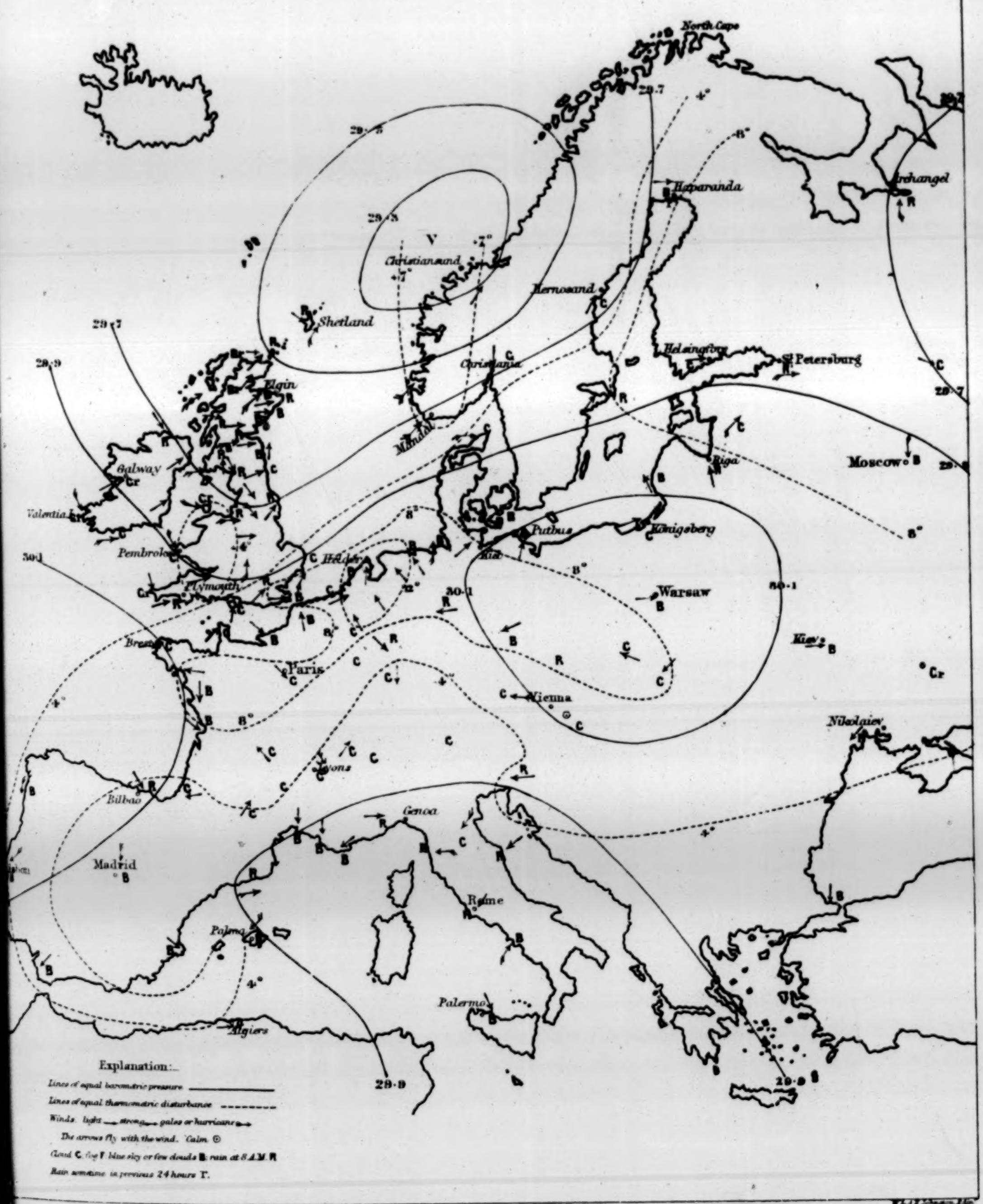
ETEOROLOGICAL CHART OF EUROPE

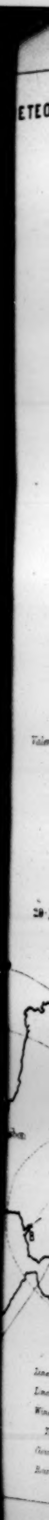
For 8 A.M. November 2nd 1863



ETEOROLOGICAL CHART OF EUROPE

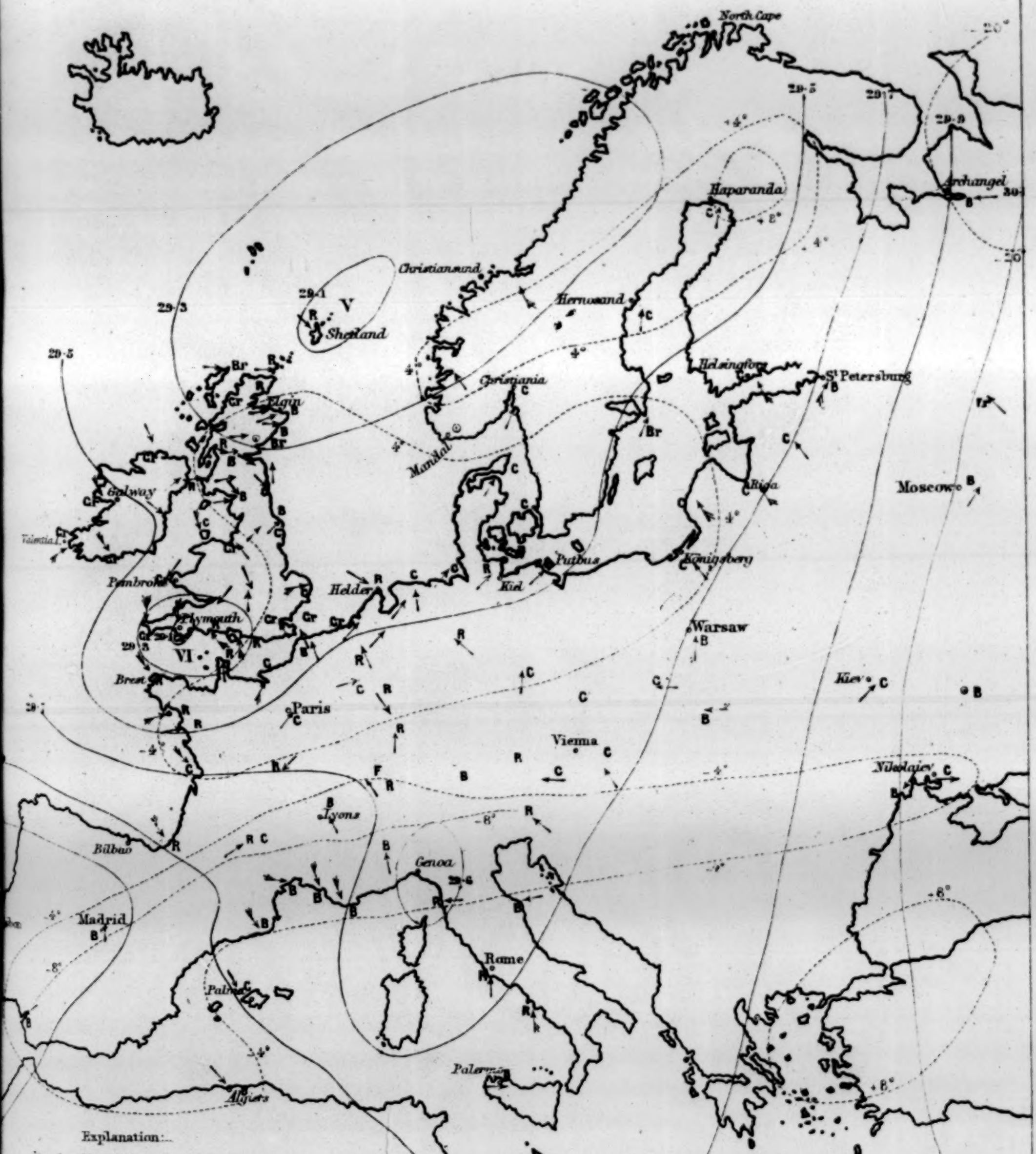
For 8 A.M. November 10th 1863




ETEO

ETEOROLOGICAL CHART OF EUROPE

For 8 A.M. November 11th 1863



Explanation:—

Lines of equal barometric pressure ————

Lines of equal thermometric disturbance - - - - - - -

Winds, light → strong → gales or hurricanes →

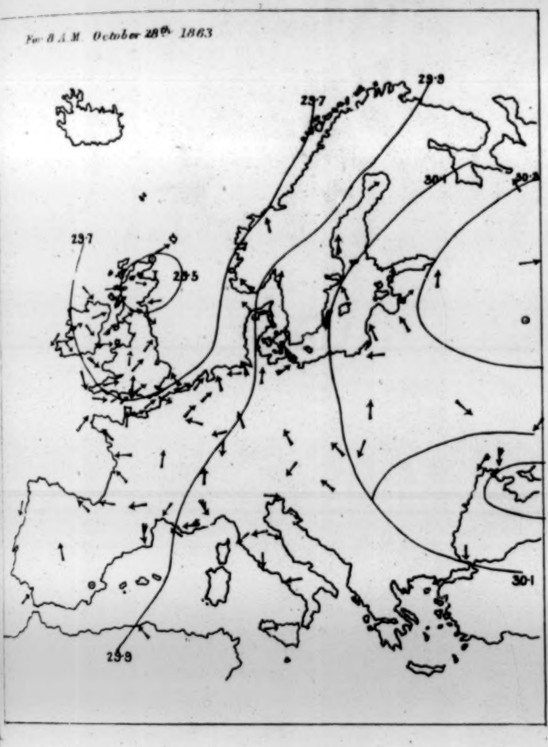
The arrows fly with the wind. Calm ⊙

Cloud C, fog F, blue sky or few clouds B, rain at 8 A.M. R

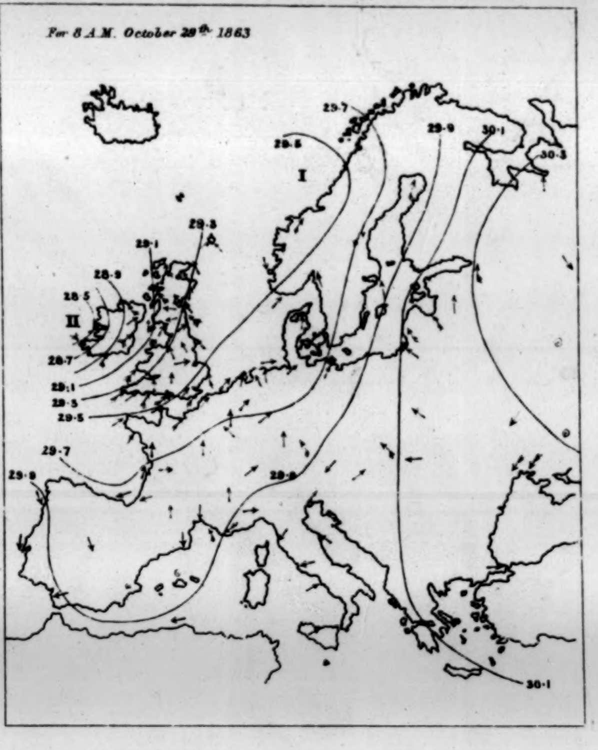
Rain sometime in previous 24 hours r.

METEOROLOGICAL CHART OF EUROPE

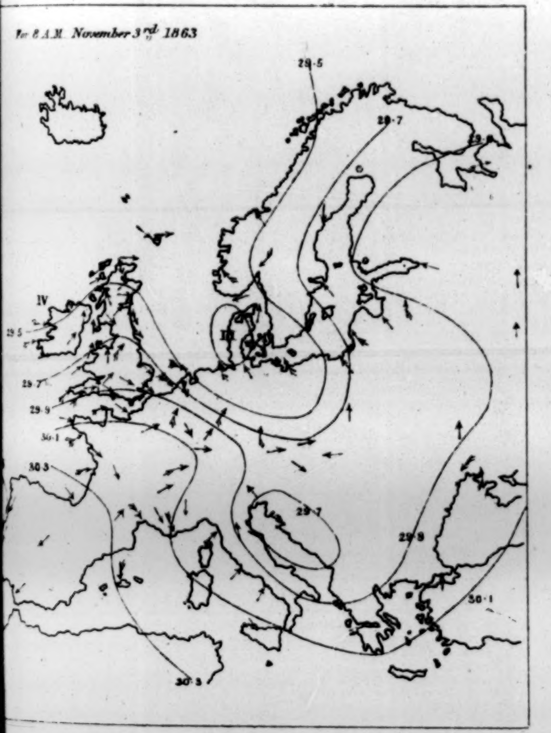
PLATE XIX Royal Soc. trans. Edin. Vol. XXIV.



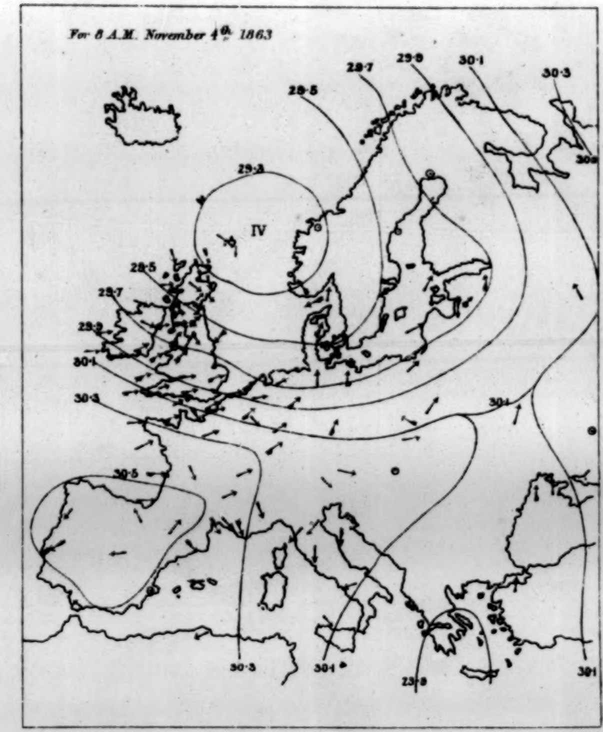
For 8 A.M. October 28th 1863



For 8 A.M. October 29th 1863



For 8 A.M. November 3rd 1863



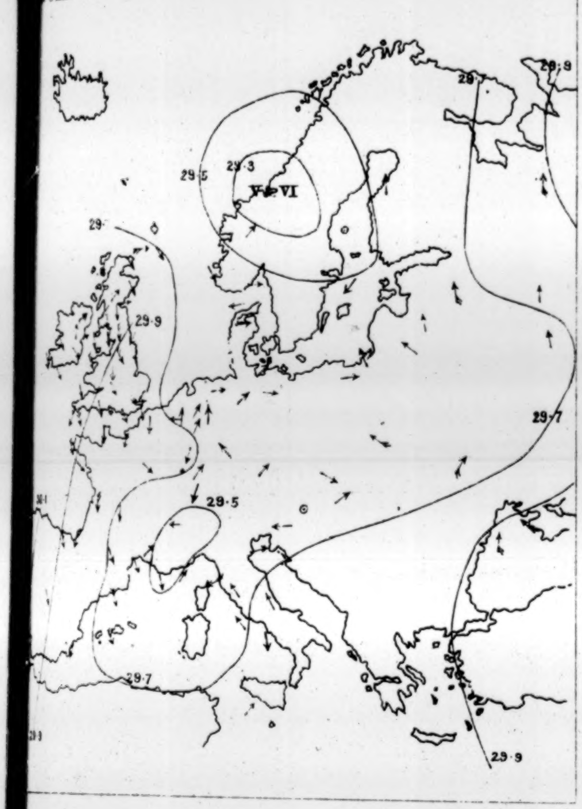
For 8 A.M. November 4th 1863

W. & A. K. Johnston, Edin^r

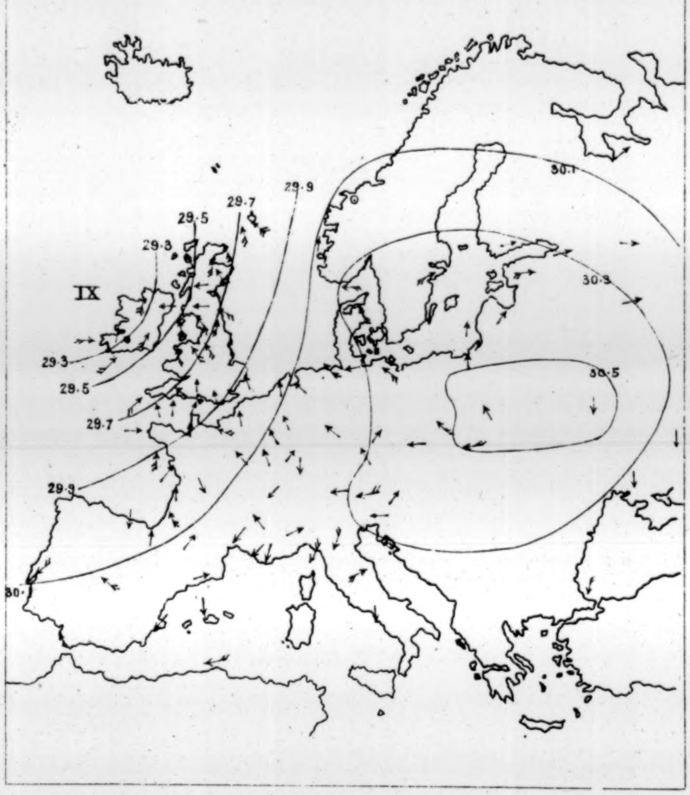
Explanation: *Lines of equal barometric pressure* ——————— *Winds. light* → *strong* → *gales or hurricanes* → *The arrows fly with the wind. Calm.* ⊙

METEOROLOGICAL CHART OF EUROPE

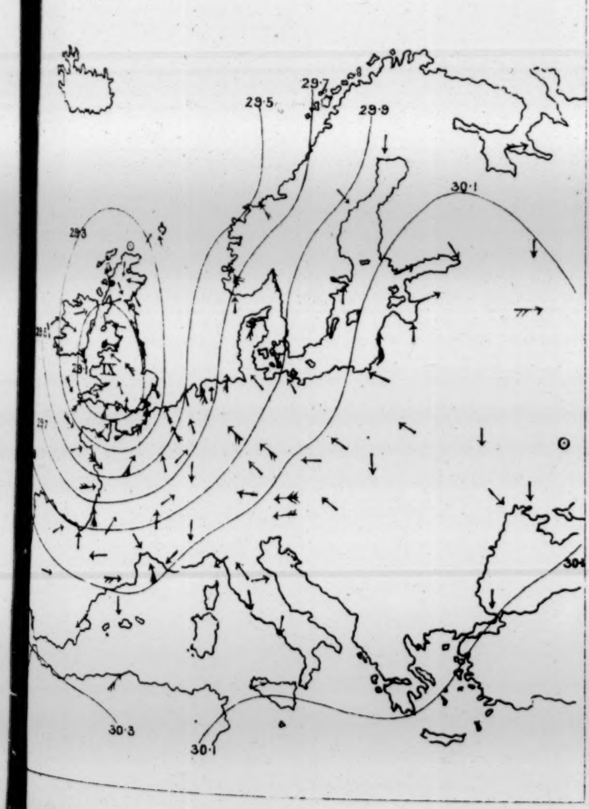
r 8 A.M. November 12th 1863



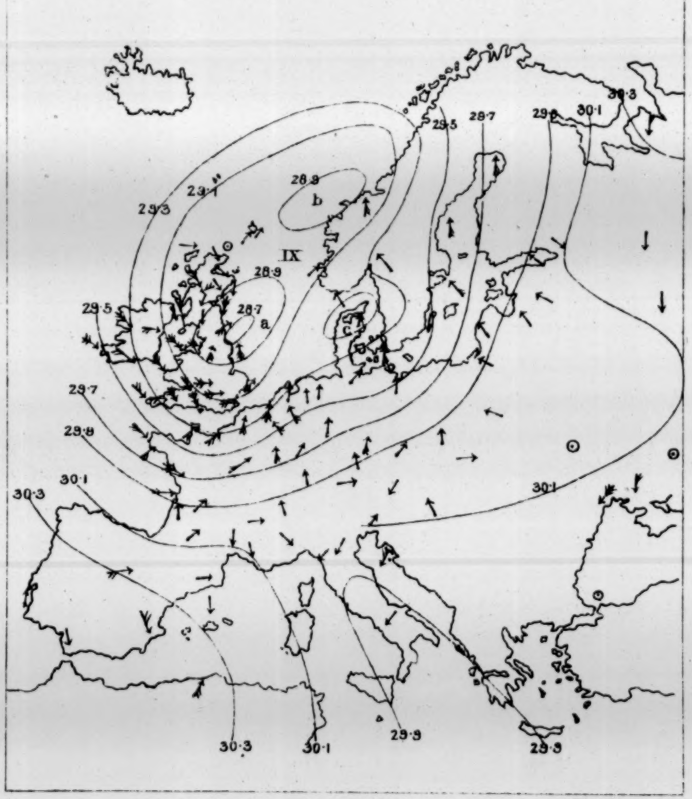
For 8 A.M. December 1st 1863



r 8 A.M. December 2nd 1863



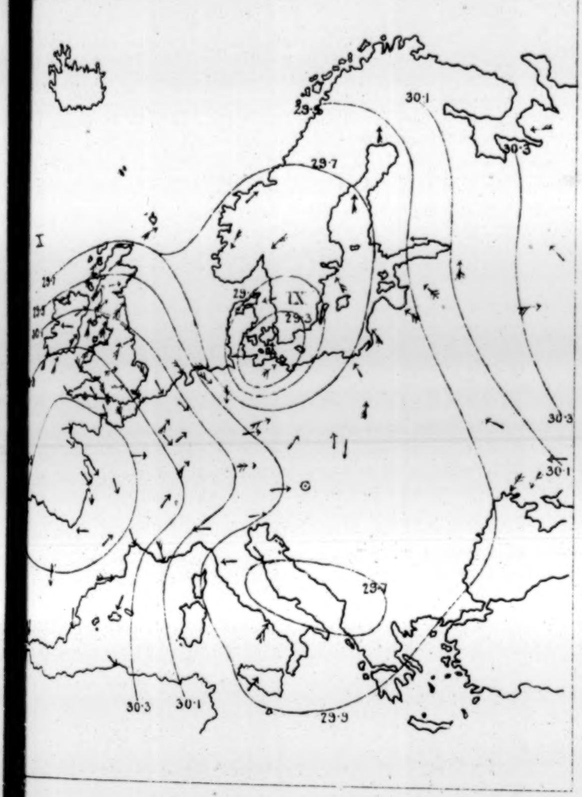
For 8 A.M. December 3rd 1863



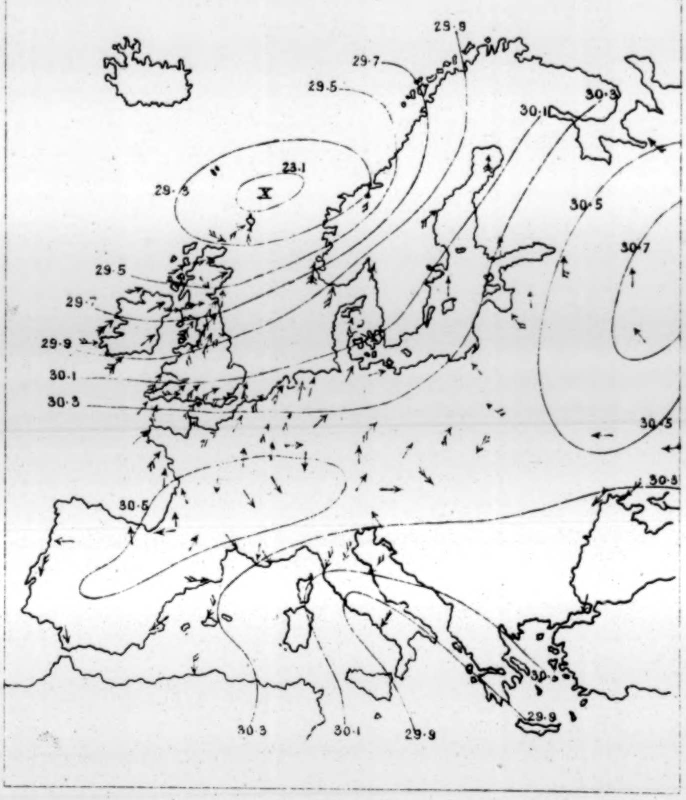
W. & A. K. Johnston, Edin.

METEOROLOGICAL CHART OF EUROPE

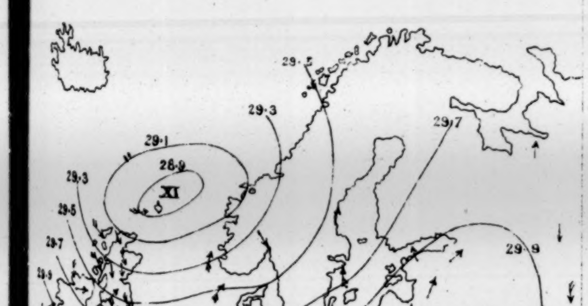
For 8 A.M. December 4th 1863



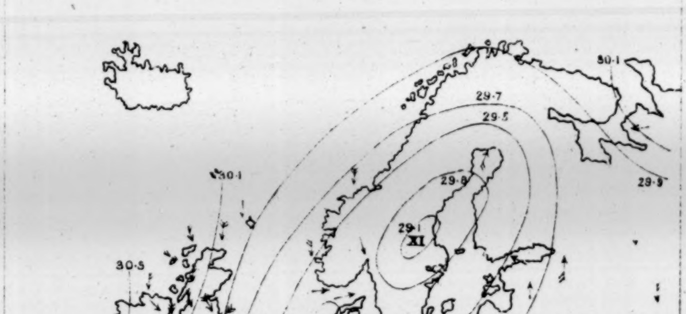
For 8 A.M. December 5th 1863.



For 8 A.M. December 16th 1863



For 8 A.M. December 17th 1863



W. & A.K. Johnston, Edin.

TABLE I.—Showing the BAROMETRIC PRESSURE, in English inches, reduced to 32° and Sea Level, at 130 places in Europe, from 26th October to 12th November, from 20th to 26th November, from 30th to 5th December, and from 14th to 18th December, at 8 A.M.

	LAT. N.	LONG.	OCTOBER.						NOVEMBER.																				DECEMBER.						
			26.	27.	28.	29.	30.	31.	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	20.	21.	22.	23.	24.	25.	26.	30.	1.	2.	3.	4.	5.	14.	15.
IRELAND.																																			
Greencastle,	55·13	6·57 W	30·11	29·94	29·58	28·96	28·93	28·95	29·24	29·19	29·48	29·60	30·10	30·52	30·13	29·87	30·41	29·72	29·39	29·96	29·77	29·18	29·14	29·65	29·74	29·48	29·91	30·06	29·28	29·23	...	30·05	29·57	30·40	30·22
Belfast,	54·37	5·56 W	03	88	52	79	92	89	33	06	47	58	05	45	06	82	33	67	34	90	78	38	40	59	66	51	94	29·99	29	10	28·97	09	54	34	18
Dublin,	53·21	6·16 W	12	96	63	90	29·02	29·21	50	16	50	78	14	50	21	30·00	39	80	51	98	94	45	66	74	78	66	30·02	30·06	42	14	29·30	20	76	47	33
Galway,	53·17	9·3 W	11	97	72	56	06	23	20	25	49	78	14	49	24	08	41	91	54	30·04	86	16	55	63	73	48	29·88	29·95	24	31	39	20	76	44	29
Valencia,	51·54	10·25 W	05	87	76	96	09	42	28·98	29	61	95	10	39	34	15	39	30·03	63	06	75	24	59	67	68	52	86	83	29	44	59	30	90	43	31
Queenstown,	51·51	8·17 W	10	97	74	90	28·88	46	29·38	12	53	97	13	47	18	29·87	41	03	63	29·92	77	16	48	61	62	47	86	86	23	25	37	23	80	37	26
SCOTLAND.																																			
East Yell,	60·33	1·3 W	02	91	55	29·37	44	28·60	08	32	40	24	29·81	14	13	30·15	31	29·43	09	45	77	47	14	56	68	76	30·05	30·17	77	34	97	29·62	17	29·90	29·85
Bressay,	60·10	1·10 W	12	88	65	47	56	66	18	38	47	29	82	24	22	19	41	53	21	91	95	46	18	66	76	84	12	24	80	38	04	71	26	30·00	94
Sandwick,	59·3	3·18 W	04	82	51	24	66	59	21	23	41	28	77	28	19	14	31	47	14	84	76	39	05	54	66	59	29·98	06	56	27	28·98	67	25	05	91
Stornoway,	58·11	6·22 W	02	82	52	12	64	63	23	34	39	36	88	34	22	04	09	69	29	89	68	32	22	50	68	47	81	02	52	08	99	66	38	16	98
Dunrobin,	57·58	3·57 W	02	84	51	31	76	63	25	27	45	36	87	31	16	09	38	55	23	86	77	45	07	54	69	60	88	29·96	51	19	98	78	37	07	98
Portree,	57·25	6·12 W	02	83	48	01	66	70	23	20	40	43	92	39	12	29·91	26	54	24	88	65	31	18	53	68	37	80	97	62	14	96	81	33	22	99
Elgin,	57·39	3·19 W	03	84	46	21	81	68	23	20	44	32	87	34	18	94	31	53	23	83	77	42	09	55	68	61	97	30·08	53	21	96	78	35	13	30·00
New Pitsligo,	57·36	2·12 W	04	86	41	20	82	68	23	20	44	35	88	34	11	97	29	53	22	81	86	48	07	58	70	69	30·04	22	62	17	94	82	40	12	01
Barry,	56·31	2·45 W	09	92	44	28	93	77	22	14	55	41	30·10	49	21	94	39	62	30	91	96	55	23	64	73	73	13	20	68	18	91	95	48	17	14
Aberdeen,	57·8	2·5 W	06	90	41	22	89	75	...	...	54	39	29.97	42	17	...	...	...	26	93	88	59	10	60	72	75	09	...	57	17	95	90	40	17	06
Trinity Gask,	56·22	3·41 W	13	98	58	38	29·04	89	38	31	62	50	06	51	26	97	41	70	40	95	99	58	29	68	75	77	09	20	67	30	29·07	99	58	27	17
Oban,	56·24	5·27 W	04	88	53	14	28·87	79	29	18	51	46	29·99	44	18	85	34	67	31	89	83	46	24	59	60	53	29·93	10	45	16	28·98	97	48	26	16
Baillieston,	55·52	4·6 W	11	96	52	23	29·00	90	37	19	60	63	30·10	54	33	94	44	70	37	94	96	57	36	71	76	71	30·11	20	60	24	95	30·05	62	25	22
Inveresk,	55·56	3·3 W	04	90	59	29	00	83	34	24	54	51	04	50	24	84	37	58	34	91	92	56	28	64	73	75	08	02	59	18	94	02	47	27	13
Thurston,	55·57	2·27 W	04	93	49	24	05	85	34	04	58	54	15	45	20	85	35	63	35	88	89	63	28	64	73	80	13	13	65	15	85	29·96	55	25	13
Milne-Graden,	55·42	2·12 W	10	95	51	36	06	85	35	12	62	51	10	53	27	92	40	62	39	92	30·06	63	36	72	75	80	12	21	76	23	93	30·03	60	24	14
Drumlanrig,	55·17	3·48 W	05	93	50	24	00	87	34	05	58	60	07	48	20	82	37	67	34	89	01	58	29	70	73	70	09	14	58	12	79	10	67	33	22
Cargen,	55·2	3·43 W	03	90	43	10	06	84	30	28·97	52	61	03	44	12	76	19	63	31	89	29·91	63	34	64	67	67	08	10	53	09	76	08	63	29	18
South Cairn,	54·58	5·8 W	08	30·10	54	20	08	96	45	29·15	64	67	15	50	20	91	47	70	46	89	96	53	48	71	80	67	10	13	50	24	29·11	12	71	38	29
ENGLAND.																																			
Silloth,	54·52	3·23 W	07	29·94	49	20	02	91	37	28·97	58	63	10	52	17	81	40	68	36	90	98	59	43	68	73	72	07	13	57	07	28·77	14	68	39	35
Shields,	55·0	1·27 W	10	94	54	36	13	91	44	94	68	...	12	53	23	...	36	67	34	84	30·05	86	55	70	79	87	24	21	81	15	81	11	75	31	25
Scarborough,	54·12	0·23 W	08	98	58	40	23	29·03	40	87	65	58	13	48	22	80	30	60	36	96	07	36	55	70	72	91	27	22	84	17	77	11	85	31	27
Leeds,	53·48	1·32 W	07	95	57	25	12	13	43	89	69	72	14	50	19	79	29	69	43	89	02	62	53	69	76	85	21	13	64	01	79	13	80	14	20
Liverpool,	53·25	2·59 W	11	99	57	30	09	15	45	85	74	73	13	53	20	81	37	77	38	86	06	68	64	75	80	82	22	17	69	04	84	20	93	45	33
Holyhead,	53·18	4·39 W	01	94	56	13	05	15	48	83	61	77	17	48	15	95	34	79	40	84	29·97	61	73	70	76	67	10	08	47	01	29·02	25	86	44	31
Nottingham,	52·56	1·8 W	11	30·01	64	42	24	32	52	82	79	72	20	54	21	80	32	79	45	91	30·13	84	64	83	86	97	36	29·86	72	02	28·83	25	97	42	35
Yarmouth,	52·37	1·43 E	06	00	81	54	46	38	58	92	72	76	18	47	23	93	16	92	44	72	09	96	68	82	93	30·03	40	30·23	92	26	96	19	30·11	38	33
Banbury,	52·4	1·21 W	07	02	68	41	32	41	55	94	82	85	20	51	23	81	26	82	35	82	17	78	77	84	86	02	35	16	77	28·99	29·05	32	08	42	34
Clifton,	51·27	2·37 W	09	29·99	65	33	29	51	57	93	83	30·01	22	53	25	83	29	85	32	90	14	75	81	84	84	29·96	29	12	70	97	25	38	14	49	40
Pembroke,	51·40	4·54 W	08	99	62	21	19	64	...	89	74	29·86	17	47	28	...	34	92	28	92	06	...	...	77	71	84	16	05	59	97	27	39	29·88	50	39
London,	51·31	0·6 W	06	30·00	70	49	45	51	60	92	86	91	21	48	28	87	17	94	41	79	19	86	84	88	89	30·04	36	18	87	29·04	28·98	36	30·18	45	39
Dover,	51·7	1·18 E	06	00	72	52	51	59	...	29·01	92	30·00	25	49	28	...	11	95	42	79	...	...	...	92	95	11	43	20	93	07	29·09	38	22	46	41
Portsmouth,	50·48	1·6 W	03	29·99	70	48	43	49	73	28·94	90	00	26	48	29	93	24	97	30	80	19	98	96	88	87	04	31	12	84	28·99	14	38	43	40	42
Weymouth,	50·37	2·27 W	07	30·00	68	44	41	58	65	96	91	06	29	49	31	89	24	94	18	83	17	83	87	88	86	29·99	24	10	78	98	25	42	23	49	42
Plymouth,	50·22	4·9 W	08	03	69	39	37	64	...	29·00	90	11	29	44	34	...	28	95	10	89	14	...	...	80	84	93	22	16	71	99	34	48	25	51	44
Penzance,	50·7	5·33 W	09	02	73	35	35	67	72	05	92	12	36	46	28	30·07	31	30·00	31	96	10	72	89	87	90	86	19	02	67	29·24	50	51	23	56	44
NORWAY AND SWEDEN.																																			
Haparanda,	65·52	24·3 E	03	02	98	82	66	43	28·86	28·98	84	29·76	29·23	29·17	29·52	29·74	29·76	29·74	41	56	29·74	40	30·01	76	94	30·15	41	20	30·11	30·03	86	29·84	29·94	65	29·74
Hernösand,	62·35	17·49 E	00	10	96	76	39	00	...	29·33	75	55	09	27	40	78	90	67	37	40	78	63	29·70	52	69	09	33	36	26	29·96	60	69	95	74	82
Christiansund,	63·7	7·46 E	29·87	29·82	70	38	28·88	66	65	36	48	29	25	46	71	30·03	30·03	28	24	31	79	71	30	31	81	29·94	13	24	12	46	28·96	67	28	78	92
Skudesnos,	59·10	5·15 E	30·14	30·02	75	44	29·08	72	29·07	39	39	20	66	30·03	90	29·98	29	66	29	60	30·04	80	23	50	94	30·00	35	52	17	52	29·10	71	68	30·05	30·05
Mandal,	58·3	7·25 E	23	15	90	56	25	86	14	35	48	47	71	29·99	81	97	32	82	34	59	15	99	45	55	88	10	47	59	33	73	22	58	96	06	08
Christiania,	59·54	10·48 E	15	06	93	55	17	49	28·92	46	58	42	43	67	63	95	22	70	35	53	29·97	84	45	42	92	08	42	53	35	88	37	53	94	29·97	09
Stockholm,	59·17	18·3 E	26	20	30·11	88	55	28·92	29·03	51	79	57	10	48	42	74	03	90	55	55	97	84	64	44	92	09	53	50	43	30·04	60	58	30·17	94	29·84
RUSSIA.																																			
Archangel,	64·33	40·33 E	28	08	21	30·31	30·31	30·13	91	82	96	30·29	54	05	54	58	29·45	58	94	89	87	82	86	30·17	30·08	12	47	20	04	29·96	30·31	30·45	41	49	72
Helsingfors,	60·14	24·57 E	24	15	17	12	29·87	29·45	25	41	98	29·54	28·89	39	23	49	73	94	61	54	87	...	63	29·53	29·77	21	52	43	32	30·17	29·91	29·93	36	72	78
St Petersburg,	59·58	30·26 E	31	23	31	33	30·18	84	58	55	30·03	69	29·08	22	26	37	63	85	80	70	81	70	69	61	71	29·97	56	40	29	15	30·08	30·12	52	59	72
Dorpat,	58·17	26·47 E	30	28	34	24	29·99	56	44	59	04	60	08	46	30	43	75	94	72	59	90	79	70	60	79	30·08	56	45	37	16	29·98	00	48	72	65
Riga,	56·57	24·3 E	38	22	29	21	30·00	61	59	69	29·97	63	30	59	32	42	81	30·19	68	72	30·06	98	91	77	66	15	37	46	46	29	98	29·94	38	74	68
Liban,	56·30	21·0 E	32	21	28	08	29·74	27	39	67	76	63	16	46	40	48	30·11	04	67	60	15	94	82	...	76	10	40	50	48	18	76	73	30	86	73
Warsaw,	52·14	21·7 E	30	31	26	16	30·04	73	84	95	77	30·13	91	87	79	76	04	24	77	60	35	30·25	30·06	98	99	24	39	69	60	28	96	85	45	92	82
Moscow,	55·42	37·39 E	26	31	38	45	46	30·46	30·12	30·03	30·20	05	86	53	38	36	29·67	29·87	30·22	65	02	29·91	29·83	85	68	29·90	36	36	31	12	30·14	30·37	30·84	52	73
Kostroma,	57·44	40·57 E	22	28	28	36	42	43	21	00	09	13	78	25	51	25	49	61	19	99	29·90	68	60	93	67	87	31	24	13	29·97	08	43	78	46	61
Catherinenbourg,	56·50	63·35 E	20	00	06	08	29	44	38	35	26	32	30·10	75	77	79	72	48	29·70	30·15	25	71	45	74	30·16	30·06	29·81	29·44	52	28	29·30	29·99	71	30·01	30·02
Slatoust,	55·11	59·40 E	15	29·86	29·90	29·96	17	37	37	35	25	31	18	90	80	79	76	50	77	17	28	70	47	80	29·97	29·95	75	50	56	41	32	88	06	29·96	29·94
Orenbourg,	51·47	55·12 E	29·57	61	72	97	29·93	29·92	17	21	18	14	29·92	81	79	72	58	62	99	27	71	71	66	76	78	61	67	52	...	...	...	...	...	...	...
Kiev,	50·27	30·34 E	30·15	30·00	30·28	30·28	30·29	30·38	11	08	29·88	11	30·17	99	66	78	98	30·02	30·32	29·60	30·18	30·32	30·09	97	98	30·18	30·43	...	60	30·16	30·18	30·11	64	50	9
Lougan	48·24	38·20 E	29·74	29·78	04	22	30	29	31	23	30·11	15	08	80	58	75	78	00	52	30·27	21	09	05	30·11	30·06	29·86	01	96	28	17	29·98	10	44	20	4

BAROMETRIC PRESSURE, in English inches, reduced to 32° and Sea Level, at 130 places in Europe, from 26th October to 12th November, from 20th to 26th November, from 30th November to 5th December, and from 14th to 18th December, at 8 A.M.

OCTOBER.						NOVEMBER.																				DECEMBER.									
26.	27.	28.	29.	30.	31.	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	20.	21.	22.	23.	24.	25.	26.	30.	1.	2.	3.	4.	5.	14.	15.	16.	17.	18.
·11	29·94	29·58	28·96	28·93	28·95	29·24	29·19	29·48	29·60	30·10	30·52	30·13	29·87	30·41	29·72	29·39	29·96	29·77	29·18	29·14	29·65	29·74	29·48	29·91	30·06	29·28	29·23	...	30·05	29·57	30·40	30·22	29·65	30·24	30·47
03	88	52	79	92	89	33	06	47	58	05	45	06	82	33	67	34	90	78	38	40	59	66	51	94	29·99	29	10	28·97	09	54	34	18	64	14	41
12	96	63	90	29·02	29·21	50	16	50	78	14	50	21	30·00	39	80	51	98	94	45	66	74	78	66	30·02	30·06	42	14	29·30	20	76	47	33	86	30	55
11	97	72	56	06	23	20	25	49	78	14	49	24	08	41	91	54	30·04	86	16	55	63	73	48	29·88	29·95	24	31	39	20	76	44	29	94	37	56
05	87	76	96	09	42	28·98	29	61	95	10	39	34	15	39	30·03	63	06	75	24	59	67	68	52	86	83	29	44	59	30	90	43	31	30·09	37	58
10	97	74	90	28·88	46	29·38	12	53	97	13	47	18	29·87	41	03	63	29·92	77	16	48	61	62	47	86	86	23	25	37	23	80	37	26	29·96	16	45
02	91	55	29·37	44	28·60	08	32	40	24	29·81	14	13	30·15	31	29·43	09	45	77	47	14	56	68	76	30·05	30·17	77	34	97	29·62	17	29·90	29·85	28·84	03	29·95
12	88	65	47	56	66	18	38	47	29	82	24	22	19	41	53	21	91	95	46	18	66	76	84	12	24	80	38	04	71	26	30·00	94	93	14	30·04
04	82	51	24	66	59	21	23	41	28	77	28	19	14	31	47	14	84	76	39	05	54	66	59	29·98	06	56	27	28·98	67	25	05	91	96	09	05
02	82	52	12	64	63	23	34	39	36	88	34	22	04	09	69	29	89	68	32	22	50	68	47	81	02	52	08	99	66	38	16	98	29·23	16	17
02	84	51	31	76	63	25	27	45	36	87	31	16	09	38	55	23	86	77	45	07	54	69	60	88	29·96	51	19	98	78	37	07	98	21	09	12
02	83	48	01	66	70	23	20	40	43	92	39	12	29·91	26	54	24	88	65	31	18	53	68	37	80	97	62	14	96	81	33	22	99	30	18	27
03	84	46	21	81	68	23	20	44	32	87	34	18	94	31	53	23	83	77	42	09	55	68	61	97	30·08	53	21	96	78	35	13	30·00	22	07	16
04	86	41	20	82	68	23	20	44	35	88	34	11	97	29	53	22	81	86	48	07	58	70	69	30·04	22	62	17	94	82	40	12	01	20	29·98	12
09	92	44	28	93	77	22	14	55	41	30·10	49	21	94	39	62	30	91	96	55	23	64	73	73	13	20	68	18	91	95	48	17	14	41	30·10	28
06	90	41	22	89	75	...	...	54	39	29.97	42	17	...	...	...	26	93	88	59	10	60	72	75	09	...	57	17	95	90	40	17	06	27	01	24
13	98	58	38	29·04	89	38	31	62	50	06	51	26	97	41	70	40	95	99	58	29	68	75	77	09	20	67	30	29·07	99	58	27	17	54	10	30
04	88	53	14	28·87	79	29	18	51	46	29·99	44	18	85	34	67	31	89	83	46	24	59	60	53	29·93	10	45	16	28·98	97	48	26	16	47	12	32
11	96	52	23	29·00	90	37	19	60	63	30·10	54	33	94	44	70	37	94	96	57	36	71	76	71	30·11	20	60	24	95	30·05	62	25	22	54	15	39
04	90	59	29	00	83	34	24	54	51	04	50	24	84	37	58	34	91	92	56	28	64	73	75	08	02	59	18	94	02	47	27	13	50	14	38
04	93	49	24	05	85	34	04	58	54	15	45	20	85	35	63	35	88	89	63	28	64	73	80	13	13	65	15	85	29·96	55	25	13	54	05	25
10	95	51	36	06	85	35	12	62	51	10	53	27	92	40	62	39	92	30·06	63	36	72	75	80	12	21	76	23	93	30·03	60	24	14	63	01	35
05	93	50	24	00	87	34	05	58	60	07	48	20	82	37	67	34	89	01	58	29	70	73	70	09	14	58	12	79	10	67	33	22	52	03	40
03	90	43	10	06	84	30	28·97	52	61	03	44	12	76	19	63	31	89	29·91	63	34	64	67	67	08	10	53	09	76	08	63	29	18	57	00	34
08	30·10	54	20	08	96	45	29·15	64	67	15	50	20	91	47	70	46	89	96	53	48	71	80	67	10	13	50	24	29·11	12	71	38	29	69	20	42
07	29·94	49	20	02	91	37	28·97	58	63	10	52	17	81	40	68	36	90	96	59	43	68	73	72	07	13	57	07	28·77	14	68	39	35	63	09	42
10	94	54	36	13	91	44	94	68	...	12	53	23	...	36	67	34	84	30·05	86	55	70	79	87	24	21	81	15	81	11	75	31	25	61	29·92	39
08	98	58	40	23	29·03	40	87	65	58	13	48	22	80	30	60	36	96	07	36	55	70	72	91	27	22	84	17	77	11	85	31	27	67	75	42
07	95	57	25	12	13	43	89	69	72	14	50	19	79	29	69	43	89	02	62	53	69	76	85	21	13	64	01	79	13	80	14	20	67	96	43
11	99	57	30	09	15	45	85	74	73	13	53	20	81	37	77	38	86	06	68	64	75	80	82	22	17	69	04	84	20	93	45	33	81	30·04	52
01	94	56	13	05	15	48	83	61	77	17	48	15	95	34	79	40	84	29·97	61	73	70	76	67	10	08	47	01	29·02	25	86	44	31	83	09	49
11	30·01	64	42	24	32	52	82	79	72	20	54	21	80	32	79	45	91	30·13	84	64	83	86	97	36	29·86	72	02	28·83	25	97	42	35	84	03	50
06	00	81	54	46	38	58	92	72	76	18	47	23	93	16	92	44	72	09	96	68	82	93	30·03	40	30·23	92	26	96	19	30·11	38	33	83	29·66	39
07	02	68	41	32	41	55	94	82	85	20	51	23	81	26	82	35	82	17	78	77	84	86	02	35	16	77	28·99	29·05	32	08	42	34	84	92	48
09	29·99	65	33	29	51	57	93	83	30·01	22	53	25	83	29	85	32	90	14	75	81	84	84	29·96	29	12	70	97	25	38	14	49	40	95	30·05	54
08	99	62	21	19	64	...	89	74	29·86	17	47	28	...	34	92	28	92	06	...	...	77	71	84	16	05	59	97	27	39	29·88	50	39	99	14	55
06	30·00	70	49	45	51	60	92	86	91	21	48	28	87	17	94	41	79	19	86	84	88	89	30·04	36	18	87	29·04	28·98	36	30·18	45	39	95	29·85	45
06	00	72	52	51	59	...	29·01	92	30·00	25	49	28	...	11	95	42	79	...	...	...	92	95	11	43	20	93	07	29·09	38	22	46	41	96	85	44
03	29·99	70	48	43	49	73	28·94	90	00	26	48	29	93	24	97	30	80	19	98	96	88	87	04	31	12	84	28·99	14	38	43	40	42	95	91	46
07	30·00	68	44	41	58	65	96	91	06	29	49	31	89	24	94	18	83	17	83	87	88	86	29·99	24	10	78	98	25	42	23	49	42	30·00	30·01	52
08	03	69	39	37	64	...	29·00	90	11	29	44	34	...	28	95	10	89	14	...	...	80	84	93	22	16	71	99	34	48	25	51	44	00	10	53
09	02	73	35	35	67	72	05	92	12	36	46	28	30·07	31	30·00	31	96	10	72	89	87	90	86	19	02	67	29·24	50	51	23	56	44	06	22	57
03	02	98	82	66	43	28·86	28·98	84	29·76	29·23	29·17	29·52	29·74	29·76	29·74	41	56	29·74	40	30·01	76	94	30·15	41	20	30·11	30·03	86	29·84	29·94	65	29·74	29·78	29·47	29·59
00	10	96	76	39	00	...	29·33	75	55	09	27	40	78	90	67	37	40	78	63	29·70	52	69	09	33	36	26	29·96	60	69	95	74	82	51	17	85
9·87	29·82	70	38	28·88	66	65	36	48	29	25	46	71	30·03	30·03	28	24	31	79	71	30	31	81	29·94	13	24	12	46	28·96	67	28	78	92	12	57	91
0·14	30·02	75	44	29·08	72	29·07	39	39	20	66	30·03	90	29·98	29	66	29	60	30·04	80	23	50	94	30·00	35	52	17	52	29·10	71	68	30·05	30·05	30	48	30·20
23	15	90	56	25	86	14	35	48	47	71	29·99	81	97	32	82	34	59	15	99	45	55	88	10	47	59	33	73	22	58	96	06	08	52	43	23
15	06	93	55	17	49	28·92	46	58	42	43	67	63	95	22	70	35	53	29·97	84	45	42	92	08	42	53	35	88	37	53	94	29·97	09	39	31	08
26	20	30·11	88	55	28·92	29·03	51	79	57	10	48	42	74	03	90	55	55	97	84	64	44	92	09	53	50	43	30·04	60	58	30·17	94	29·84	61	31	29·86
28	08	21	30·31	30·31	30·13	91	82	96	30·29	54	05	54	58	29·45	58	94	89	87	82	86	30·17	30·08	12	47	20	04	29·96	30·31	30·45	41	49	72	96	30·13	88
24	15	17	12	29·87	29·45	25	41	98	29·54	28·89	39	23	49	73	94	61	54	87	...	63	29·53	29·77	21	52	43	32	30·17	29·91	29·93	36	72	78	85	29·50	46
31	23	31	33	30·18	84	58	55	30·03	69	29·08	22	26	37	63	85	80	70	81	70	69	61	71	29·97	56	40	29	15	30·08	30·12	52	59	72	96	86	61
30	28	34	24	29·99	56	44	59	04	60	08	46	30	43	75	94	72	59	90	79	70	60	79	30·08	56	45	37	16	29·98	00	48	72	65	94	64	58
38	22	29	21	30·00	61	59	69	29·97	63	30	59	32	42	81	30·19	68	72	30·06	98	91	77	66	15	37	46	38	29	98	29·94	38	74	68	30·01	74	68
32	21	28	08	29·74	27	39	67	76	63	16	46	40	48	30·11	04	67	60	15	94	82	...	76	10	40	50	48	18	76	73	30	86	73	29·98	53	...
30	31	26	16	30·04	73	84	95	77	30·13	91	87	79	76	04	24	77	60	35	30·25	30·06	98	99	24	39	69	60	28	96	85	45	92	82	30·10	75	92
26	31	38	45	46	30·46	30·12	30·03	30·20	05	86	53	38	36	29·67	29·87	30·22	65	02	29·91	29·83	85	68	29·90	36	36	31	12	30·14	30·37	30·84	52	73	29·89	87	30·00
22	28	28	36	42	43	21	00	09	13	78	25	51	25	49	61	19	99	29·90	68	60	93	67	87	31	24	13	29·97	08	43	78	46	61	84	29·75	29·92
20	00	06	08	29	44	38	35	26	32	30·10	75	77	79	72	48	29·70	30·15	25	71	45	74	30·16	30·06	29·81	29·44	52	28	29·30	29·99	71	30·01	30·02	30·02	30·13	97
15	29·86	29·90	29·96	17	37	37	35	25	31	18	90	80	79	76	50	77	17	28	70	47	80	29·97	29·95	75	50	56	41	32	88	06	29·96	29·94	29·97	03	74
9·57	61	72	97	29·93	29·92	17	21	18	14	29·92	81	79	72	58	62	99	27	71	71	66	76	78	61	67	52	...	...	...	...	...	...	...	...	...	...
0·15	30·00	30·28	30·28	30·29	30·38	11	08	29·88	11	30·17	99	66	78	98	30·02	30·32	29·60	30·18	30·32	30·09	97	98	30·18	30·43	...	60	30·16	30·18	30·11	64	50	91	30·11	29·92	88
·74	29·78	04	22	30	29	31	23	30·11	15	08	80	58	75	78	00	52	30·27	21	09	05	30·11	30·06	29·86	01	96	28	17	29·98	10	44	20	45	29·61	70	30·02
																[illegible]	[illegible]	24	80	19	18	07	98	91	30·99	28	36	91	29·94	29	49	70	87	30·05	03

Nottingham,	52·56	1·8 W	11	30·01	64	42	24	32	52	82	79	72	20	54	21	80																
Yarmouth,	52·37	1·13 E	06	00	81	54	46	38	58	92	72	76	18	47	23	93	16	92	44	72	09	96	68	82	93	30·03	40	30·23	92	26	96	
Banbury,	52·4	1·21 W	07	02	68	41	32	41	55	94	82	85	20	51	23	81	26	82	35	82	17	78	77	84	86	02	35	16	77	28·99	29·05	3
Clifton,	51·27	2·37 W	09	29·99	65	33	29	51	57	93	83	30·01	22	53	25	83	29	85	32	90	14	75	81	84	84	29·96	29	12	70	97	25	3
Pembroke,	51·40	4·54 W	08	99	62	21	19	64	...	89	74	29·86	17	47	28	...	34	92	28	92	06	...	...	77	71	84	16	05	59	97	27	38
London,	51·31	0·6 W	06	30·00	70	49	45	51	60	92	86	91	21	48	28	87	17	94	41	79	19	86	84	88	89	30·04	36	18	87	29·04	28·98	36
Dover,	51·7	1·18 E	06	00	72	52	51	59	...	29·01	92	30·00	25	49	28	...	11	95	42	79	...	...	...	92	95	11	43	20	93	07	29·09	38
Portsmouth,	50·48	1·6 W	03	29·99	70	48	43	49	73	28·94	90	00	26	48	29	93	24	97	30	80	19	98	96	88	87	04	31	12	84	28·99	14	36
Weymouth,	50·37	2·27 W	07	30·00	68	44	41	58	65	96	91	06	29	49	31	89	24	94	18	83	17	83	87	88	86	29·99	24	10	78	98	25	42
Plymouth,	50·22	4·9 W	08	03	69	39	37	64	...	29·00	90	11	29	44	34	...	28	95	10	89	14	...	...	80	84	93	22	16	71	99	34	48
Penzance,	50·7	5·33 W	09	02	73	35	35	67	72	05	92	12	36	46	28	30·07	31	30·00	31	96	10	72	89	87	90	86	19	02	67	29·24	50	51
NORWAY AND SWEDEN.																																
Haparanda,	65·52	24·3 E	03	02	98	82	66	43	28·86	28·98	84	29·76	29·23	29·17	29·52	29·74	29·76	29·74	41	56	29·74	40	30·01	76	94	30·15	41	20	30·11	30·03	86	29·84
Hernösand,	62·35	17·49 E	00	10	96	76	39	00	...	29·33	75	55	09	27	40	78	90	67	37	40	78	63	29·70	52	69	09	33	36	26	29·96	60	69
Christiansund,	63·7	7·46 E	29·87	29·82	70	38	28·88	66	65	36	48	29	25	46	71	30·03	30·03	28	24	31	79	71	30	31	81	29·94	13	24	12	46	28·96	67
Skudesnos,	59·10	5·15 E	30·14	30·02	75	44	29·08	72	29·07	39	39	20	66	30·03	90	29·98	29	66	29	60	30·04	80	23	50	94	30·00	35	52	17	52	29·10	71
Mandal,	58·3	7·25 E	23	15	90	56	25	86	14	35	48	47	71	29·99	81	97	32	82	34	59	15	99	45	55	88	10	47	59	33	73	22	58
Christiania,	59·54	10·48 E	15	06	93	55	17	49	28·92	46	58	42	43	67	63	95	22	70	35	53	29·97	84	45	42	92	08	42	53	35	88	37	53
Stockholm,	59·17	18·3 E	26	20	30·11	88	55	28·92	29·03	51	79	57	10	48	42	74	03	90	55	55	97	84	64	44	92	09	53	50	43	30·04	60	58
RUSSIA.																																
Archangel,	64·33	40·33 E	28	08	21	30·31	30·31	30·13	91	82	96	30·29	54	05	54	58	29·45	58	94	89	87	82	86	30·17	30·08	12	47	20	04	29·96	30·31	30·45
Helsingfors,	60·14	24·57 E	24	15	17	12	29·87	29·45	25	41	98	29·54	28·89	39	23	49	73	94	61	54	87	...	63	29·53	29·77	21	52	43	32	30·17	29·91	29·93
St Petersburg,	59·58	30·26 E	31	23	31	33	30·18	84	58	55	30·03	69	29·08	22	26	37	63	85	80	70	81	70	69	61	71	29·97	56	40	29	15	30·08	30·12
Dorpat,	58·17	26·47 E	30	28	34	24	29·99	56	44	59	04	60	08	46	30	43	75	94	72	59	90	79	70	60	79	30·08	56	45	37	16	29·98	00
Riga,	56·57	24·3 E	38	22	29	21	30·00	61	59	69	29·97	63	30	59	32	42	81	30·19	68	72	30·06	98	91	77	66	15	37	46	46	29	98	29·94
Libau,	56·30	21·0 E	32	21	28	08	29·74	27	39	67	76	63	16	46	40	48	30·11	04	67	60	15	94	82	...	76	10	40	50	48	18	76	73
Warsaw,	52·14	21·7 E	30	31	26	16	30·04	73	84	95	77	30·13	91	87	79	76	04	24	77	60	35	30·25	30·06	98	99	24	39	69	60	28	96	85
Moscow,	55·42	37·39 E	26	31	38	45	46	30·46	30·12	30·03	30·20	05	86	53	38	36	29·67	29·87	30·22	65	02	29·91	29·83	85	68	29·90	36	36	31	12	30·14	30·37
Kostroma,	57·44	40·57 E	22	28	28	36	42	43	21	00	09	13	78	25	51	25	49	61	19	99	29·90	68	60	93	67	87	31	24	13	29·97	08	43
Catherinenbourg,	56·50	63·35 E	20	00	06	08	29	44	38	35	26	32	30·10	75	77	79	72	48	29·70	30·15	25	71	45	74	30·16	30·06	29·81	29·44	52	28	29·30	29·99
Slatoust,	55·11	59·40 E	15	29·86	29·90	29·96	17	37	37	35	25	31	18	90	80	79	76	50	77	17	28	70	47	80	29·97	29·95	75	50	56	41	32	88
Orenbourg,	51·47	55·12 E	29·57	61	72	97	29·93	29·92	17	21	18	14	29·92	81	79	72	58	62	99	27	71	71	66	76	78	61	67	52	...	...	...	...
Kiev,	50·27	30·34 E	30·15	30·00	30·28	30·28	30·29	30·38	11	08	29·88	11	30·17	99	66	78	98	30·02	30·32	29·60	30·18	30·32	30·09	97	98	30·18	30·43	...	60	30·16	30·18	30·11
Lougan	48·24	38·20 E	29·74	29·78	04	22	30	29	31	23	30·11	15	08	80	58	75	78	00	52	30·27	21	09	05	30·11	30·06	29·86	01	96	28	17	29·98	10
Nicolaiev,	46·58	32·0 E	46	75	29·91	20	30	26	11	05	06	06	01	98	85	84	85	05	18	29·98	24	20	19	18	07	98	21	30·29	28	36	91	29·94
Odessa,	46·28	30·43 E	39	74	95	17	25	23	29·93	29·92	29·98	29·97	04	91	85	84	90	01	06	94	20	22	16	11	29·96	30·05	12	24	32	26	93	88
Tiflis,	41·42	44·48 E	83	98	30·15	23	19	18	30·52	30·62	30·53	30·28	10	30·03	30·11	30·03	30·19	30	64	30·53	43	49	46	46	30·21	05	23	21	34	09	30·17	30·20
DENMARK.																																
Iviglut (Greenland),	61·12	35·44 W	65	92	29·59	29·80	29·98	29·63	29·90	29·72	29·91	44	15	29·60	29·84	29·21	29·51	29·99	29·29	29·51	29·43	29·12	29·11	28·81	29·07	29·34	29·65	29·10	28·92	29·02	29·01	29·11
Skagen, or the Skaw,	57·42	10·34 E	30·27	30·14	91	61	41	28·80	16	51	49	43	29·69	80	70	90	30·22	85	29	53	30·15	30·04	55	29·40	95	30·00	30·47	30·62	30·40	82	16	51
Copenhagen,	55·40	12·35 E	30	18	96	70	53	29·46	40	62	49	56	72	89	71	92	22	30·04	81	71	23	08	73	62	98	16	57	56	36	89	30	21
Kiel,	54·19	10·8 E	25	13	88	67	55	37	44	58	36	56	85	30·02	91	91	15	09	45	61	23	07	74	67	94	06	47	45	26	80	30	18
Heligoland (England),	54·11	7·53 E	14	06	81	62	50	29	...	...	27	53	94	06	99	...	19	05	39	59	25	...	...	72	95	06	45	...	18	72	...	...
PRUSSIA.																																
Königsberg,	54·10	20·30 E	36	34	30·26	30·10	90	49	62	77	77	29·85	59	29·64	53	63	29·99	16	67	62	28	15	93	83	94	18	47	63	47	30.23	88	73
Putbus,	54·23	13·27 E	33	19	29·97	29·74	59	45	45	63	46	63	76	83	71	85	30·11	09	49	58	25	11	80	70	95	10	46	50	35	29·91	46	16
Paderborn,	51·43	8·45 E	17	05	85	65	70	63	69	61	45	77	30·07	30·17	30·20	78	29·93	06	53	63	27	13	88	88	30·01	07	43	34	15	77	54	73
Breslau,	51·6	17·2 E	24	18	30·05	93	84	62	79	81	69	30·00	29·96	29·89	29·98	86	96	14	61	55	35	21	30·00	94	03	16	33	45	31	30·08	83	69
AUSTRIAN EMPIRE.																																
Prague,	50·5	14·26	17	22	05	96	92	77	87	87	75	06	30·13	30·02	30·19	30·00	93	14	68	63	44	34	17	30·05	12	20	41	42	37	15	90	80
Cracow,	50·4	19·56 E	21	24	17	30·11	30·03	80	87	89	74	15	09	29·87	06	29·87	97	22	71	55	45	32	15	10	09	26	37	51	50	25	97	82
Vienna,	48·13	16·22 E	20	19	06	02	29·99	30·13	92	92	93	21	16	30·11	24	30·03	91	12	76	58	46	25	25	15	10	38	...	40	45	19	30·06	87
Kremsmunster,	48·3	14·7 E	21	15	11	29·94	99	08	92	87	93	19	32	21	25	05	82	01	71	56	48	34	22	13	16	27	38	27	25	10	03	91
Klagenfurt,	46·37	14·19 E	14	11	02	30·00	99	29·82	77	75	76	16	19	08	12	29·95	73	29·99	63	52	21	44	31	21	20	32	38	41	42	25	10	90
GERMAN STATES.																																
Leipzig,	51·21	12·21 E	32	20	00	29·83	81	71	86	83	72	29·96	15	20	20	30·06	30·07	30·13	63	75	41	25	03	00	18	12	...	31	32	02	29·82	78
Bamberg,	49·53	10·52 E	21	14	00	86	80	81	87	80	74	30·02	22	18	27	29·77	29·77	03	68	63	42	31	11	00	12	18	45	34	24	04	88	92
Munich,	48·9	11·33 E	17	15	29·96	89	94	91	93	79	90	14	35	24	23	99	79	29·93	69	59	41	28	15	01	14	21	41	22	18	29·93	96	30·04
SWITZERLAND.																																
Aarau,	47·24	8·3 E	05	04	88	81	80	84	94	72	97	18	42	38	25	30·01	69	85	66	47	33	32	11	29·98	02	15	33	10	15	90	94	09
Geneva,	46·12	6·9 E	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
NETHERLANDS.																																
Groningen,	53·13	6·34 E	15	05	72	61	57	42	53	40	40	29·65	04	29	15	29·95	30·18	30·00	43	67	26	05	29·78	82	29·95	10	48	37	08	60	37	29·78
Leeuwarden,	53·12	5·47 E	15	06	76	62	56	43	54	35	44	68	12	34	18	97	20	01	44	69	27	04	79	83	30·00	11	49	37	13	57	36	90
Helder,	52·57	4·45 E	09	02	73	59	53	38	50	26	48	66	12	34	19	95	14	00	41	67	24	00	75	82	29·96	07	45	32	08	50	41	98
Amsterdam,	52·22	4·53 E	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
Utrecht,	52·5	5·7 E	11	03	76	62	59	54	64	28	63	78	17	35	23	95	10	00	47	70	27	05	87	88	30·01	12	45	30	10	51	31	30·08
Hellevoetsluis,	51·50	4·7 E	09	03	76	63	59	54	64	19	69	81	20	39	26	95	10	00	49	70	26	03	86	88	01	11	45	28	06	44	27	13
Breda,	51·35	4·46 E	05	00	74	66	59	50	63	20	69	78	16	32	19	94	29·99	05	45	67	21	00	82	84	29·98	04	37	29	02	48	24	12
Flushing,	51·26	3·35 E	09	03	77	63	63	61	69	14	61	88	24	42	28	92	97	02	50	71	27	03	89	90	30·02	13	48	27	05	38	25	22
BELGIUM.																																
Brussels,	50·52	4·21 E	12	04	83	70	70	73	80	26	91	94	28	44	32	98	97	03	55	72	32	09	90	95	07	17	47	28	10	49	39	28
Maastricht,	50·51	5·42 E	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
Luxemburg,	49·37	6·8 E	15	09	91	80	83	87	84	62	97	30·07	43	39	36	30·04	82	04	68	70	39	23	30·06	95	14	21	45	29	16	85	57	32
FRANCE.																																
Dunkirk,	51·7	2·39 E	00	09	76	60	64	62	72	10	86	29·91	30	44	20	29·99	30·04	04	48	76	24	29·96	29·88	88	00	13	47	28	00	27	21	27
Mezières,	49·45	4·43 E	10	05	80	76	76	87	86	42	95	30·08	38	43	36	30·03	29·80	03	60	72	32	30·16	30·03	96	07	18	48	19	13	60	57	31
Havre,	49·29	0·6 E	02	00	76	64	68	80	81	15	30·03	18	40	46	34	07	30·02	...	45	82	27	01	00	30·00	00	14	38	21	29·92	18	28	47
Cherbourg,	49·38	1·37 W	03	29·99	75	54	54	71	74	06	00	15	39	47	36	05	12	02	24	80	30	00	04	04	02	20	39	19	88	11	29	51
Paris,	48·50	2·20 E	06	99	81	70	75	87	88	29	07	17	44	42	36	08	29·84	03	58	70	30	11	29·81	29·99	04	16	40	17	30·03	32	50	41

	00	81	54	46	38	58	92	72	76	18	47	23	93	16	92	44	72	09	96	00	82	93	30·00	40	30·25	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
07	02	68	41	32	41	55	94	82	85	20	51	23	81	26	82	35	82	17	78	77	84	86	02	35	16	77	28·99	29·05	32	08	42	34	84	92	46
09	29·99	65	33	29	51	57	93	83	30·01	22	53	25	83	29	85	32	90	14	75	81	84	84	29·96	29	12	70	97	25	38	14	49	40	95	30·05	54
08	99	62	21	19	64	...	89	74	29·86	17	47	28	...	34	92	28	92	06	...	...	77	71	84	16	05	59	97	27	39	29·88	50	39	99	14	55
36	30·00	70	49	45	51	60	92	86	91	21	48	28	87	17	94	41	79	19	86	84	88	89	30·04	36	18	87	29·04	28·98	36	30·18	45	39	95	29·85	45
06	00	72	52	51	59	...	29·01	92	30·00	25	49	28	...	11	95	42	79	...	...	...	92	95	11	43	20	93	07	29·09	38	22	46	41	96	85	44
03	29·99	70	48	43	49	73	28·94	90	00	26	48	29	93	24	97	30	80	19	98	96	88	87	04	31	12	84	28·99	14	38	43	40	42	95	91	46
07	30·00	68	44	41	58	65	96	91	06	29	49	31	89	24	94	18	83	17	83	87	88	86	29·99	24	10	78	98	25	42	23	49	42	30·00	30·01	52
08	03	69	39	37	64	...	29·00	90	11	29	44	34	...	28	95	10	89	14	...	...	80	84	93	22	16	71	99	34	48	25	51	44	00	10	53
09	02	73	35	35	67	72	05	92	12	36	46	28	30·07	31	30·00	31	96	10	72	89	87	90	86	19	02	67	29·24	50	51	23	56	44	06	22	57
03	02	98	82	66	43	28·86	28·98	84	29·76	29·23	29·17	29·52	29·74	29·76	29·74	41	56	29·74	40	30·01	76	94	30·15	41	20	30·11	30·03	86	29·84	29·94	65	29·74	29·78	29·47	29·59
00	10	96	76	39	00	...	29·33	75	55	09	27	40	78	90	67	37	40	78	63	29·70	52	69	09	33	36	26	29·96	60	69	95	74	82	51	17	85
·87	29·82	70	38	28·88	66	65	36	48	[illegible]	25	46	71	30·03	30·03	28	24	31	79	71	30	31	81	29·94	13	24	12	46	28·96	67	28	78	92	12	57	91
·14	30·02	75	44	29·08	72	29·07	39	39	20	66	30·03	90	29·98	29	66	29	60	30·04	80	23	50	94	30·00	35	52	17	52	29·10	71	68	30·05	30·05	30	48	30·20
23	15	90	56	25	86	14	35	48	47	71	29·99	81	97	32	82	34	59	15	99	45	55	88	10	47	59	33	73	22	58	96	06	08	52	43	23
15	06	93	55	17	49	28·92	46	58	42	43	67	63	95	22	70	35	53	29·97	84	45	42	92	08	42	53	35	88	37	53	94	29·97	09	39	31	08
26	20	30·11	88	55	28·92	29·03	51	79	57	10	48	42	74	03	90	55	55	97	84	64	44	92	09	53	50	43	30·04	60	58	30·17	94	29·84	61	31	29·86
28	08	21	30·31	30·31	30·13	91	82	96	30·29	54	05	54	58	29·45	58	94	89	87	82	86	30·17	30·08	12	47	20	04	29·96	30·31	30·45	41	49	72	96	30·13	88
24	15	17	12	29·87	29·45	25	41	98	29·54	28·89	39	23	49	73	94	61	54	87	...	63	29·53	29·77	21	52	43	32	30·17	29·91	29·93	36	72	78	85	29·50	46
31	23	31	33	30·18	84	58	55	30·03	69	29·08	22	26	37	63	85	80	70	81	70	69	61	71	29·97	56	40	29	15	30·08	30·12	52	59	72	96	86	61
30	28	34	24	29·99	56	44	59	04	60	08	46	30	43	75	94	72	59	90	79	70	60	79	30·08	56	45	37	16	29·98	00	48	72	65	94	64	58
38	22	29	21	30·00	61	59	69	29·97	63	30	59	32	42	81	30·19	68	72	30·06	98	91	77	66	15	37	46	46	29	98	29·94	38	74	68	30·01	74	68
32	21	28	08	29·74	27	39	67	76	63	16	46	40	48	30·11	04	67	60	15	94	82	...	76	10	40	50	48	18	76	73	30	86	73	29·98	53	...
30	31	26	16	30·04	73	84	95	77	30·13	91	87	79	76	04	24	77	60	35	30·25	30·06	98	99	24	39	69	60	28	96	85	45	92	82	30·10	75	92
26	31	38	45	46	30'46	30·12	30·03	30·20	05	86	53	38	36	29·67	29·87	30·22	65	02	29·91	29·83	85	68	29·90	36	36	31	12	30·14	30·37	30·84	52	73	29·89	87	30·00
22	28	28	36	42	43	21	00	09	13	78	25	51	25	49	61	19	99	29·90	68	60	93	67	87	31	24	13	29·97	08	43	78	46	61	84	29·75	29·92
20	00	06	08	29	44	38	35	26	32	30·10	75	77	79	72	48	29·70	30·15	25	71	45	74	30·16	30·06	29·81	29·44	52	28	29·30	29·99	71	30·01	30·02	30·02	30·13	97
15	29·86	29·90	29·96	17	37	37	35	25	31	18	90	80	79	76	50	77	17	28	70	47	80	29·97	29·95	75	50	56	41	32	88	06	29·96	29·94	29·97	03	74
·57	61	72	97	29·93	29·92	17	21	18	14	29·92	81	79	72	58	62	99	27	71	71	66	76	78	61	67	52	...	...	...	...	...	...	...	...	...	...
·15	30·00	30·28	30·28	30·29	30·38	11	08	29·88	11	30·17	99	66	78	98	30·02	30·32	29·60	30·18	30·32	30·09	97	98	30·18	30·43	...	60	30·16	30·18	30·11	64	50	91	30·11	29·92	88
·74	29·78	04	22	30	29	31	23	30·11	15	08	80	58	75	78	00	52	30·27	21	09	05	30·11	30·06	29·86	01	96	28	17	29·98	10	44	20	45	29·61	70	30·02
46	75	29·91	20	30	26	11	05	06	06	01	98	85	84	85	05	18	29·98	24	20	19	18	07	98	21	30·29	28	36	91	29·94	29	49	70	87	30·05	03
39	74	95	17	25	23	29·93	29·92	29·98	29·97	04	91	85	84	90	01	06	94	20	22	16	11	29·96	30·05	12	24	32	26	93	88	25	50	74	92	04	29·98
83	98	30·15	23	19	18	30·52	30·62	30·53	30·28	10	30·03	30·11	30·03	30·19	30	64	30·53	43	49	46	46	30·21	05	23	21	34	09	30·17	30·20	56	49	91	30·09	21	30·37
65	92	29·59	29·80	29·98	29·63	29·90	29·72	29·91	44	15	29·60	29·84	29·21	29·51	29·99	29·29	29·51	29·43	29·12	29·11	28·81	29·07	29·34	29·05	29·10	28·92	29·02	29·01	29·11	...	30·24	30·12	29·53	12	29·53
·27	30·14	91	61	41	28·80	16	51	49	43	29·69	80	70	90	30·22	85	29	53	30·15	30·04	55	29·40	95	30·00	30·47	30·62	30·40	82	16	51	03	29·96	29·96	50	29·43	30·07
30	18	96	70	53	29·46	40	62	49	56	72	89	71	92	22	30·04	81	71	23	08	73	62	98	16	57	56	36	89	30	21	17	30·05	30·04	75	53	13
25	13	88	67	55	37	44	58	36	56	85	30·02	91	91	15	09	45	61	23	07	74	67	94	06	47	45	26	80	30	18	15	07	08	84	46	09
14	06	81	62	50	29	...	...	27	53	94	06	99	...	19	05	39	59	25	...	...	72	95	06	45	...	18	72	...	...	...	12	18	86	35	19
36	34	30·26	30·10	90	49	62	77	77	29·85	59	29·64	53	63	29·99	16	67	62	28	15	93	83	94	18	47	63	47	30.23	88	73	37	29·90	29·74	30·00	63	29·76
33	19	29·97	29·74	59	45	45	63	46	63	76	83	71	85	30·11	09	49	58	25	11	80	70	95	10	46	50	35	29·91	46	16	19	30·03	91	29·84	51	92
17	05	85	65	70	63	69	61	45	77	30·07	30·17	30·20	78	29·93	06	53	63	27	13	88	88	30·01	07	43	34	15	77	54	73	32	20	27	30·08	70	30·07
24	18	30·05	93	84	62	79	81	69	30·00	29·96	29·89	29·98	86	96	14	61	55	35	21	30·00	94	03	16	33	45	31	30·08	83	69	30	29·93	29·85	05	69	29·86
17	22	05	96	92	77	87	87	75	06	30·13	30·02	30·19	30·00	93	14	68	63	44	34	17	30·05	12	20	41	42	37	15	90	80	38	30·09	30·12	16	77	93
21	24	17	30·11	30·03	80	87	89	74	15	09	29·87	06	29·87	97	22	71	55	45	32	15	10	09	26	37	51	50	25	97	82	40	29·77	29·95	14	77	96
20	19	06	02	29·99	30·13	92	92	93	21	16	30·11	[illegible]	30·03	91	12	76	58	46	25	25	15	10	38	...	40	45	19	30·06	87	45	30·03	30·06	17	84	30·02
21	15	11	29·94	99	08	92	87	93	19	32	21	25	05	82	01	71	56	48	34	22	13	16	27	38	27	25	10	03	91	51	19	28	24	90	00
14	11	02	30·00	99	29·82	77	75	76	16	19	08	12	29·95	73	29·99	63	52	21	44	31	21	20	32	38	41	42	25	10	90	42	10	19	23	93	29·93
32	20	00	29·83	81	71	86	83	72	29·96	15	20	20	30·06	30·07	30·13	63	75	41	25	03	00	18	12	...	31	32	02	29·82	78	35	17	23	12	78	30·08
11	14	00	86	80	81	87	80	74	30·02	22	18	27	29·77	29·77	03	68	63	42	31	11	00	12	18	45	34	24	04	88	92	33	24	30	15	81	29·99
17	15	29·96	89	94	91	93	79	90	14	35	24	23	99	79	29·93	69	59	41	28	15	04	14	21	41	22	18	29·93	96	30·04	47	27	33	15	85	30·02
05	04	88	81	80	84	94	72	97	18	42	38	25	30·01	69	85	66	47	33	32	11	29·98	02	15	33	10	15	90	94	09	46	33	33	16	82	29·98
...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
15	05	72	61	57	42	53	40	40	29·65	04	29	15	29·95	30·18	30·00	43	67	26	05	29·78	82	29·95	10	48	37	08	60	37	29·78	19	22	27	29·86	44	30·26
15	06	76	62	56	43	54	35	44	68	12	34	18	97	20	01	44	69	27	04	79	83	30·00	11	49	37	13	57	36	90	20	26	30	89	45	28
09	02	73	59	53	38	50	26	48	66	12	34	19	95	14	00	41	67	24	00	75	82	29·96	07	45	32	08	50	41	98	17	28	30	88	46	28
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	03	76	62	59	54	64	28	63	78	17	35	23	95	10	00	47	70	27	05	87	88	30·01	12	45	30	10	51	31	30·08	27	32	35	97	57	28
	03	76	63	59	54	64	19	69	81	20	39	26	95	10	00	49	70	26	03	86	88	01	11	45	28	06	44	27	13	27	35	36	97	62	30
	00	74	66	59	50	63	20	69	78	16	32	19	94	29·99	05	45	67	21	00	82	84	29·98	04	37	29	02	48	24	12	27	31	30	99	60	20
	03	77	63	63	61	69	14	61	88	24	42	28	92	97	02	50	71	27	03	89	90	30·02	13	48	27	05	38	25	22	30	40	40	30·01	69	33
	04	83	70	70	73	80	26	91	94	28	44	32	98	97	03	55	72	32	09	90	95	07	17	47	28	10	49	39	26	40	44	39	12	75	33
	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
	09	91	80	83	87	84	62	97	30·07	43	39	36	30·04	82	04	68	70	39	23	30·06	95	14	21	45	29	16	85	57	32	47	42	46	18	84	21
	09	76	60	64	62	72	10	86	29·91	30	44	20	29·99	30·04	04	48	76	24	29·96	29·88	88	00	13	47	28	00	27	21	27	30	47	42	04	76	35
	05	80	76	76	87	86	42	95	30·08	38	43	36	30·03	29·80	03	60	72	32	30·16	30·03	96	07	18	48	19	13	60	57	31	44	...	43	35	83	28
	00	76	64	68	80	81	15	30·03	18	40	46	34	07	30·02	...	45	82	27	01	00	30·00	00	14	38	21	29·92	18	28	47	38	52	44	14	95	41
	29·99	75	54	54	71	74	06	00	15	39	47	36	05	12	02	24	80	30	00	04	04	02	20	39	19	88	11	29	51	36	53	47	06	30·04	44
	99	81	70	75	87	88	29	07	17	44	42	36	08	29·84	03	58	70	30	11	29·81	29·99	04	16	40	17	30·03	32	50	41	48	49	46	18	29·91	32
	30·12	97	...	86	97	30·00	70	06	15	45	53	38	06	88	01	74	67	43	32	...	30·00	14	22	47	25	24	89	87	26	56	...	47	22	89	17
	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	51	45	19	30·18	09	37	98	12	29·85	96	29·94	29·92	29·97	20	29·99	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	50	43	20	30·20	47

Utrecht,	52·5	5·7 E	11	03	76	62	59	54	64	28	63	78	17	35	23	95	10	00	47	70	27	03	87	88	30·01	12	45	30	10	51	31
Hellevoetsluis,	51·50	4·7 E	09	03	76	63	59	54	64	19	69	81	20	39	26	95	10	00	49	70	26	03	86	88	01	11	45	28	06	44	27
Breda,	51·35	4·46 E	05	00	74	66	59	50	63	20	69	78	16	32	19	94	29·99	05	45	67	21	00	82	84	29·98	04	37	29	02	48	24
Flushing,	51·26	3·35 E	09	03	77	63	63	61	69	14	61	88	24	42	28	92	97	02	50	71	27	03	89	90	30·02	13	48	27	05	38	25
Belgium.																															
Brussels,	50·52	4·21 E	12	04	83	70	70	73	80	26	91	94	28	44	32	98	97	03	55	72	32	09	90	95	07	17	47	28	10	49	39
Maastricht,	50·51	5·42 E	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
Luxemburg,	49·37	6·8 E	15	09	91	80	83	87	84	62	97	30·07	43	39	36	30·04	82	04	68	70	39	23	30·06	95	14	21	45	29	16	85	57
France.																															
Dunkirk,	51·7	2·39 E	00	09	76	60	64	62	72	10	86	29·91	30	44	20	29·99	30·04	04	48	76	24	29·96	29·88	88	00	13	47	28	00	27	21
Mezières,	49·45	4·43 E	10	05	80	76	76	87	86	42	95	30·08	38	43	36	30·03	29·80	03	60	72	32	30·16	30·03	96	07	18	48	19	13	60	37
Havre,	49·29	0·6 E	02	00	76	64	68	80	81	15	30·03	18	40	46	34	07	30·02	...	45	82	27	01	00	30·00	00	14	38	21	29·92	18	28
Cherbourg,	49·38	1·37 W	03	29·99	75	54	54	71	74	06	00	15	39	47	36	05	12	02	24	80	30	00	04	04	02	20	39	19	88	11	29
Paris,	48·50	2·20 E	06	99	81	70	75	87	88	29	07	17	44	42	36	08	29·84	03	58	70	30	11	29·81	29·99	04	16	40	17	30·03	32	50
Strasbourg,	48·36	7·42 E	18	30·12	97	...	86	97	30·00	70	06	15	45	53	38	06	88	01	74	67	43	32	...	30·00	14	22	47	25	24	89	87
Brest,	48·23	4·27 W	06	01	76	51	68	84	29·88	22	12	32	49	51	45	19	30·18	09	37	92	12	29·85	96	29·94	29·92	29·97	20	29·99	29·82	35	52
L'Orient,	47·44	3·21 W	02	00	78	60	66	88	93	27	16	29	48	48	42	22	08	13	43	80	12	96	30·01	98	95	30·02	32	30·03	91	41	47
Napoleon Vendée,	46·41	1·25 W	05	01	78	74	78	98	30·00	46	30	...	...	...	47	30	00	15	61	...	...	30·11	09	30·04	30·04	13	30	05	30·03	53	76
Besançon,	47·17	6·5 E	07	04	88	84	84	97	02	67	13	28	53	48	36	17	29·76	29·92	69	52	36	24	11	10	09	21	38	20	18	77	90
Lyons,	45·45	4·49 E	14	29·98	93	85	94	99	10	79	25	41	59	51	42	21	88	30·00	87	51	43	32	24	16	18	18	41	27	24	87	30·06
Limoges,	45·52	1·17 E	02	30·00	76	78	87	90	00	53	28	43	53	47	35	23	85	01	72	62	47	13	13	08	02	21	32	07	09	63	29·88
Rochfort,	45·56	0·57 W	09	29·96	76	76	80	96	00	49	28	43	55	51	36	...	92	08	69	76	24	04	04	00	00	08	24	96	00	57	92
Bordeaux,	44·50	0·31 W	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	17	29·93	09	08	03	14	26	03	04	72	91
Montauban,	44·1	1·22 E	01	93	77	85	92	30·00	08	73	28	52	56	49	41	28	88	06	82	70	29	30·18	17	13	08	18	25	06	10	82	30·05
Bayonne,	43·31	1·26 W	29·92	88	57	72	...	29·96	04	69	32	51	...	43	47	20	30·00	18	84	76	16	08	08	08	00	08	20	29·88	00	81	...
Montpellier,	43·36	3·54 E	30·07	30·05	89	88	97	30·04	00	89	27	39	43	35	24	29·97	29·84	29·91	74	54	39	33	24	14	13	24	34	30·17	22	57	10
Marseilles,	43·17	5·22 E	06	06	89	86	99	03	29·93	99	17	33	34	23	15	30·00	78	82	77	54	39	34	...	16	12	24	33	12	23	30·04	10
Toulon,	43·7	5·56 E	04	08	88	88	30·00	04	96	92	08	33	32	28	...	29·96	76	76	69	49	44	32	...	25	21	28	41	12	20	08	...
Spain and Portugal.																															
Bilbao,	43·15	2·54 W	29·92	29·87	87	81	29·86	04	30·13	86	...	59	50	51	48	30·30	30·09	30·13	...	88	20	...	09	06	02	08	21	01	00	29·93	01
Barcelona,	41·23	2·9 E	30·02	98	78	81	96	08	29·92	94	26	40	41	31	21	14	29·83	29·87	85	55	34	30	19	13	21	17	24	08	15	30·03	12
Oporto,	41·7	8·33 W	29·84	89	88	90	...	13	30·26	30·15	50	54	44	44	35	26	30·20	30·22	...	30·14	13	29·81	16	04	07	...	10	29·95	...	22	37
Lisbon,	38·41	9·10 W	77	84	84	94	30·12	19	24	23	43	49	43	41	30	23	09	14	30·27	17	16	30·20	18	00	06	12	09	93	11	26	39
Madrid,	40·25	3·43 W	96	97	90	93	12	19	11	15	43	65	56	54	42	32	12	10	19	29·99	45	56	28	09	19	30	32	30·09	19	25	35
Alicante,	38·23	0·25 W	96	88	82	87	06	13	00	07	...	48	46	39	...	25	29·94	29·95	...	78	38	36	26	17	14	...	26	05	19	21	25
San Fernando,	36·27	6·13 W	85	86	77	86	08	16	10	20	32	48	36	36	25	18	30·05	99	20	30·03	24	26	15	07	08	13	08	29·98	05	29	40
Palma (Majorca),	39·35	2·43 E	30·04	30·04	...	84	06	13	00	05	31	41	46	34	25	22	29·88	94	29·91	29·63	48	36	30	19	15	31	...	30·08	23	16	20
Africa.																															
Algiers,	36·43	3·3 E	02	02	89	94	14	29·88	...	14	36	48	54	42	32	31	63	30·07	30·09	76	43	44	36	29	15	...	...	06	31	27	34
Italy.																															
Turin,	45·3	7·41 E	...	08	30·02	30·08	08	30·03	06	29·80	29·94	25	32	20	18	...	...	29·93	29·62	56	46	49	25	19	14	26	33	26	28	17	05
Leghorn,	43·33	10·18 E	06	10	04	29·98	03	08	29·96	89	90	22	33	21	12	07	69	80	62	52	28	31	39	13	14	24	...	...	22	16	29·98
Florence,	43·47	11·14 E	09	12	04	...	07	10	...	...	94	28	...	...	12	...	71	81	...	59	40	32	23	...	...	26	34	17	20	17	96
Ancona,	43·37	13·31 E	...	...	29·83	82	29·84	29·78	76	62	64	...	06	01	29·91	29·82	58	81	70	80	14	16	26	21	17	25	...	19	23	26	96
Rome,	41·54	12·59 E	09	12	30·04	30·06	30·12	30·17	30·07	98	97	15	27	22	30·10	30·04	78	78	68	65	32	29	21	20	18	25	34	10	20	19	83
Naples,	40·50	14·15 E	04	04	00	05	03	22	16	30·05	03	12	25	22	12	06	84	86	81	75	30	30	...	21	19	26	34	10	14	18	82
Palermo (Sicily),	38·7	13·22 E	04	03	29·98	02	07	13	07	29·99	17	11	11	12	04	03	88	83	78	76	16	19	11	17	14	15	23	02	04	10	...
Turkey.																															
Constantinople,	41·0	28·59 E	00	10	30·08	18	43	...	17	30·09	28	04	16	06	01	29·96	93	30·00	30·07	94	22	26	18	14	06	13	22	24	22	12	93
Greece.																															
Athens,	37·58	23·43 E	01	29·96	29·99	14	31	21	09	09	04	29·92	00	05	29·92	87	88	29·92	29·96	78	10	08	06	02	29·98	08	14	14	15	15	96

03	76	62	59	54	64	28	63	78	17	35	23	95	10	00	47	70	27	05	87	88	30·01	12	45	30	10	51	31	30·08	27	32	35	97	57	28
03	76	63	59	54	64	19	69	81	20	39	26	95	10	00	49	70	26	03	86	88	01	11	45	28	06	44	27	13	27	35	36	97	62	30
00	74	66	59	50	63	20	69	78	16	32	19	94	29·99	05	45	67	21	00	82	84	29·98	04	37	29	02	48	24	12	27	31	30	99	60	20
03	77	63	63	61	69	14	61	88	24	42	28	92	97	02	50	71	27	03	89	90	30·02	13	48	27	05	38	25	22	30	40	40	30·01	69	33
04	83	70	70	73	80	26	91	94	28	44	32	98	97	03	55	72	32	09	90	95	07	17	47	28	10	49	39	23	40	44	39	12	75	33
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09	91	80	83	87	84	62	97	30·07	43	39	36	30·04	82	04	68	70	39	23	30·06	95	14	21	45	29	16	85	57	32	47	42	46	18	84	21
09	76	60	64	62	72	10	86	29·91	30	44	20	29·99	30·04	04	48	76	24	29·96	29·88	88	00	13	47	28	00	27	21	27	30	47	42	04	76	35
05	80	76	76	87	86	42	95	30·08	38	43	36	30·03	29·80	03	60	72	32	30·16	30·03	96	07	18	48	19	13	60	57	31	44	...	43	35	83	28
00	76	64	68	80	81	15	30·03	18	40	46	34	07	30·02	...	45	82	27	01	00	30·00	00	14	38	21	29·92	18	28	47	38	52	44	14	95	41
29·99	75	54	54	71	74	06	00	15	39	47	36	05	12	02	24	80	30	00	04	04	02	20	39	19	88	11	29	51	36	53	47	06	30·04	44
99	81	70	75	87	88	29	07	17	44	42	36	08	29·84	03	58	70	30	11	29·81	29·99	04	16	40	17	30·03	32	50	41	48	49	46	18	29·91	32
30·12	97	...	86	97	30·00	70	06	15	45	53	38	06	88	01	74	67	43	32	...	30·00	14	22	47	25	24	89	87	26	56	...	47	22	89	17
01	76	51	68	84	29·88	22	12	32	49	51	45	19	30·18	09	37	92	12	29·85	96	29·94	29·92	29·97	20	29·99	29·82	35	52	52	37	50	43	20	30·20	47
00	78	60	66	88	93	27	16	29	48	48	42	22	08	13	43	80	12	96	30·01	98	95	30·02	32	30·03	91	41	47	58	43	51	43	20	15	43
01	78	74	78	98	30·00	46	30	...	...	...	47	30	00	15	61	...	...	30·11	09	30·04	30·04	13	30	05	30·03	53	76	63	55	52	46	...	15	39
04	88	84	84	97	02	67	13	28	53	48	36	17	29·76	29·92	69	52	36	24	11	10	09	21	38	20	18	77	90	36	50	43	41	24	29·91	09
29·98	93	85	94	99	10	79	25	41	59	51	42	21	88	30·00	87	51	43	32	24	16	18	18	41	27	24	87	30·06	...	57	47	39	20	30·02	09
30·00	76	78	87	90	00	53	28	43	53	47	35	23	85	01	72	62	47	13	13	08	02	21	32	07	09	63	29·88	54	56	40	34	23	04	18
29·96	76	76	80	96	00	49	28	43	55	51	36	...	92	08	69	76	24	04	04	00	00	08	24	96	00	57	92	63	52	43	35	28	12	28
...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	17	29·93	09	08	03	14	26	03	04	72	91	63	54	42	36	29	14	30
93	77	85	92	30·00	08	73	28	52	56	49	41	28	88	06	82	70	29	30·18	17	13	08	18	25	06	10	82	30·05	53	57	41	38	34	06	14
88	57	72	...	29·96	04	69	32	51	...	43	47	20	30·00	18	84	76	16	08	08	08	00	08	20	29·88	00	81	...	55	47	36	36	32	12	20
30·05	89	88	97	30·04	00	89	27	39	43	35	24	29·97	29·84	29·91	74	54	39	33	24	14	13	24	34	30·17	22	57	10	42	38	27	30	29	29·97	29·93
06	89	86	99	03	29·93	99	17	33	34	23	15	30·00	78	82	77	54	39	34	...	16	12	24	33	12	23	30·04	10	25	29	17	26	24	95	82
08	88	88	30·00	04	96	92	08	33	32	28	...	29·96	76	76	69	49	44	32	...	25	21	28	41	12	20	08	...	12	20	12	24	24	92	84
29·87	87	81	29·86	04	30·13	86	...	59	50	51	48	30·30	30·09	30·13	...	88	20	...	09	06	02	08	21	01	00	29·93	01	...	44	43	35	35	30·21	30·30
98	78	81	96	08	29·92	94	26	40	41	31	21	14	29·83	29·87	85	55	34	30	19	13	21	17	24	08	15	30·03	12	19	25	21	24	25	29·96	29·90
89	88	90	...	13	30·26	30·15	50	54	44	44	35	26	30·20	30·22	...	30·14	13	29·81	16	04	07	...	10	29·95	...	22	37	51	47	36	36	35	30·24	30·28
84	84	94	30·12	19	24	23	43	49	43	41	30	23	09	14	30·27	17	16	30·20	18	00	06	12	09	93	11	26	39	47	45	36	32	30	17	42
97	90	93	12	19	11	15	43	65	56	54	42	32	12	10	19	29·99	45	56	28	09	19	30	32	30·09	19	25	35	04	59	45	41	41	24	25
88	82	87	06	13	00	07	...	48	46	39	...	25	29·94	29·95	...	78	38	36	26	17	14	...	26	05	19	21	25	35	38	31	30	31	04	00
86	77	86	08	16	10	20	32	48	36	36	25	18	30·05	99	20	30·03	24	26	15	07	08	13	08	29·98	05	29	40	38	39	34	28	28	19	08
30·04	...	84	06	13	00	05	31	41	46	34	25	22	29·88	94	29·91	29·63	48	36	30	19	15	31	...	30·08	23	16	20	36	32	27	24	29	03	29·87
02	89	94	14	29·88	...	14	36	48	54	42	32	31	63	30·07	30·09	76	43	44	36	29	15	...	...	06	31	27	34	34	38	38	36	35	19	98
08	30·02	30·08	08	30·03	06	29·80	29·94	25	32	20	18	...	...	29·93	29·62	56	46	49	25	19	14	26	33	26	28	17	05	29·98	28	15	19	26	29·96	89
10	04	29·98	03	08	29·96	89	90	22	33	21	12	07	69	80	62	52	28	31	39	13	14	24	...	...	22	16	29·98	76	10	15	24	26	96	82
12	04	...	07	10	...	...	94	28	...	...	12	...	71	81	...	59	40	32	23	...	...	26	34	17	20	17	96	...	22	15	24	26	30·12	86
...	29·83	82	29·84	29·78	76	62	64	...	06	01	29·91	29·82	58	81	70	80	14	16	26	21	17	25	...	19	23	26	96	30·01	10	15	22	27	10	98
12	30·04	30·06	30·12	30·17	30·07	98	97	15	27	22	30·10	30·04	78	78	68	65	32	29	21	20	18	25	34	10	20	19	83	29·74	29·86	17	22	28	09	91
04	00	05	03	22	16	30·05	03	12	25	22	12	06	84	86	81	75	30	30	...	21	19	26	34	10	14	18	82	75	87	18	26	31	13	98
03	29·98	02	07	13	07	29·99	17	11	11	12	04	03	88	83	78	76	16	19	11	17	14	15	23	02	04	10	...	87	...	14	19	21	06	89
10	30·08	18	43	...	17	30·09	28	04	16	06	01	29·96	93	30·00	30·07	94	22	26	18	14	06	13	22	24	22	12	93	93	30·18	29·68	29·96	00	19	30·15
29·96	29·99	14	31	21	09	09	04	29·92	00	05	29·92	87	88	29·92	29·96	78	10	08	06	02	29·98	08	14	14	15	15	96	92	03	87	30·06	03	13	29·71

TABLE II.—Showing the Temperature (Fahr.) and State of the Sky in the Morning at 110 Places in Europe, from 26th October to 12th November, from 20th to 26th November, f
5th December, and 14th to 18th December 1863.

NOTE.—B represents blue sky, or sky little more than half covered with clouds ; C, a much clouded or overcast sky ; F, fog ; R, rain at the time of observation ; and r, rain sometime du

	OCTOBER.						NOVEMBER.																				DECEM				
	26.	27.	28.	29.	30.	31.	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	20.	21.	22.	23.	24.	25.	26.	30.	1.	2.	3.	4.	5.
IRELAND.																															
Greencastle,	46 c	47 c	44 cr	44 R	40 cr	44 cr	39	43 cr	41 R	51 cr	43 cr	36 cr	44 cr	48	37 c	43 R	41 cr	43 cr	52 cr	...	49	46 cr	43 cr	51 R	54 R	46 c	49 cr	40 cr	...	44 cr	47 R
Galway,	46 R	49 c	44 Br	51 R	42 cr	44 R	33	42 cr	52 R	58 R	48 R	37 cr	52 cr	44	37 Br	43 cr	42 cr	41 cr	54 cr	...	37	49 cr	46 cr	50 cr	53 cr	49 c	47 cr	40 cr	32	34	48
Valencia,	53 c	49 c	43 cr	48	46	45 R	45 R	46 R	53	54 cr	53 R	54 cr	54 cr	48 F	42 cr	48 cr	44 cr	43 Br	54 cr	...	50	50 cr	46 Br	50 c	54 R	49 cr	48	44 Br	41 R	...	49 cr
Queenstown,	56 c	51 c	48 cr	51 cr	44 cr	46 c	...	43 cr	51 R	55 cr	55 cr	50 cr	53 cr	...	45 c	47 c	43 cr	44 B	55 R	...	...	49 c	50 c	52 R	54 c	50 c	59 c	42 cr	...	46 cr	52 R
SCOTLAND.																															
*East Yell,	47	45	42 R	37 R	39 R	41 R	41 cr	41 R	40 R	42 R	39 R	36 R	36 R	35	34 R	42 R	43 R	42 R	48 R	48 R	47 R	48	47	46	49	44 R	42 R	38 R	36	47 R	44 R
Sandwick,	44 Br	45 cr	41 cr	41 cr	40 Br	42 B	40 Br	38 Br	41 Br	44 Br	40 R	40 R	36 Br	40 Br	35 cr	43 R	42 R	37 cr	49 B	49 Br	48 R	46 cr	45 B	47 R	46 cr	42 B	44 B	40 cr	36 cr	47 cr	45 cr
Stornoway,	39 B	42 cr	45 R	45 cr	38 B	43 B	45 cr	42 B	35 B	44 R	39 R	34 cr	40 cr	43 Br	34 Br	38 Br	37 Br	36 cr	53 R	52 cr	49 R	46 R	45 Br	53 R	54 cr	42 B	49 B	39 R	35 Br	48 cr	43 R
Portree,	38 Br	41 cr	42 cr	43 cr	40 cr	42 R	35 cr	31 Br	34 cr	45 cr	42 cr	36 Br	41 c	40 cr	36 B	41 Br	39 cr	35 cr	52 cr	51 cr	45 R	43 cr	41 cr	53 R	54 cr	41 B	48 cr	39 R	34 R	43 R	46 R
Elgin,	42 c	44 B	42 R	38 cr	38 B	40 c	38 c	32 B	36 B	46 cr	37 cr	33 B	38 cr	34 cr	34 B	39 B	37 B	36 B	48 c	49 cr	44 cr	46 cr	40 B	50 c	50 B	40 B	41 B	37 cr	35 c	42 B	48 c
*Barry,	44 cr	46 cr	49 cr	42 cr	41 B	38 B	40 B	39 c	38 cr	50 cr	40 c	35 B	36 cr	40 cr	38 B	40 Br	36 B	39 B	44 c	52 cr	46 cr	43 cr	45 cr	48 cr	51 c	43 c	42 cr	42 cr	35 cr	43 c	52 Br
Otter House,	37 B	48 R	44 cr	40 c	40 R	43 R	41 B	36 Br	42 c	49 cr	42 R	33 B	48 R	39 cr	32 B	42 R	40 R	40 Br	50 c	52 cr	45 R	46 cr	43 B	52 cr	54 cr	37 c	40 R	39 R	35 R	45 c	47 R
*Baillieston,	41 cr	46 Br	47 cr	38 c	37 B	40 cr	36 Br	35 c	36 Br	39 cr	40 c	28 B	35 cr	37 cr	30 c	39 Br	34 B	32 B	45 cr	51 cr	43 Br	43 cr	52 c	51 c	51 c	30 c	36 cr	37 cr	33 c	41 cr	48 cr
*Milne-Graden,	44 c	47 R	45 R	35 R	41	40	37	40 R	37 c	51 B	40 c	30 c	33 R	40 R	38 c	41 c	35 c	36 c	40 c	53 c	44	43 c	47 R	46	56 B	39 c	39 c	36 R	33 R	38	51
ENGLAND.																															
Silloth,	45 c	48 R	48 R	42 c	41 c	46 R	42 B	40 R	45 R	52 R	46 R	31 B	37 R	41 R	39 R	43 R	39 B	38 B	40 B	53 c	46 R	47 c	50 R	54 c	56 R	34 B	40 B	40 R	38 R	45 c	51 R
Shields,	46 B	50 c	49 c	44 Br	43 R	48 c	49 c	49 R	41 B	...	45 c	38 B	39 c	44 B	49 cr	45 c	40 c	40 R	41 c	48 c	49 R	48 cr	50 c	50 c	57 c	46 c	49 c	47 R	44 R	41 cr	52 c
Scarborough,	46 B	46 B	44 c	42 B	41 R	44 cr	43	46 cr	46 Br	55 cr	44 c	43 B	37 c	44	44 cr	38 R	38 B	40 R	42 c	48	44	46 c	47 c	50 c	51 c	43 c	43 c	47 R	43 R	39 Br	49 c
Liverpool,	46 c	46 c	48 R	45 cr	44 R	47 c	43	43 R	43 cr	57 cr	48 R	40 cr	45 cr	51	39 Br	47 R	41 cr	41 Br	44 c	42	48	48 cr	49 c	54 c	54 c	49 c	49 c	47 R	44 R	44	41 c
Holyhead,	51 c	51 R	50 cr	48 cr	45 R	47 c	B	45 cr	46 cr	55 cr	48 R	45 cr	52 R	B	48 cr	48 cr	46 c	47 c	52 c	c	B	51 cr	51 cr	54 c	55 cr	45 c	48 cr	46 R	43 cr	46 c	51 cr
Yarmouth,	44 c	47 c	51 c	50 c	46 c	43 Br	43 B	50 c	41 B	46 cr	49 R	43 c	39 c	48 R	46 c	39 c	36 c	37 B	45 c	48 c	43 B	49 c	48 c	51 c	50 c	43 c	45 B	49 R	49 R	38 B	46 c
Pembroke,	50 c	51 c	50 c	55 R	51 R	46 R	B	47 cr	52 R	55 cr	54 R	45 cr	52 cr	B	42 Br	49 c	43 cr	45 Br	51 c	c	B	51 cr	50 cr	55 c	55 cr	45 c	50 R	50 R	44 Br	46 cr	51 cr
London,	42 c	42 c	50 c	48 R	49 cr	44 Br	42	42 R	40 cr	57 cr	53 R	38 Br	45 c	50	42 cr	36 R	35 cr	36 B	47 c	51	44	45 cr	51 R	54 cr	51 c	34 c	43 c	49 cr	45 cr	35 B	48 c
Dover,	51 c	51 c	55 c	54 R	54 cr	50 Br	...	48 R	46 cr	57 cr	56 c	46 Br	48 c	...	48 Br	44 B	46 cr	41 Br	52 c	...	...	50 cr	52 R	55 cr	53 c	41 c	44 c	51 R	50 R	...	52 c
Portsmouth,	48 c	48 c	53 c	51 R	53 cr	49 cr	47 B	47 cr	47 cr	55 cr	55 c	47 cr	49 c	54 R	46 cr	41 c	45 R	40 cr	49 c	52 c	49 B	49 cr	53 R	53 cr	51 c	45 c	49 c	49 cr	47 R	43 cr	50 c
Weymouth,	45 c	54 c	54 c	52 R	52 R	50 cr	47 c	45 cr	48 R	56 cr	55 c	47 c	52 cr	54 c	43 Br	48 R	46 R	40 Br	49 c	53 c	50 c	52 cr	54 cr	54 c	53 c	45 c	50 R	51 cr	48 R	44 Br	51 c
Plymouth,	48 c	45 c	51 c	52 R	51 R	49 cr	...	45 cr	51 R	55 cr	55 c	51 R	56 cr	...	45 c	48 R	47 cr	44 B	51 c	...	...	52 cr	52 c	55 c	55 B	45 c	50 R	50 R	46 cr	44 c	51 c
Penzance,	50 B	50 cr	51 c	52 cr	51 R	46 cr	47 B	45 cr	51 R	54 cr	54 c	53 cr	53 cr	51	48 cr	48 cr	46 cr	45 cr	53 c	52	50	51 cr	48 c	54 c	53 cr	49 c	52 cr	45 c	45 R	46 cr	51 c
NORWAY and SWEDEN.																															
*Haparanda,	38 c	30 B	37 cr	42 c	40 c	41 cr	42 c	35 c	30 B	34 c	27 c	24 cr	27 c	26 c	13 B	11 Br	29 c	34 cr	23 c	22 cr	—3 B	9 c	—6 B	—11 B	31 cr	32 B	30 B	32 c	32 c	34 c	36 cr
Hernösand,	41 c	30 c	34 c	42 c	46 c	46 c	...	38 B	36 c	39 c	37 c	31 B	32 c	27 c	26 B	29 c	36 c	37 B	39 c	45 B	28 c	22 B	19 c	25 c	35 c	26 c	49 B	30 c	38 c	40 c	40 c
Christiansund,	43	42	43	44	47	47	42	39	44	43	39	36	41	34	34	43	38	39	51	49	44	42	43	43	44	42	28	46	47	39	44
Skudesnos,	50	49	46	46	46	44	45	43	46	45	42	37	43	34	31	43	40	40	49	47	47	44	45	48	49	39	42	43	40	40	47
Mandal,	49	48	46	49	44	45	43	36	45	43	40	33	38	31	30	43	34	35	48	45	47	38	37	43	47	37	39	41	40	39	44
Christiania,	38	44	41	43	45	40	35	32	40	40	39	36	31	36	22	31	35	34	39	38	42	37	35	35	41	34	36	34	35	36	37
Stockholm,	32 B	39 c	43 B	44 B	47 R	48 cr	45 Br	42 R	42 cr	41 cr	39 R	33 B	32 Br	30 B	24 B	32 cr	32 Br	33 c	47 c	46 c	40 cr	40 cr	31 c	38 cr	37 c	38 c	34 c	36 c	39 B	42 c	41 cr
RUSSIA.																															
Archangel,	26 F	35 F	35 F	35 F	32 F	26 F	34 F	32 R	34 B	25 F	36 F	33 R	32 R	24 F	25 R	15 R	4 B	31 R	26 F	22 F	4 B	1 B	—1 B	1 F	28 F	31 F	32 F	32 F	31 F	0 B	18 F
Helsingfors,	31 B	41 B	44 c	37 c	38 B	41 R	45 R	44 B	34 F	44 R	44 c	33 B	28 R	35 c	24 B	27 B	44 c	40 c	44 c	39 B	44 R	44 c	30 R	26 c	25 c	33 F	36 c	38 c	35 B	34 c	33 c
St Petersburg,	36 L	38 B	33 B	31 B	26 B	38 B	37 B	42 B	39 c	37 c	42 c	35 B	28 Br	30 R	28 B	24 B	32 B	33 B	40 R	42 B	36 R	35 c	37 c	35 cr	25 B	36 c	38 c	35 c	28 c	28 c	29 c
Dorpat,	41	39	38	37	40	41	35	36	32	38	37	33	31	29	23	22	30	34	45	44	43	42	34	32	30	39	33	27	21	21	26
Riga,	38 B	34 B	28 B	29 B	31 B	38 c	36 B	40 R	36 c	44 c	47 B	34 c	34 R	34 c	32 R	23 B	29 B	36 Fr	45 c	43 c	41 c	37 B	41 c	37 c	34 B	41 c	38 c	30 B	27 B	28 c	33 B
Libau,	34	37 c	34 B	32 B	40 B	46 R	48 c	47 B	41 B	48 B	48 B	43 B	40 B	35 c	36 Br	36 B	28 c	35 cr	37 c	45 c	45 cr	46 c	44 c	40 B	41 B	42 c	33 c	32 c	33 B	31 B	33 B
Moscow,	21 B	36 c	30	31 c	29 c	32 B	29 c	32 c	36 c	33 c	45 B	35 B	34 R	31 c	25 c	13 B	30 B	34 R	35 c	23 c	40 c	36 c	41 c	40 R	45 c	34 c	31 F	36 R	30 c	12	12 B
Warsaw,	36 B	32 F	34 B	37 B	42 B	55 c	39 c	44 c	41 F	33 R	54 R	39 R	31 B	35 c	25 B	26 B	31 c	37 F	45 c	35 B	34 B	41 c	43 R	41 F	44 c	35 c	26 c	22 B	20 B	27 c	27 c
*Kostroma,	28 c	26 c	32 c	33 B	32 c	33 c	25 B	29 R	34 c	33 c	40 R	38 R	31 B	32 R	26 R	17 c	10 F	23 R	32 R	38 c	40 R	17 R	28 R	34 F	16 c	28 R	34 c	30 R	28 c	16 c	15 B
*Orenbourg,	B	B	B	B	B	B	B	B	20 R	33 B	35 B	33 B	B	32 B	28 R	33 R	24 B	9 B	29 B	B	36 B	30 R	33 R	37 B	37 B	25 B	24 B	29 c	28 B	11 R	2 R
*Slatoust,	18 B	23 c	20 B	14 B	16 B	12 R	8 B	12 B	5 B	19 R	32 R	38 B	32 B	17 B	30 B	26 R	23 R	15 B	29 R	25 R	27 R	10 R	4 R	4 R	7 R	27 c	19 R	23 R	10 R	16 c	—36 B
*Catherinenbourg,	22 c	19 R	9 R	0 B	19 c	11 B	19 c	11 B	7 B	19 R	29 c	36 B	33 B	26 B	28 B	27 R	19 c	10 c	25 c	17 c	20 c	2 R	—3 c	—9 B	3 R	24 c	20 R	23 R	26 c	9 R	22 B
*Kiev,	34 B	27 B	41 B	42 B	18 B	22 B	24 R	46 B	52 F	52 c	46 c	44 B	31 R	34 B	29 B	31 B	27 B	39 c	43 c	41 B	32 B	30 B	28 F	39 B	32 B	...	23 c	26 F	F	25 c	24 c
Lougan,	36 R	35 cr	24 Br	18 B	32 c	32 c	25 B	34 c	42 Br	40 c	45 c	46 B	49 R	31 B	33 c	29 cr	18 B	22 B	34 c	42 F	44 B	41 B	43 c	34 F	40 c	32 c	27 cr	18 F	23 F	23 c	21 c
Nicolaiev,	46 R	37 c	36 c	29 c	26 B	32 B	38 c	47 c	50 B	54 B	40 c	47 B	43 B	36 B	35 B	34 c	34 c	41 c	43 B	46 B	38 B	47 B	48 c	39 c	31 B	32 c	31 c	22 B	27 c	34 F	29 c
Odessa,	35 c	36 c	35 c	29 c	26 B	42 B	48 R	49 c	53 B	54 B	47 F	50 c	39	38 B	32 B	34 R	c	54 B	42 B	44 c	40 B	48 B	51 c	39 B	43 B	30 B	30 c	21 c	26 c	40 R	29 c
DENMARK.																															
Iviglut, Greenland,	21 B	16Br	16 B	15 B	12 B	22 cr	17 B	14 B	9 B	9 B	27 c	40 c	37 B	37 cr	37 c	26 B	24 cr	24 B	37 cr	36 c	28 c	17 B	24 B	19 B	9 B	—1 B	1 B	—1 B	—6 B	—3 B	...
Skagen, or the Skaw,	43 B	43 B	47 B	48 cr	45 B	41 c	44 B	41 B	46 Br	44 B	42 B	36 B	40 B	43 B	34 B	38 c	40 cr	39 c	47 c	44 c	45 B	45 B	41 F	46 F	46 F	39 c	43 c	42 B	42 cr	42 B	46 c
Copenhagen,	44 c	45 c	46 c	44 c	50 R	44 c	45 R	42 B	45 R	44 R	45 B	38 c	41 c	30 B	32 B	34 c	34 c	39 B	43 c	43 c	44 c	46 c	44 c	45 cr	42 c	39 c	37 c	33 B	38 cr	41 R	34 c
*Kiel,	44	44	45	50 R	53 R	51	39 R	39 R	40 R	47 R	43 R	38 R	40 R	40 R	36	34	37 R	38	42	41	48	49 R	49 R	46 R	45	33	28	29	37 R	37 R	41 R
Heligoland (Eng.),	44 c	44 c	43 c	48 R	50 c	46 R	...	...	45 R	51 cr	...	44 R	43 cr	...	39 B	36 B	40 c	41 cr	45 c	...	...	49 c	47 cr	47 cr	47 cr	...	33 c	40 c	...	...	...
PRUSSIA.																															
Königsberg,	52	39	38	42	50	51	42	30	40	42	45	35	35	33	32	30	31	33	42	39	40	45	45	46	44	42	34	32	34	35	36
*Putbus,	46	47	47	51	58 R	51	39	38	38	44 R	41	36 R	38 R	36	33	31	34	36	40 R	37	41 R	46	46 R	43	42 R	35 R	28	26	34	36 R	34 R
*Paderborn,	47	49	51	58 R	60 R	54	42	41	41 R	50 R	46 R	37 R	37	37 R	35	33 R	38 R	33	47	42	50 R	45 R	49 R	53 R	51	32	23	35	38 R	37 R	41 R
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	53 R	54 R	42 c	41 c	45 R	43 R	49 c	36 R	33 R	36 c	30 c	27 c	39 c	37 c	42 B	42 B	43 B	45 R	47 c	47 R	46 R	34 c	24 B	25 B	31 R	37 B	36 c

howing the Temperature (Fahr.) and State of the Sky in the Morning at 110 Places in Europe, from 26th October to 12th November, from 20th to 26th November, from 30th November to 5th December, and 14th to 18th December 1863.

s blue sky, or sky little more than half covered with clouds ; C, a much clouded or overcast sky ; F, fog ; R, rain at the time of observation ; and r, rain sometime during the previous 24 hours.

OCTOBER.						NOVEMBER.																				DECEMBER.									
26.	27.	28.	29.	30.	31.	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	20.	21.	22.	23.	24.	25.	26.	30.	1.	2.	3.	4.	5.	14.	15.	16.	17.	18.
6 c	47 c	44 cr	44 R	40 cr	44 cr	39	43 cr	41 R	51 cr	43 cr	36 cr	44 cr	48	37 c	43 R	41 cr	43 cr	52 cr	...	49	46 cr	43 cr	51 R	54 R	46 c	49 cr	40 cr	...	44 cr	47 R	48 cr	48 c	39 cr	41 cr	44 c
6 R	49 c	44 Br	51 R	42 cr	44 R	33	42 cr	52 R	58 R	48 R	37 cr	52 cr	44	37 Br	43 cr	42 cr	41 cr	54 cr	...	37	49 cr	46 cr	50 cr	53 cr	49 c	47 cr	40 cr	32	34	48	48 cr	48 cr	41 cr	42 cr	45 R
3 c	49 c	43 cr	48	46	45 R	45 R	46 R	53	54 cr	53 R	54 cr	54 cr	48 F	42 cr	48 cr	44 cr	43 Br	54 cr	...	50	50 cr	46 Br	50 c	54 R	49 cr	48	44 Br	41 R	...	49 cr	49 R	48 cr	47 cr	44 cr	43 c
6 c	51 c	48 cr	51 cr	44 cr	46 c	...	43 cr	51 R	55 cr	55 cr	50 cr	53 cr	...	45 c	47 c	43 cr	44 B	55 R	...	...	49 c	50 c	52 R	54 c	50 c	59 c	42 cr	...	46 cr	52 R	52 c	52 c	47 c	44 cr	40 B
7	45	42 R	37 R	39 R	41 R	41 cr	41 R	40 R	42 R	39 R	36 R	36 R	35	34 R	42 R	43 R	42 R	48 R	48 R	47 R	48	47	46	49	44 R	42 R	38 R	36	47 R	44 R	46 R	46 R	34 R	34 R	47 R
4 Br	45 cr	41 cr	41 cr	40 Br	42 B	40 Br	38 Br	41 Br	44 Br	40 R	40 R	36 Br	40 Br	35 cr	43 R	42 R	37 cr	49 B	49 Br	48 R	46 cr	45 B	47 R	46 cr	42 B	44 B	40 cr	36 cr	47 cr	45 cr	48 cr	47 cr	42 R	48 cr	47 cr
9 B	42 cr	45 R	45 cr	38 B	43 B	45 cr	42 B	35 B	44 R	39 R	34 cr	40 cr	43 Br	34 Br	38 Br	37 Br	36 cr	53 R	52 cr	49 R	46 R	45 Br	53 R	54 cr	42 B	49 B	39 R	35 Br	48 cr	43 R	48 cr	48 cr	41 R	40 Br	50 cr
8 Br	41 cr	42 cr	43 cr	40 cr	42 R	35 cr	31 Br	34 cr	45 cr	42 cr	36 Br	41 c	40 cr	36 B	41 Br	39 cr	35 cr	52 cr	51 cr	45 R	43 cr	41 cr	53 R	54 cr	41 B	48 cr	39 R	34 R	43 R	46 R	47 R	47 cr	39 Br	40 cr	47 R
2 c	44 B	42 R	38 cr	38 B	40 c	38 c	32 B	36 B	46 cr	37 cr	33 B	38 cr	34 cr	34 B	39 B	37 B	36 B	48 c	49 cr	44 cr	46 cr	40 B	50 c	50 B	40 B	41 B	37 cr	35 c	42 B	48 c	45 B	46 B	40 Br	39 cr	44 R
4 cr	46 cr	49 cr	42 cr	41 B	38 B	40 B	39 c	38 cr	50 cr	40 c	35 B	36 cr	40 cr	38 B	40 Br	36 B	39 B	44 c	52 cr	46 cr	43 cr	45 cr	48 cr	51 c	43 c	42 cr	42 cr	35 cr	43 c	52 Br	45 B	47 c	40 Br	44 B	42 Br
7 B	48 R	44 cr	40 c	40 R	43 R	41 B	36 Br	42 c	49 cr	42 R	33 B	48 R	39 cr	32 B	42 R	40 R	40 Br	50 c	52 cr	45 R	46 cr	43 B	52 cr	54 cr	37 c	40 R	39 R	35 R	45 c	47 R	49 cr	48 c	41 R	40 Br	46 R
1 cr	46 Br	47 cr	38 c	37 B	40 cr	36 Br	35 c	36 Br	39 cr	40 c	28 B	35 cr	37 cr	30 c	39 Br	34 B	32 B	45 cr	51 cr	43 Br	43 cr	52 c	51 c	51 c	30 c	36 cr	37 cr	33 c	41 cr	48 cr	47 cr	47 c	38 Br	37 B	43 c
4 c	47 R	45 R	35 R	41	40	37	40 R	37 c	51 B	40 c	30 c	33 R	40 R	38 c	41 c	35 c	36 c	40 c	53 c	44	43 c	47 R	46	56 B	39 c	39 c	36 R	33 R	38	51	48	49 R	40 R	41	38
5 c	48 R	48 R	42 c	41 c	46 R	42 B	40 R	45 R	52 R	46 R	31 B	37 R	41 R	39 R	43 R	39 B	38 B	40 B	53 c	46 R	47 c	50 R	54 c	56 R	34 B	40 B	40 R	38 R	45 c	51 R	48 B	47 c	43 R	39 R	42 c
6 B	50 c	49 c	44 Br	43 R	48 c	49 c	49 R	41 B	...	45 c	38 B	39 c	44 B	49 cr	45 c	40 c	40 R	41 c	48 c	49 R	48 cr	50 c	50 c	57 c	46 c	49 c	47 R	44 R	41 cr	52 c	50 c	51 c	46 B	45 c	42 c
6 B	46 B	44 c	42 B	41 R	44 cr	43	46 cr	46 Br	55 cr	44 c	43 B	37 c	44	44 cr	38 R	38 B	40 R	42 c	48	44	46 c	47 c	50 c	51 c	43 c	43 c	47 R	43 R	39 Br	49 c	46 B	45 c	46 B	45 cr	37 c
6 c	46 c	48 R	45 cr	44 R	47 c	43	43 R	43 cr	57 cr	48 R	40 cr	45 cr	51	39 Br	47 R	41 cr	41 Br	44 c	42	48	48 cr	49 c	54 c	54 c	49 c	49 c	47 R	44 R	44	41 c	50 c	50 c	47 cr	45 cr	40 c
1 c	51 R	50 cr	48 cr	45 R	47 c	B	45 cr	46 cr	55 cr	48 R	45 cr	52 R	B	48 cr	48 cr	46 c	47 c	52 c	c	B	51 cr	51 cr	54 c	55 cr	45 c	48 cr	46 R	43 cr	46 c	51 cr	51 c	50 c	46 cr	47 c	43 c
4 c	47 c	51 c	50 c	46 c	43 Br	43 B	50 c	41 B	46 cr	49 R	43 c	39 c	48 R	46 c	39 c	36 c	37 B	45 c	48 c	43 B	49 c	48 c	51 c	50 c	43 c	45 B	49 R	49 R	38 B	46 c	42 B	43 c	46 c	42 R	41 c
0 c	51 c	50 c	55 R	51 R	46 R	B	47 cr	52 R	55 cr	54 R	45 cr	52 cr	B	42 Br	49 c	43 cr	45 Br	51 c	c	B	51 cr	50 cr	55 c	55 cr	45 c	50 R	50 R	44 Br	46 cr	51 cr	51 c	50 R	48 cr	46 c	36 c
2 c	42 c	50 c	48 R	49 cr	44 Br	42	42 R	40 cr	57 cr	53 R	38 Br	45 c	50	42 cr	36 R	35 cr	36 B	47 c	51	44	45 cr	51 R	54 cr	51 c	34 c	43 c	49 cr	45 cr	35 B	48 c	38 c	42 c	47 c	43 c	35 c
1 c	51 c	55 c	54 R	54 cr	50 Br	...	48 R	46 cr	57 cr	56 c	46 Br	48 c	...	48 Br	44 B	46 cr	41 Br	52 c	...	...	50 cr	52 R	55 cr	53 c	41 c	44 c	51 R	50 R	...	52 c	48 B	48 c	54 c	45 cr	38 B
8 c	48 c	53 c	51 R	53 cr	49 cr	47 B	47 cr	47 cr	55 cr	55 c	47 cr	49 c	54 R	46 cr	41 c	45 R	40 cr	49 c	52 c	49 B	49 cr	53 R	53 cr	51 c	45 c	49 c	49 cr	47 R	43 cr	50 c	46 c	48 c	50 cr	45 c	37 c
5 c	54 c	54 c	52 R	52 R	50 cr	47 c	45 cr	48 R	56 cr	55 c	47 c	52 cr	54 c	43 Br	48 R	46 R	40 Br	49 c	53 c	50 c	52 cr	54 cr	54 c	53 c	45 c	50 R	51 cr	48 R	44 Br	51 c	49 c	49 c	51 R	45 cr	34 B
8 c	45 c	51 c	52 R	51 R	49 cr	...	45 cr	51 R	55 cr	55 c	51 R	56 cr	...	45 c	48 R	47 cr	44 B	51 c	...	...	52 cr	52 c	55 c	55 B	45 c	50 R	50 R	46 cr	44 c	51 c	51 c	52 c	50 c	45 B	37 B
0 B	50 cr	51 c	52 cr	51 R	46 cr	47 B	45 cr	51 R	54 cr	54 c	53 cr	53 cr	51	48 cr	48 cr	46 cr	45 cr	53 c	52	50	51 cr	48 c	54 c	53 cr	49 c	52 cr	45 c	45 R	46 cr	51 c	50 c	50 c	49 cr	46 cr	40 B
3 c	30 B	37 cr	42 c	40 c	41 cr	42 c	35 c	30 B	34 c	27 c	24 cr	27 c	26 c	13 B	11 Br	29 c	34 cr	23 c	22 cr	—3 B	9 c	—6 B	—11 B	31 cr	32 B	30 B	32 c	32 c	34 c	36 cr	10 B	7 B	1 c	22 cr	17 c
1 c	30 c	34 c	42 c	46 c	46 c	...	38 B	36 c	39 c	37 c	31 B	32 c	27 c	26 B	29 c	36 c	37 B	39 c	45 B	28 c	22 B	19 c	25 c	35 c	26 c	49 B	30 c	38 c	40 c	40 c	16 c	11 B	29 c	35 c	20 B
3	42	43	44	47	47	42	39	44	43	39	36	41	34	34	43	38	39	51	49	44	42	43	43	44	42	28	46	47	39	44	35	31	36	34	32
	49	46	46	46	44	45	43	46	45	42	37	43	34	31	43	40	40	49	47	47	44	45	48	49	39	42	43	40	40	47	38	42	46	41	35
	48	46	49	44	45	43	36	45	43	40	33	38	31	30	43	34	35	48	45	47	38	37	43	47	37	39	41	40	39	44	34	41	44	37	30
3	44	41	43	45	40	35	32	40	46	39	36	31	36	22	31	35	34	39	38	42	37	35	35	41	34	36	34	35	36	37	18	23	33	36	32
2 B	39 c	43 B	44 B	47 R	48 cr	45 Br	42 R	42 cr	41 cr	39 R	33 B	32 Br	30 B	24 B	32 cr	32 Br	33 c	47 c	46 c	40 cr	40 cr	31 c	38 cr	37 c	38 c	34 c	36 c	39 B	42 c	41 cr	19 B	22 B	32 cr	37 cr	24 B
6 F	35 F	35 F	35 F	32 F	26 F	34 F	32 R	34 B	25 F	36 F	33 R	32 R	24 F	25 R	15 R	4 B	31 R	26 F	22 F	4 B	1 B	—1 B	1 F	28 F	31 F	32 F	32 F	31 F	0 B	18 F	18 F	16 F	3 F	0 F	7 F
1 B	41 B	44 c	37 c	38 B	41 R	45 R	44 B	34 F	44 R	44 c	33 B	28 R	35 c	24 B	27 B	44 c	40 c	44 c	39 B	44 R	44 c	30 R	26 c	25 c	33 F	36 c	38 c	35 B	34 c	33 c	20 B	20 c	29 c	32 c	32 c
5 B	38 B	33 B	31 B	26 B	38 B	37 B	42 B	39 c	37 c	42 c	35 B	28 Br	30 R	28 B	24 B	32 B	33 B	40 R	42 B	36 R	35 c	37 c	35 cr	25 B	36 c	38 c	35 c	28 c	28 c	29 c	22 B	15 F	22 B	19 c	28 c
1	39	38	37	40	41	35	36	32	38	37	33	31	29	23	22	30	34	45	44	43	42	34	32	30	39	33	27	21	21	26	21	23	15	27	26
8 B	34 B	28 B	29 B	31 B	38 c	36 B	40 R	36 c	44 c	47 B	34 c	34 R	34 c	32 R	23 B	29 B	36 Fr	45 c	43 c	41 c	37 B	41 c	37 c	34 B	41 c	38 c	30 B	27 B	28 c	33 B	28 c	30 c	23 B	31 cr	34 c
4	37 c	34 B	32 B	40 B	46 R	48 c	47 B	41 B	48 B	48 B	43 B	40 B	35 c	36 Br	36 B	28 c	35 cr	37 c	45 c	45 cr	46 c	44 c	40 B	41 B	42 c	33 c	32 c	33 B	31 B	33 B	28 c	33 R	31 B	36 R	R
1 B	36 c	30	31 c	29 c	32 B	29 c	32 c	36 c	33 c	45 B	35 B	34 R	31 c	25 c	13 B	30 B	34 R	35 c	23 c	40 c	36 c	41 c	40 R	45 c	34 c	31 F	36 R	30 c	12	12 B	35 R	20	11 c	4 c	5 c
6 B	32 F	34 B	37 B	42 B	55 c	39 c	44 c	41 F	33 R	54 R	39 R	31 B	35 c	25 B	26 B	31 c	37 F	45 c	35 B	34 B	41 c	43 R	41 F	44 c	35 c	26 c	22 B	20 B	27 c	27 c	22 c	26 R	27 F	34 c	35 c
8 c	26 c	32 c	33 B	32 c	33 c	25 B	29 R	34 c	33 c	40 R	38 R	31 B	32 R	26 R	17 c	10 F	23 R	32 R	38 c	40 R	17 R	28 R	34 F	16 c	28 R	34 c	30 R	28 c	16 c	15 B	27 c	22 c	12 c	8 c	16 c
B	B	B	B	B	B	B	B	20 R	33 B	35 B	33 B	B	32 B	28 R	33 R	24 B	9 B	29 B	B	36 B	30 R	33 R	37 B	37 B	25 B	24 B	29 c	28 B	11 R	2 R	13 R	19 B	25 B	17 R	15 R
3 B	23 c	20 B	14 B	16 B	12 R	8 B	12 B	5 B	19 R	32 R	38 B	32 R	17 B	30 B	26 R	23 R	15 B	29 R	25 R	27 R	10 R	4 R	4 R	7 R	27 c	19 R	23 R	10 R	16 c	—36 B	6 c	10 B	12 B	—1 B	3 R
2 c	19 R	9 R	0 B	19 c	11 B	19 c	11 B	7 B	19 R	29 c	36 B	33 B	26 B	28 B	27 R	19 c	10 c	25 c	17 c	20 c	2 R	—3 c	—9 B	3 R	24 c	20 R	23 R	26 c	9 R	22 B	2 P	3 c	8 c	—9 B	—3 c
4 B	27 B	41 B	42 B	18 B	22 B	24 R	46 B	52 F	52 c	46 c	44 B	31 R	34 B	29 B	31 B	27 B	39 c	43 c	41 B	32 B	30 B	28 F	39 B	32 B	...	23 c	26 F	F	25 c	24 c	24 B	20 c	10 B	13 c	21 c
6 R	35 cr	24 Br	18 B	32 c	32 c	25 B	34 c	42 Br	40 c	45 c	46 B	49 R	31 B	33 c	29 cr	18 B	22 B	34 c	42 F	44 B	41 B	43 c	34 F	40 c	32 c	27 cr	18 F	23 F	23 c	21 c	33 cr	23 c	15 B	12 c	9 c
6 R	37 c	36 c	29 c	26 B	32 B	38 c	47 c	50 B	54 B	40 c	47 B	43 B	36 B	35 B	34 c	34 c	41 c	43 B	46 B	38 B	47 B	48 c	39 c	31 B	32 c	31 c	22 B	27 c	34 F	29 c	29 c	25 c	18 c	11 B	11 c
5 c	36 c	35 c	29 c	26 B	42 B	48 R	49 c	53 B	54 B	47 F	50 c	39	38 B	32 B	34 R	c	54 B	42 B	44 c	40 B	48 B	51 c	39 B	43 B	30 B	30 c	21 c	26 c	40 R	29 c	26 B	23 B	17 c	17 B	25 c
B	16 Br	16 B	15 B	12 B	22 cr	17 B	14 B	9 B	9 B	27 c	40 c	37 B	37 cr	37 c	26 B	24 cr	24 B	37 cr	36 c	28 c	17 B	24 B	19 B	9 B	—1 B	1 B	—1 B	—6 B	—3 B	...	4 B	16 c	37 cr	32 cr	31 cr
B	43 B	47 B	48 cr	45 B	41 c	44 B	41 B	46 Br	44 B	42 B	36 B	40 B	43 B	34 B	38 c	40 cr	39 c	47 c	44 c	45 B	45 B	41 F	46 F	46 F	39 c	43 c	42 B	42 cr	42 B	46 c	35 B	39 B	47 cr	37 B	36 B
c	45 c	46 c	44 c	50 R	44 c	45 R	42 B	45 R	44 R	45 B	38 c	41 c	30 B	32 B	34 c	34 c	39 B	43 c	43 c	44 c	46 c	44 c	45 cr	42 c	39 c	37 c	33 B	38 cr	41 R	34 c	31 B	28 c	43 c	37 B	32 B
	44	45	50 R	53 R	51	39 R	39 R	40 R	47 R	43 R	38 R	40 R	40 R	36	34	37 R	38	42	41	48	49 R	49 R	46 R	45	33	28	29	37 R	37 R	41 R	44	46	44	40	37
c	44 c	43 c	48 R	50 c	46 R	...	...	45 R	51 cr	...	44 R	43 cr	...	39 B	36 B	40 c	41 cr	45 c	...	...	49 c	47 cr	47 cr	47 cr	...	33 c	40 c	...	...	...	45 c	46 c	44 c	42 R	38 B
	39	38	42	50	51	42	30	40	42	45	35	35	33	32	30	31	33	42	39	40	45	45	46	44	42	34	32	34	35	36	32	34	35	38	38
	47	47	51	58 R	51	39	38	38	44 R	41	36 R	38 R	36	33	31	34	36	40 R	37	41 R	46	46 R	43	42 R	35 R	28	26	34	36 R	34 R	37 R	42	42 R	37 R	33
	49	51	58 R	60 R	54	42	41	41 R	50 R	46 R	37 R	37	37 R	35	33 R	38 R	33	47	42	50 R	45 R	49 R	53 R	51	32	23	35	38 R	37 R	41 R	44 R	44	43 R	39 R	36
	[illegible]	[illegible]	[illegible]	52 R	54 R	42 c	41 c	45 R	43 R	49 c	36 R	33 R	36 c	30 c	27 c	39 c	37 c	42 B	42 B	43 B	45 R	47 c	17 R	46 R	34 c	24 B	25 B	31 R	37 B	36 c	33 B	41 R	41 c	39 c	36 c

TABLE III.—Showing for 69 places in Europe the Deviations from the Mean Temperature (Fahr.) of each day from 26th October to 12th November, from 20th to 26th November, from 30th November to 5th December, and from 14th to 18th December 1863.

NOTE.—The Temperatures marked with a minus sign are under the Mean, the others are above the Mean.

	OCTOBER.						NOVEMBER.																				DECEMBER.									
	26.	27.	28.	29.	30.	31.	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	20.	21.	22.	23.	24.	25.	26.	30.	1.	2.	3.	4.	5.	14.	15.	16.	17.	18.
IRELAND.																																				
Greencastle,	2°	3°	0°	0°	−4°	−0°	−5°	−1°	−2°	7°	−1°	−8°	0°	4°	−7°	−1°	−3°	−1°	11°	...°	8°	5°	−1°	10°	13°	5°	8°	−1°	...°	3°	6°	7°	7°	−1°	1°	4°
Valencia,	6	2	−4	1	−1	−2	−2	−1	6	7	6	7	7	2	−4	2	−2	−3	9	...	5	5	2	6	10	5	4	1	−4	...	5	5	3	2	0	−1
SCOTLAND.																																				
Bressay,	3	1	−1	−6	−4	−2	−2	−1	−2	0	−3	−6	−5	−6	−7	1	2	1	8	8	7	8	7	6	9	4	2	−2	−4	7	4	5	5	3	−7	6
Stornoway,	1	2	−1	−1	−2	−0	−2	−4	−1	2	−2	−2	−6	−2	−7	2	−0	−5	9	9	8	6	5	7	6	2	4	−2	−4	7	5	7	6	1	7	6
Elgin,	−2	−0	−5	−5	−5	−3	−5	−11	−7	3	−6	−10	−4	−8	−8	−3	−5	−6	8	9	4	6	0	10	10	1	2	−3	−5	2	8	5	6	0	−1	4
Barry,	−1	1	5	−3	−4	−0	−4	−4	−5	7	−3	−8	−6	−2	−4	−2	−6	−3	4	12	6	3	5	8	11	2	3	3	−4	4	13	5	7	0	5	3
ENGLAND.																																				
Silloth,	1	2	3	−3	−4	1	−3	−3	2	9	3	−8	−5	−1	−3	1	−3	−4	1	14	7	8	11	15	17	−5	1	1	−1	6	12	9	8	4	0	3
Shields,	1	5	5	0	−1	4	5	7	−1	...	3	−4	−3	2	7	3	−2	−2	2	9	10	9	11	11	18	7	11	9	6	3	14	12	13	8	7	4
Holyhead,	3	3	3	1	0	2	...	1	2	11	4	1	8	...	4	4	2	3	11	...	...	10	10	13	14	5	8	6	3	6	11	12	11	7	8	4
Pembroke,	2	3	2	7	3	−3	...	2	7	10	9	0	7	...	−2	5	−1	−1	9	...	...	9	8	13	14	4	9	9	3	5	10	12	11	9	7	−3
Yarmouth,	−3	−0	5	4	0	−3	−3	5	−3	2	5	−1	−5	4	2	−5	−6	−6	4	7	2	8	7	10	9	3	5	9	9	−2	6	3	4	7	3	2
London,	−6	−6	3	1	3	−2	−2	−2	−4	13	9	−6	1	6	−2	−8	−9	−7	6	10	3	4	10	13	10	−6	3	9	5	−5	8	−1	3	8	4	−4
Weymouth,	−4	5	5	3	3	1	−2	−2	1	9	8	0	7	9	−2	3	1	−4	7	11	8	10	12	12	11	4	9	10	7	3	10	9	9	11	5	−6
Penzance,	0	0	2	3	2	−3	−2	−2	4	7	7	6	7	5	1	1	−1	−3	10	9	7	8	5	11	10	7	10	3	3	4	9	9	9	8	5	−1
NORWAY AND SWEDEN.																																				
Haparanda,	9	1	2	13	12	13	14	8	3	8	1	−1	3	2	−9	−9	8	14	5	6	−19	−7	−21	−26	17	18	17	18	18	21	23	0	−2	−8	14	9
Hernösand,	6	−5	0	8	13	13	...	6	4	8	6	0	1	−3	−4	−1	6	8	13	19	2	−4	−6	0	10	3	16	7	15	17	18	−4	−9	9	15	0
Christiansund,	3	3	4	6	9	9	4	1	6	5	2	−1	4	−3	−2	7	2	4	18	16	11	9	10	11	12	11	−3	13	16	9	14	7	3	8	6	4
Skudesnos,	10	9	6	6	6	4	6	4	7	6	4	−1	5	−4	−7	6	3	3	12	10	10	7	8	12	13	3	6	7	5	5	12	4	8	12	7	1
Mandal,	9	8	6	10	5	6	5	−2	7	5	3	−4	1	−6	−6	7	−2	0	13	10	12	3	2	9	13	3	5	7	6	5	11	2	9	12	5	−2
Christiania,	−1	6	3	6	8	3	−1	−4	4	4	4	1	−4	1	−13	−3	1	1	8	7	11	6	4	4	11	5	7	5	6	8	11	−8	−3	7	10	−4
Stockholm,	−11	−4	1	2	6	7	4	1	2	1	−1	−7	−7	−9	−15	−8	−6	−5	13	12	6	6	−2	5	4	9	5	7	11	14	14	−5	−2	8	14	1
SSIA.																																				
Archangel,	−5	4	5	5	4	−1	6	5	6	−2	8	6	5	−2	−1	−10	−21	6	4	0	−17	−19	−20	−17	10	13	16	16	7	−14	3	3	2	−11	−12	−5
St Petersburg,	−1	1	−3	−4	−9	3	2	8	6	4	9	2	−3	−1	−3	−7	−1	0	14	16	11	18	12	10	0	11	10	10	3	3	4	−2	−8	−2	−3	6
Dorpat,	−0	−3	−4	−4	−2	−1	4	5	1	8	7	2	−0	−2	−7	−9	−1	4	14	13	12	12	4	1	−1	9	8	2	−4	−4	2	−4	−2	−9	2	2
Moscow,	−18	−3	−7	−6	−7	−4	−7	−3	2	−1	11	2	1	−0	−6	−17	1	6	10	1	17	13	18	17	21	11	8	14	8	−10	−10	16	1	−6	−11	−7
arsaw,	−3	−13	−11	−8	−3	10	6	7	9	0	19	6	−2	1	−10	−7	−2	4	12	1	1	8	10	10	11	2	0	−8	−5	1	1	−3	1	−3	8	10
iev,	−8	−18	−14	1	−22	−18	−9	−4	14	10	4	3	−6	−3	−8	−5	−7	4	11	10	0	−1	−2	7	2	...	−6	−3	...	−1	−2	...	−3	−13	−10	−2
icolaiev,	−4	−11	−10	−16	−20	−12	−6	3	7	11	−3	5	2	3	−6	−7	−6	1	6	9	2	10	12	3	−4	−3	−3	−9	−6	3	−2	−1	−4	−11	−17	−17
MARK.																																				
penhagen,	−2	−1	2	0	6	0	2	0	3	2	3	−3	−0	−10	−8	−5	−4	2	7	7	8	10	8	10	7	3	1	−3	2	6	−0	−3	−6	9	4	−1
el,	−5	−5	−3	1	4	1	1	0	1	9	5	0	1	1	−3	−4	−1	0	4	3	9	11	10	8	6	−5	−5	−3	4	4	7	10	13	11	7	4
SIA.																																				
igsberg,	−4	−7	−8	−4	4	5	5	2	4	6	8	−1	−1	−4	−4	−6	−6	−4	5	2	3	9	8	9	7	6	−0	−2	−0	−1	−2	−2	−0	1	4	4
bus,	−3	−2	−2	3	9	3	2	2	1	7	4	−1	1	−1	−4	−6	−3	−0	3	0	4	9	9	6	4	−2	−4	−6	2	4	2	6	10	10	6	1
erborn,	−4	−2	−0	8	10	3	3	2	2	11	7	−2	−2	−2	−4	−7	−2	−7	8	3	10	6	10	14	12	−8	−6	1	4	5	7	10	9	8	5	1
slau,	−2	−7	−2	−2	5	6	5	4	8	6	12	−1	−4	−2	−7	−10	2	1	5	5	6	8	10	10	8	−3	−6	−5	1	6	6	3	11	10	9	6
IAN EMPIRE.																																				
gue,	−5	−8	−5	−0	5	7	2	2	3	6	13	3	−3	−1	−4	−6	0	0	2	3	3	6	7	9	7	−4	−5	−8	5	5	2	8	13	9	7	5
ow,	−3	−5	−10	−7	1	7	4	2	5	4	10	1	−4	−3	−7	−8	2	3	4	0	1	2	7	8	8	−2	−6	−9	−5	2	−1	−2	2	2	2	6
na,	−1	−5	−7	−6	1	5	3	3	5	−5	13	11	−6	−0	−5	−7	−1	2	3	3	2	0	1	3	4	−9	−6	−6	−4	−1	5	8	11	8	2	5
smunster,	−7	−5	−6	−2	−0	6	4	3	2	6	13	7	−0	−0	−2	−0	2	3	3	−0	−3	−2	4	5	10	−7	−7	−6	1	4	3	10	12	6	3	6
enfurt,	−10	−10	−11	−8	−1	7	10	−3	−6	−6	−4	−4	−5	−6	−7	−9	−8	−2	3	−5	−5	−5	−1	0	2	−3	−3	−5	1	4	3	13	14	10	10	14
STATES.																																				
ig,	−10	−7	−10	2	14	4	0	−2	−2	7	6	−5	−7	−5	−8	−10	−4	−4	2	−2	5	3	7	5	6	−8	−8	−6	3	3	3	7	9	6	5	−1
ch,	−7	−7	−2	8	9	7	1	2	1	6	11	4	1	7	3	−2	−0	−3	−2	−2	6	12	12	12	11	−6	−6	−0	7	1	1	10	10	5	4	5
TZERLAND.																																				
Aarau,	−4	−2	2	7	11	4	−1	−3	−1	4	8	4	4	8	4	−0	−2	−3	−4	−3	0	9	11	7	6	−3	−2	−3	5	−1	−3	7	1	0	1	3
Geneva,	−2	−1	1	6	11	9	−3	−2	−3	0	1	−2	1	4	2	−1	−4	−4	−4	−7	3	11	7	4	4	−3	−4	4	8	−2	−3	0	0	0	1	3
NETHERLANDS.																																				
Groningen,	−11	−10	−6	0	2	2	1	−3	2	10	4	1	2	−1	−5	−11	−2	−7	−2	−1	7	7	7	12	6	−9	−14	−0	1	7	4	12	10	11	8	3
Helder,	−11	−4	−5	0	3	2	2	−3	3	8	5	0	1	−2	−3	−15	−1	−4	−1	−7	6	7	6	9	6	−9	−12	−3	1	2	7	9	8	10	7	5
BELGIUM.																																				
Brussels,	−12	−14	−0	1	2	5	−2	−0	−1	10	10	4	−8	3	−5	−13	−5	−9	−0	1	5	4	6	13	11	−13	−10	5	5	4	4	5	3	4	2	−1
FRANCE.																																				
Havre,	−1	−1	1	5	2	2	−2	2	2	8	5	6	5	8	2	−3	−2	−1	0	4	6	4	8	8	5	−7	−1	−9	8	3	6	10	8	−5	11	5
Cherbourg,	...	4	1	2	5	2	−2	−2	0	9	7	2	7	6	4	−4	−3	3	0	4	3	1	8	6	5	−4	2	6	4	3	7	8	7	8	6	1
Brest,	−4	−7	−1	3	1	−3	−7	−6	0	5	4	3	2	3	−3	−4	−1	−5	−0	7	−1	1	7	7	2	−5	6	3	4	−1	6	5	3	5	1	−2
L'Orient,	−3	−10	−7	3	3	−2	−8	−6	3	4	2	2	3	2	−5	−13	−4	−8	−4	5	−5	−2	6	4	0	0	6	4	9	−8	4	3	−6	−3	−5	−3
Paris,	−9	−5	−4	−1	−1	−3	−3	−2	−3	8	8	5	0	5	−5	−7	−9	−4	−9	−7	5	4	5	6	3	−10	−8	6	9	−3	−2	2	3	1	1	−1
Strasbourg,	−10	−7	−1	4	9	5	3	0	4	6	11	9	1	8	8	0	2	−1	−0	1	...	16	13	14	5	−2	3	2	12	9	7	...	10	3	7	10
Rochfort,	−5	−8	1	−1	2	1	−3	−3	−5	1	0	0	...	...	−4	−9	−4	...	...	−2	2	5	2	−1	[illegible]	[illegible]										

AND SWEDEN.																																				
Haparanda,	9	1	2	13	12	13	14	8	3	8	1	−1	3	2	−9	−9	8	14	5	6	−19	−7	−21	−20	17	10	17	18	18	21	20	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
Hernösand,	6	−5	0	8	13	13	...	6	4	8	6	0	1	−3	−4	−1	6	8	13	19	2	−4	−6	0	10	3	16	7	15	17	18	−4	−9	9	15	0
Christiansund,	3	3	4	6	9	9	4	1	6	5	2	−1	4	−3	−2	7	2	4	18	16	11	9	10	11	12	11	−3	13	16	9	14	7	3	8	6	4
Skudesnos,	10	9	6	6	6	4	6	4	7	6	4	−1	5	−4	−7	6	3	3	12	10	10	7	8	12	13	3	6	7	5	5	12	4	8	12	7	1
Mandal,	9	8	6	10	5	6	5	−2	7	5	3	−4	1	−6	−6	7	−2	0	13	10	12	3	2	9	13	3	5	7	6	5	11	2	9	12	5	−2
Christiania,	−1	6	3	6	8	3	−1	−4	4	4	4	1	−4	1	−13	−3	1	1	8	7	11	6	4	4	11	5	7	5	6	8	11	−8	−3	7	10	−4
Stockholm,	−11	−4	1	2	6	7	4	1	2	1	−1	−7	−7	−9	−15	−8	−6	−5	13	12	6	6	−2	5	4	9	5	7	11	14	14	−5	−2	8	14	1
SSIA.																																				
Archangel,	−5	4	5	5	4	−1	6	5	6	−2	8	6	5	−2	−1	−10	−21	6	4	0	−17	−19	−20	−17	10	13	16	16	7	−14	3	3	2	−11	−12	−5
St Petersburg,	−1	1	−3	−4	−9	3	2	8	6	4	9	2	−3	−1	−3	−7	−1	0	14	16	11	18	12	10	0	11	10	10	3	3	4	−2	−8	−2	−3	6
Dorpat,	−0	−3	−4	−4	−2	−1	4	5	1	8	7	2	−0	−2	−7	−9	−1	4	14	13	12	12	4	1	−1	9	8	2	−4	−4	2	−4	−2	−9	2	2
Moscow,	−18	−3	−7	−6	−7	−4	−7	−3	2	−1	11	2	1	−0	−6	−17	1	6	10	1	17	13	18	17	21	11	8	14	8	−10	−10	16	1	−6	−11	−7
Warsaw,	−3	−13	−11	−8	−3	10	6	7	9	0	19	6	−2	1	−10	−7	−2	4	12	1	1	8	10	10	11	2	0	−8	−5	1	1	−3	1	−3	8	10
Kiev,	−8	−18	−14	1	−22	−18	−9	−4	14	10	4	3	−6	−3	−8	−5	−7	4	11	10	0	−1	−2	7	2	...	−6	−3	...	−1	−2	...	−3	−13	−10	−2
Nicolaiev,	−4	−11	−10	−16	−20	−12	−6	3	7	11	−3	5	2	3	−6	−7	−6	1	6	9	2	10	12	3	−4	−3	−3	−9	−6	3	−2	−1	−4	−11	−17	−17
NMARK.																																				
Copenhagen,	−2	−1	2	0	6	0	2	0	3	2	3	−3	−0	−10	−8	−5	−4	2	7	7	8	10	8	10	7	3	1	−3	2	6	−0	−3	−6	9	4	−1
Kiel,	−5	−5	−3	1	4	1	1	0	1	9	5	0	1	1	−3	−4	−1	0	4	3	9	11	10	8	6	−5	−5	−3	4	4	7	10	13	11	7	4
SSIA.																																				
Königsberg,	−4	−7	−8	−4	4	5	5	2	4	6	8	−1	−1	−4	−4	−6	−6	−4	5	2	3	9	8	9	7	6	−0	−2	−0	−1	−2	−2	−0	1	4	4
Putbus,	−3	−2	−2	3	9	3	2	2	1	7	4	−1	1	−1	−4	−6	−3	−0	3	0	4	9	9	6	4	−2	−4	−6	2	4	2	6	10	10	6	1
Paderborn,	−4	−2	−0	8	10	3	3	2	2	11	7	−2	−2	−2	−4	−7	−2	−7	8	3	10	6	10	14	12	−8	−6	1	4	5	7	10	9	8	5	1
Breslau,	−2	−7	−2	−2	5	6	5	4	8	6	12	−1	−4	−2	−7	−10	2	1	5	5	6	8	10	10	8	−3	−6	−5	1	6	6	3	11	10	9	6
TRIAN EMPIRE.																																				
ague,	−5	−8	−5	−0	5	7	2	2	3	6	13	3	−3	−1	−4	−6	0	0	2	3	3	6	7	9	7	−4	−5	−8	5	5	2	8	13	9	7	5
acow,	−3	−5	−10	−7	1	7	4	2	5	4	10	1	−4	−3	−7	−8	2	3	4	0	1	2	7	8	8	−2	−6	−9	−5	2	−1	−2	2	2	2	6
enna,	−1	−5	−7	−6	1	5	3	3	5	−5	13	11	−6	−0	−5	−7	−1	2	3	3	2	0	1	3	4	−9	−6	−6	−4	−1	5	8	11	8	2	5
emsmunster,	−7	−5	−6	−2	−0	6	4	3	2	6	13	7	−0	−0	−2	−0	2	3	3	−0	−3	−2	4	5	10	−7	−7	−6	1	4	3	10	12	6	3	6
agenfurt,	−10	−10	−11	−8	−1	7	10	−3	−6	−6	−4	−4	−5	−6	−7	−9	−8	−2	3	−5	−5	−5	−1	0	2	−3	−3	−5	1	4	3	13	14	10	10	14
IAN STATES.																																				
ipzig,	−10	−7	−10	2	14	4	0	−2	−2	7	6	−5	−7	−5	−8	−10	−4	−4	2	−2	5	3	7	5	6	−8	−8	−6	3	3	3	7	9	6	5	−1
nich,	−7	−7	−2	8	9	7	1	2	1	6	11	4	1	7	3	−2	−0	−3	−2	−2	6	12	12	12	11	−6	−6	−0	7	1	1	10	10	5	4	5
ITZERLAND.																																				
Aarau,	−4	−2	2	7	11	4	−1	−3	−1	4	8	4	4	8	4	−0	−2	−3	−4	−3	0	9	11	7	6	−3	−2	−3	5	−1	−3	7	1	0	1	3
Geneva,	−2	−1	1	6	11	9	−3	−2	−3	0	1	−2	1	4	2	−1	−4	−4	−4	−7	3	11	7	4	4	−3	−4	4	8	−2	−3	0	0	0	1	3
NETHERLANDS.																																				
Groningen,	−11	−10	−6	0	2	2	1	−3	2	10	4	1	2	−1	−5	−11	−2	−7	−2	−1	7	7	7	12	6	−9	−14	−0	1	7	4	12	10	11	8	3
Helder,	−11	−4	−5	0	3	2	2	−3	3	8	5	0	1	−2	−3	−15	−1	−4	−1	−7	6	7	6	9	6	−9	−12	−3	1	2	7	9	8	10	7	5
BELGIUM.																																				
Brussels,	−12	−14	−0	1	2	5	−2	−0	−1	10	10	4	−8	3	−5	−13	−5	−9	−0	1	5	4	6	13	11	−13	−10	5	5	4	4	5	3	4	2	−1
FRANCE.																																				
Havre,	−1	−1	1	5	2	2	−2	2	2	8	5	6	5	8	2	−3	−2	−1	0	4	6	4	8	8	5	−7	−1	−9	8	3	6	10	8	−5	11	5
Cherbourg,	...	4	1	2	5	2	−2	−2	0	9	7	2	7	6	4	−4	−3	3	0	4	3	1	8	6	5	−4	2	6	4	3	7	8	7	8	6	1
Brest,	−4	−7	−1	3	1	−3	−7	−6	0	5	4	3	2	3	−3	−4	−1	−5	−0	7	−1	1	7	7	2	−5	6	3	4	−1	6	5	3	5	1	−2
L'Orient,	−3	−10	−7	3	3	−2	−8	−6	3	4	2	2	3	2	−5	−13	−4	−8	−4	5	−5	−2	6	4	0	0	6	4	9	−8	4	3	−6	−3	−5	−3
Paris,	−9	−5	−4	−1	−1	−3	−3	−2	−3	8	8	5	0	5	−5	−7	−9	−4	−9	−7	5	4	5	6	3	−10	−8	6	9	−3	−2	2	3	1	1	−1
Strasbourg,	−10	−7	−1	4	9	5	3	0	4	6	11	9	1	8	8	0	2	−1	−0	1	...	16	13	14	5	−2	3	2	12	9	7	...	10	3	7	10
Rochfort,	−5	−8	1	−1	2	1	−3	−3	−5	1	0	0	...	...	−4	−9	−4	...	...	−2	2	5	2	−1	−3	−2	−1	8	12	−3	−7	2	5	5	5	5
Bayonne,	2	2	6	4	...	8	0	0	0	2	...	−2	−0	1	−9	−2	−0	−2	−4	3	3	3	3	1	0	−1	5	8	...	5	−2	5	5	3	6	4
Montauban,	−5	−5	0	6	2	5	2	−6	−3	−1	−1	−5	−4	2	−2	−5	−7	−6	−9	−5	3	3	3	1	2	−1	3	7	9	−2	−6	3	−0	−3	−0	−1
Marseilles,	5	6	7	7	8	8	8	−6	−3	1	−2	−4	−5	3	3	−1	−6	−6	−2	−2	...	5	8	9	5	−2	1	5	2	−1	−7	−1	1	6	1	−3
SPAIN AND PORTUGAL.																																				
Oporto,	6	8	1	2	...	5	−2	1	−4	−3	4	3	1	8	−1	−5	...	−0	7	9	9	4	6	...	4	5	...	6	6	...	−1	−5	−3	−11	−5	−9
Lisbon,	2	0	6	3	0	3	−2	1	−3	−4	−2	1	0	3	2	−2	−2	−0	−0	3	0	0	7	7	2	1	0	5	6	...	−5	−4	−9	−5	−0	−5
Barcelona,	4	4	7	8	6	7	6	0	0	0	0	−1	2	2	2	−1	−3	−3	−2	−1	3	2	4	0	5	8	4	7	0	3	0	1	1	1	−3	−4
Alicante,	7	8	3	4	3	4	3	2	...	−3	−4	−4	...	4	3	0	−7	−5	−3	−0	−1	4	2	...	−7	7	6	2	5	6	−6	−5	−6	−7	−7	−8
San Fernando,	3	4	2	2	2	0	2	−5	−5	−9	−2	−5	−4	2	−4	1	−8	−2	−2	2	7	12	5	−2	5	7	8	5	9	2	−7	−7	−8	−7	−8	2
Palma (Majorca),	7	7	...	7	6	8	11	5	0	2	...	0	1	6	4	−3	3	−2	−2	−1	0	1	4	...	3	3	0	−1	5	1	−10	−3	−3	1	1	−4
AFRICA.																																				
Algiers,	3	7	8	4	...	...	...	2	−2	−1	1	2	−0	−2	−0	−3	−5	−11	1	0	0	0	4	...	2	−2	3	10	9	6	0	1	1	1	−3	−6
ITALY.																																				
Leghorn,	3	−2	−3	−3	8	10	7	7	4	−4	−4	−4	−2	−5	5	−2	−4	−5	−5	−9	−6	−2	−0	−2	...	−4	−4	3	−6	...	−0	2	2	0	5	2
Rome,	−5	−8	−12	−4	−1	2	4	5	3	−6	−9	−8	−11	−11	1	0	1	−5	−1	−11	−13	−14	−4	4	−5	−4	−3	−8	−4	−2	−4	−10	−4	−14	−11	2
Naples,	−5	−7	−8	−5	−3	−6	1	6	2	−5	−6	−4	−2	−0	1	1	0	−2	−8	−5	...	−4	−3	−2	−2	−5	...	−5	−3	2	−5	−3	−0	−2	−1	−0
Palermo (Sicily),	−0	−1	−1	−2	2	1	4	4	1	−0	−0	−1	−0	−1	0	2	0	−3	2	0	−2	−2	−5	...	−2	−7	−5	−5	−2	2	...	−0	−1	−1	−1	−0
TURKEY.																																				
Constantinople,	...	−14	−14	−14	−6	−0	−4	0	4	4	5	1	−1	1	−1	1	9	7	3	3	4	5	4	2	2	−3	3	0	0	3	1	−0	−4	−2	−3	7
GREECE.																																				
Athens,	−6	−8	−9	−9	−3	−2	−0	2	4	4	3	2	2	1	4	6	7	4	0	0	−1	3	4	1	1	−5	−8	−10	−8	−4	1	−6	−7	−0	3	6

TABLE IV.—Showing the Amount of the Rainfall each Day, in English inches, at 56 places in Europe, from 26th October to 12th November, from 20th to 26th November, from 30th November to 5th December, and from 14th to 18th December 1863.

	OCTOBER.						NOVEMBER.																				DECEMBER.									
	26.	27.	28.	29.	30.	31.	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	20	21.	22.	23.	24.	25.	26.	30.	1.	2.	3.	4.	5.	14.	15.	16.	17.	18.
Ireland.																																				
Greencastle,	...	...	·50	·12	·40	·43	...	·30	·13	·15	·18	·08	·43	...	1·32	·40	·36	.27	...	...	...	·66	·19	·12	·12	...	·10	·50	...	·18	·30	...	...	·19	·29	...
Galway,	·17	·17	·50	·18	·35	·60	...	·80	·61	·65	·53	·11	·25	...	·17	·28	·40	·16	...	...	...	·62	·35	·45	·10	·07	·50	·52	...	...	...	·13	·13	·25	·27	·25
Valencia,	·14	·14	·52	...	...	...	...	...	...	·35	...	·93	·57	...	·45	·24	·30	·15	...	...	...	·52	·35	·60	·15	...	...	1·27	...	...	·35	...	·20	·22	·09	...
Queenstown,	·09	·09	·42	·25	·37	...	...	·24	·30	·16	·36	·16	·20	...	...	...	·61	...	...	...	...	...	...	·30	...	...	...	·42	...	·22	·16	...	...	...	·07	...
Scotland.																																				
East Yell,	...	...	·37	·92	·22	·19	·07	·10	·24	·10	·10	·26	·04	...	·14	·20	·36	·30	·10	·20	·03	...	...	...	...	·05	·35	·50	...	·20	·30	·56	·50	·40	·07	·18
Sandwick,	·11	·05	·08	·13	·82	·02	·06	·02	·03	·34	·13	·08	·11	·07	·01	·29	·41	·25	...	·07	·11	·01	...	·03	·04	...	...	·10	·32	·05	·22	·26	·03	·24	·11	·13
Stornoway,	...	·03	·42	·50	...	...	·18	...	...	·26	·23	·10	·05	·20	·06	·39	.25	·07	·08	·10	·93	·03	·10	...	·11	...	...	·43	·03	·17	·97	·03	·75	·06	·10	·03
Portree,	·47	·43	·10	·04	1·21	1·09	·10	·16	·05	1·00	·11	·04	...	·58	...	1·10	·66	·16	·28	·80	1·02	·10	·20	1·12	·20	...	·80	·64	·38	·38	12·48	·32	·48	1·20	·28	·32
New Pitsligo,	·16	·06	·83	·11	...	·01	...	·06	·08	·12	·10	...	·26	·29	...	·10	...	...	...	·05	·41	·08	...	...	...	...	·07	·45	·22	...	...	...	...	·12	·14	·02
Barry,	·06	·01	·46	·13	·02	·01	...	·01	·10	·06	...	...	·01	·48	...	·16	...	...	...	·12	·41	·02	·02	...	·02	...	·10	·61	·64	...	·10	...	...	·02	...	·02
Inveresk,	·01	·30	·23	·40	·09	...	...	·03	...	...	·01	...	·24	·50	·60	...	...	...	...	·45	...	...	·06	...	...	...	·10	·36	·67	...	·26	...	·15	...	...	...
South Cairn,	·30	·55	·20	·30	·30	·35	·10	...	·35	·15	...	·25	·75	·20	·35	·25	·35	·15	...	·30	·20	...	·10	·10	·10	...	·30	·98	·35	·30	·20	...	...	·15	...	...
England.																																				
Silloth,	...	·21	1·25	...	·38	·38	...	·11	·08	·35	·12	...	·17	1·25	·05	·16	...	...	...	...	·64	...	·41	...	·05	...	...	·30	·29	...	·10	...	...	·05	·09	...
Shields,	...	·20	...	...	...	...	...	...	...	...	...	...	...	...	2·80	...	...	·25	...	...	·36	·43	...	...	...	...	...	·32	·70	·40	...	...	...	...	...	...
Scarborough,	...	...	...	...	...	·52	...	·70	1·40	·13	...	...	...	...	1·06	...	...	...	...	...	...	...	...	...	...	...	...	·60	...	1·02	...	...	...	...	·17	...
Liverpool,	...	...	·04	.03	·34	·68	...	1·02	·20	·26	·15	·15	·03	...	·59	·09	·31	·17	...	·20	·30	·23	·02	...	...	...	·01	·08	·30	·05	...	...	...	·06	·12	...
Holyhead,	...	·20	1·20	·15	·50	·12	·15	·50	·12	·30	·45	·18	·45	·03	·35	·35	...	...	...	...	...	·50	·35	...	·20	...	·09	·30	·42	...	·15	...	...	·07	...	...
Pembroke,	...	...	...	·46	·33	·45	...	·68	·35	·43	·23	·22	·18	...	·34	...	1·07	·13	·12	...	...	·75	·20	...	·30	...	·15	·50	·53	·11	·10	...	...	·13	...	...
London,	...	...	...	·11	·16	·33	...	·52	·11	·07	·18	·06	...	...	·42	...	·10	...	...	...	...	·16	·14	·09	...	...	...	·21	·63	...	·02	...	...	...	...	...
Dover,	...	...	...	·35	·35	·35	...	1·00	·50	·60	...	·25	...	...	1·00	...	·25	·50	...	...	...	·06	·07	·09	...	...	...	·50	1·02	...	...	...	...	...	·07	...
Portsmouth,	...	...	...	·29	·19	·43	...	·47	·10	·21	...	·07	...	...	·09	...	·28	·33	...	...	...	·25	·50	·19	...	...	...	·70	·60	...	·08	...	...	·14	...	...
Weymouth,	...	...	...	·35	·20	·32	·37	·42	·07	·18	...	...	·07	·02	·08	...	·50	·33	...	·18	·03	·26	·22	·17	...	...	·07	·69	·87	...	·07	...	...	...	·07	...
Plymouth,	...	...	...	...	·17	·80	...	·50	...	·60	...	...	·13	...	...	...	·60	...	...	...	...	·25	...	...	...	...	·21	·45	·25	...	...	...	...	...	...	...
Penzance,	...	...	·15	·24	·27	·52	...	·80	·27	·65	...	·25	·25	...	·60	·15	·55	·20	...	...	...	·45	...	...	·20	...	·15	·85	·55	·23	...	...	...	·15	·15	...
France.																																				
Brest,	...	...	...	·04	·24	·12	...	2·05	·71	·32	...	...	...	...	·75	...	1·18	·32	...	...	...	·51	·20	·04	...	...	·08	...	1·94	·16	...	...	...	...	...	...
L'Orient,	...	...	...	...	·16	·35	...	·87	·32	·20	·02	...	·08	...	·20	...	·87	·04	...	...	...	·28	·12	...	...	...	·04	·98	·35	·12	...	...	...	...	·08	...
Rochfort,	...	...	·04	·16	·20	·35	...	·48	·20	...	·12	...	...	...	·08	...	·28	·59	...	...	...	·04	...	...	...	...	...	·28	...	·12	...	...	...	...	·12	...
Dijon,	...	...	...	·14	·08	·30	·39	·10	...	·32	...	...	...	...	·26	...	·08	·28	...	...	...	·16	·04	...	...	...	...	·49	·39	...	...	...	...	...	·14	...
Belgium.																																				
Brussels,	...	...	·01	·19	·09	·27	·10	·18	·11	·32	·06	·14	...	·41	·55	...	·06	...	...	...	...	...	...	·01	...	...	...	·18	·55	...	...	...	...	·24	·04	...
Luxemburg,	...	...	...	...	·41	1·16	...	·36	.39	·07	·01	...	·11	·20	...	·07	·01	...	...	...	·05	·48	·01	·08	·01	...	...	·56	·96	...	...	...	...	·21	·18	...
Netherlands.																																				
Groningen,	...	...	·10	·17	...	·21	·08	·10	·26	·22	·07	·11	...	·24	·04	·04	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
Helder,	...	...	·10	·06	·22	·02	·20	·42	·30	·01	·09	...	·05	·52	...	·21	·05	·05	·02	...	...	·03	·28	...	...	...	...	·11	·70	·02	...	...	·15	·03	·04	...
Utrecht,	...	...	·01	·26	·14	·01	...	·06	·16	·09	·43	...	·01	·42	·07	·01	·03	...	...	·01	·01	...	·07	...	·01	...	·01	·03	·72	·44	·01	...	...	·90	·18	...
Flushing,	...	...	...	·14	·02	·07	·04	·10	·27	·16	·06	·23	·02	·14	·32	...	·10	...	...	·01	...	...	...	...	...	...	·04	·22	·12	·02	...	...	·01	...	·04	·01
Denmark.																																				
Kiel,	...	...	·06	·19	...	·08	·27	·02	·06	·04	...	...	...	·02	·15	...	·13	...	...	...	...	·30	·03	...	...	...	...	·38	·20	·12	...	·09	...	·21	...	...
Heligoland (England),	...	...	...	·10	...	·15	...	...	·05	·25	...	...	·09	...	·03	...	...	·30	...	...	...	·08	·05	·05	·04	...	...	...	...	...	...	...	...	...	...	...
Sweden.																																				
Haparanda,	·15	...	·09	...	...	·70	...	...	...	...	...	·17	...	...	...	·09	...	·32	...	·23	...	...	...	...	·03	...	...	...	...	...	·42	...	...	...	·39	...
Stockholm.	...	...	...	...	·06	70	·05	·02	·29	·10	·27	...	·01	...	...	·02	·18	...	...	...	·01	·01	...	·002	...	...	...	...	...	...	·16	...	...	·11	·06	...
Russia.																																				
Kostroma,	...	...	...	...	...	...	...	·02	...	...	·38	·23	...	·34	·23	...	·09	·04	·06	...	·02	·08	·12	·06	...	·02	...	·02	...	...	...	...	...	...	...	...
Lougan,	·62	·60	·14	...	...	...	...	...	·01	...	...	...	...	...	...	·01	...	...	...	...	...	...	...	...	...	...	·01	...	...	...	...	...	...	...	...	...
Orenbourg,	...	...	...	...	...	...	...	...	·44	...	...	...	...	...	·06	·20	...	...	...	...	...	·06	·04	...	...	...	...	...	...	·14	·04	·16	...	...	·18	·26
Slatoust,	...	...	...	...	...	·01	...	...	...	·04	·01	...	...	...	...	·14	·04	...	·02	·57	·51	·01	·02	·07	·03	...	·05	·11	·20	...	...	...	...	...	·04	...
Catherinenbourg,	...	·10	·09	...	...	...	...	...	...	·05	...	...	...	...	...	·06	...	...	...	...	...	·07	...	...	·02	...	·01	...	...	...	...	·01	...	...	...	...
Prussia.																																				
Putbus,	...	...	...	...	·32	...	...	...	...	·59	...	·25	·17	...	...	...	...	...	·01	...	·06	...	·07	...	·08	·03	...	...	...	·08	·37	·11	·03	·11	...	...
Paderborn,	...	...	...	·20	·35	...	...	...	·07	·28	·92	·01	...	·43	...	·04	·01	...	...	...	·01	·18	·11	·06	...	...	...	...	·43	·08	·01	·10	·09	·02	...	...
Breslau,	...	...	...	·24	·20	...	...	...	·02	...	...	·04	·52	...	...	...	...	...	...	...	...	·34	...	·14	·04	...	...	...	·35	...	...	·62	...	...	...	...
Austrian Empire.																																				
Cracow,	·01	...	...	...	...	·04	·08	·31	...	...	·02	·45	·01	·03	...	...	...	·62	...	...	...	...	·04	·13	·04	...	...	...	...	·15	...	·09	·31	...	...	...
Prague,	...	...	...	...	11	·07	·04	...	...	...	...	·04	...	13	·04	·02	...	...	...	...	...	·11	...	·19	...	...	...	...	·02	...	·41	·06	...	...	...	...
Kremsmunster,	...	...	...	...	...	...	·19	...	·18	...	·10	·15	...	1·15	...	...	...	...	...	...	...	·15	...	·16	·06	...	...	...	...	...	...	·28	...	...	...	...
Klagenfurt,	...	...	...	...	...	...	1·07	·86	...	...	...	...	...	...	·60	·34	·20	·02	...	...	...	...	...	...	·10	...	...	...	...	...	...	...	...	...	·20	...
German States.																																				
Munich,	...	·01	·01	...	...	·09	·45	·20	·11	·02	·18	...	·13	·16	·21	...	·17	·04	...	...	...	...	·13	·01	·04	...	...	·01	...	·08	...	·01	...	·03	...	...
Dresden,	...	...	...	...	·03	·27	...	...	...	...	·11	...	·11	...	·03	...	...	...	...	...	...	·62	...	·51	·08	...	...	...	·04	·16	·28	·41	·16	...	...	...
Switzerland.																																				

Pembroke,	50 c	51 c	50 c	55 R	51 R	46 R	B	47 cr	52 R	55 cr	54 R	45 cr	52 cr	B	42 Br	49 c	43 cr	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
London,	42 c	42 c	50 c	48 R	49 cr	44 Br	42	42 R	40 cr	57 cr	53 R	38 Br	45 c	50	42 cr	36 R	35 cr	36 B	47 c	51	44	45 cr	51 R	54 cr	51 c	34 c	43 c	49 cr	45 cr	35 B	48 c	38 c	[illegible]
Dover,	51 c	51 c	55 c	54 R	54 cr	50 Br	...	48 R	46 cr	57 cr	56 c	46 Br	48 c	...	48 Br	44 B	46 cr	41 Br	52 c	...	...	50 cr	52 R	55 cr	53 c	41 c	44 c	51 R	50 R	...	52 c	48 B	[illegible]
Portsmouth,	48 c	48 c	53 c	51 R	53 cr	49 cr	47 B	47 cr	47 cr	55 cr	55 c	47 cr	49 c	54 R	46 cr	41 c	45 R	40 cr	49 c	52 c	49 B	49 cr	53 R	53 cr	51 c	45 c	49 c	49 cr	47 R	43 cr	50 c	46 c	[illegible]
Weymouth,	45 c	54 c	54 c	52 R	52 R	50 cr	47 c	45 cr	48 R	56 cr	55 c	47 c	52 cr	54 c	43 Br	48 R	46 R	40 Br	49 c	53 c	50 c	52 cr	54 cr	54 c	53 c	45 c	50 R	51 cr	48 R	44 Br	51 c	49 c	[illegible]
Plymouth,	48 c	45 c	51 c	52 R	51 R	49 cr	...	45 cr	51 R	55 cr	55 c	51 R	56 cr	...	45 c	48 R	47 cr	44 B	51 c	...	...	52 cr	52 c	55 c	55 B	45 c	50 R	50 R	46 cr	44 c	51 c	51 c	[illegible]
Penzance,	50 B	50 cr	51 c	52 cr	51 R	46 cr	47 B	45 cr	51 R	54 cr	54 c	53 cr	53 cr	51	48 cr	48 cr	46 cr	45 cr	53 c	52	50	51 cr	48 c	54 c	53 cr	49 c	52 cr	45 c	45 R	46 cr	51 c	50 c	[illegible]
NORWAY and SWEDEN.																																	
*Haparanda,	38 c	30 B	37 cr	42 c	40 c	41 cr	42 c	35 c	30 B	34 c	27 c	24 cr	27 c	26 c	13 B	11 Br	29 c	34 cr	23 c	22 cr	—3 B	9 c	—6 B	—11 B	31 cr	32 B	30 B	32 c	32 c	34 c	36 cr	10 B	7 B
Hernösand,	41 c	30 c	34 c	42 c	46 c	46 c	...	38 B	36 c	39 c	37 c	31 B	32 c	27 c	26 B	29 c	36 c	37 B	39 c	45 B	28 c	22 B	19 c	25 c	35 c	26 c	49 B	30 c	38 c	40 c	40 c	16 c	11 B
Christiansund,	43	42	43	44	47	47	42	39	44	43	39	36	41	34	34	43	38	39	51	49	44	42	43	43	44	42	28	46	47	39	44	35	[illegible]
Skudesnos,	50	49	46	46	46	44	45	43	46	45	42	37	43	34	31	43	40	40	49	47	47	44	45	48	49	39	42	43	40	40	47	38	[illegible]
Mandal,	49	48	46	49	44	45	43	36	45	43	40	33	38	31	30	43	34	35	48	45	47	38	37	43	47	37	39	41	40	39	44	34	41
Christiania,	38	44	41	43	45	40	35	32	40	40	39	36	31	36	22	31	35	34	39	38	42	37	35	35	41	34	36	34	35	36	37	18	[illegible]
Stockholm,	32 B	39 c	43 B	44 B	47 R	48 cr	45 Br	42 R	42 cr	41 cr	39 R	33 B	32 Br	30 B	24 B	32 cr	32 Br	33 c	47 c	46 c	40 cr	40 cr	31 c	38 cr	37 c	38 c	34 c	36 c	39 B	42 c	41 cr	19 B	[illegible]
RUSSIA.																																	
Archangel,	26 F	35 F	35 F	35 F	32 F	26 F	34 F	32 R	34 B	25 F	36 F	33 R	32 R	24 F	25 R	15 R	4 B	31 R	26 F	22 F	4 B	1 B	—1 B	1 F	28 F	31 F	32 F	32 F	31 F	0 B	18 F	18 F	16
Helsingfors,	31 B	41 B	44 c	37 c	38 B	41 R	45 R	44 B	34 F	44 R	44 c	33 B	28 R	35 c	24 B	27 B	44 c	40 c	44 c	39 B	44 R	44 c	30 R	26 c	25 c	33 F	36 c	38 c	35 B	34 c	33 c	20 B	20 c
St Petersburg,	36 B	38 B	33 B	31 B	26 B	38 B	37 B	42 B	39 c	37 c	42 c	35 B	28 Br	30 R	28 B	24 B	32 B	33 B	40 R	42 B	36 R	35 c	37 c	35 cr	25 B	36 c	38 c	35 c	28 c	28 c	29 c	22 B	15
Dorpat,	41	39	38	37	40	41	35	36	32	38	37	33	31	29	23	22	30	34	45	44	43	42	34	32	30	39	33	27	21	21	26	21	23
Riga,	38 B	34 B	28 B	29 B	31 B	38 c	36 B	40 R	36 c	44 c	47 B	34 c	34 R	34 c	32 R	23 B	29 B	36 Fr	45 c	43 c	41 c	37 B	41 c	37 c	34 B	41 c	38 c	30 B	27 B	28 c	33 R	28 c	30
Libau,	34	37 c	34 B	32 B	40 B	46 R	48 c	47 B	41 B	48 B	48 B	43 B	40 B	35 c	36 Br	36 B	28 c	35 cr	37 c	45 c	45 cr	46 c	44 c	40 B	41 B	42 c	33 c	32 c	33 B	31 B	33 B	28 c	33
Moscow,	21 B	36 c	30	31 c	29 c	32 B	29 c	32 c	36 c	33 c	45 B	35 B	34 R	31 c	25 c	13 B	30 B	34 R	35 c	23 c	40 c	36 c	41 c	40 R	45 c	34 c	31 F	36 R	30 c	12	12 B	35 R	20
Warsaw,	36 B	32 F	34 B	37 B	42 B	55 c	39 c	44 c	41 F	33 R	54 R	39 R	31 B	35 c	25 B	26 B	31 c	37 F	45 c	35 B	34 B	41 c	43 R	41 F	44 c	35 c	26 c	22 B	20 B	27 c	27 c	22 c	26
*Kostroma,	28 c	26 c	32 c	33 B	32 c	33 c	25 B	29 R	34 c	33 c	40 R	38 R	31 B	32 R	26 R	17 c	10 F	23 R	32 R	38 c	40 R	17 R	28 R	34 F	16 c	28 R	34 c	30 R	28 c	16 c	15 B	27 c	22
*Orenbourg,	B	B	B	B	B	B	B	B	20 R	33 R	35 B	33 B	B	32 B	28 R	33 R	24 B	9 B	29 B	B	36 B	30 R	33 R	37 B	37 B	25 B	24 B	29 c	28 B	11 R	2 R	13 R	19
*Slatoust,	18 B	23 c	20 B	14 B	16 B	12 R	8 B	12 B	5 B	19 R	32 R	38 B	32 B	17 B	30 B	26 R	23 R	15 B	29 R	25 R	27 R	10 R	4 R	4 R	7 R	27 c	19 R	23 R	10 R	16 c	—36 B	6 c	10
*Catherinenbourg,	22 c	19 R	9 R	0 B	19 c	11 B	19 c	11 B	7 B	19 R	29 c	36 B	33 B	26 B	28 B	27 R	19 c	10 c	25 c	17 c	20 c	2 R	—3 c	—9 B	3 R	24 c	20 R	23 R	26 c	9 R	22 B	2 B	3
*Kiev,	34 B	27 B	41 B	42 B	18 B	22 B	24 R	46 B	52 F	52 c	46 c	44 B	31 R	34 B	29 B	31 B	27 B	39 c	43 c	41 B	32 B	30 B	28 F	39 B	32 B	...	23 c	26 F	F	25 c	24 c	24 B	20
Loúgan,	36 R	35 cr	24 Br	18 B	32 c	32 c	25 B	34 c	42 Br	40 c	45 c	46 B	49 R	31 B	33 c	29 cr	18 B	22 B	34 c	42 F	44 B	41 B	43 c	34 F	40 c	32 c	27 cr	18 F	23 F	23 c	21 c	33 cr	23
Nicolaiev,	46 R	37 c	36 c	29 c	26 B	32 B	38 c	47 c	50 B	54 B	40 c	47 B	43 B	36 B	35 B	34 c	34 c	41 c	43 B	46 B	38 B	47 B	48 c	39 c	31 B	32 c	31 c	22 B	27 c	34 F	29 c	29 c	25
Odessa,	35 c	36 c	35 c	29 c	26 B	42 B	48 R	49 c	53 B	54 B	47 F	50 c	39	38 B	32 B	34 R	c	54 B	42 B	44 c	40 B	48 B	51 c	39 R	43 B	30 B	30 c	21 c	26 c	40 R	29 c	26 B	23
DENMARK.																																	
Ivigtut, Greenland,	21 B	16 Br	16 B	15 B	12 B	22 cr	17 B	14 B	9 B	9 B	27 c	40 c	37 B	37 cr	37 c	26 B	24 cr	24 B	37 cr	36 c	28 c	17 B	24 B	19 B	9 B	—1 B	1 B	—1 B	—6 B	—3 B	...	4 B	16
Skagen, or the Skaw,	43 B	43 B	47 B	48 cr	45 B	41 c	44 B	41 B	46 Br	44 B	42 B	36 B	40 B	43 B	34 B	38 c	40 cr	39 c	47 c	44 c	45 B	45 B	41 F	46 F	46 F	39 c	43 c	42 B	42 cr	42 B	46 c	35 B	39
Copenhagen,	44 c	45 c	46 c	44 c	50 R	44 c	45 R	42 B	45 R	44 R	45 B	38 c	41 c	30 B	32 B	34 c	34 c	39 B	43 c	43 c	44 c	46 c	44 c	45 cr	42 c	39 c	37 c	33 B	38 cr	41 R	34 c	31 B	28
*Kiel,	44	44	45	50 R	53 R	51	39 R	39 R	40 R	47 R	43 R	38 R	40 R	40 R	36	34	37 R	38	42	41	48	49 R	49 R	46 R	45	33	28	29	37 R	37 R	41 R	44	46
Heligoland (Eng.),	44 c	44 c	43 c	48 R	50 c	46 R	...	...	45 R	51 cr	...	44 R	43 cr	...	39 B	36 B	40 c	41 cr	45 c	...	...	49 c	47 cr	47 cr	47 cr	...	33 c	40 c	...	...	...	45 c	46
PRUSSIA.																																	
Königsberg,	52	39	38	42	50	51	42	30	40	42	45	35	35	33	32	30	31	33	42	39	40	45	45	46	44	42	34	32	34	35	36	32	34
*Putbus,	46	47	47	51	58 R	51	39	38	38	44 R	41	36 R	38 R	36	33	31	34	36	40 R	37	41 R	46	46 R	43	42 R	35 R	28	26	34	36 R	34 R	37 R	42
*Paderborn,	47	49	51	58 R	60 R	54	42	41	41 R	50 R	46 R	37 R	37	37 R	35	33 R	38 R	33	47	42	50 R	45 R	49 R	53 R	51	32	23	35	38 R	37 R	41 R	44 R	44
Breslau,	45 c	42 B	45 c	46 B	53 R	54 R	42 c	41 c	45 R	43 B	49 c	36 R	33 R	36 c	30 c	27 c	39 c	37 c	42 B	42 B	43 B	45 R	47 c	47 R	46 R	34 c	24 B	25 B	31 R	37 B	36 c	33 B	41
AUSTRIA.																																	
Prague,	45 c	43 c	46 c	51 B	55 R	57 R	41 R	41 B	42 c	45 B	52 c	42 R	36 B	40 R	35 R	33 R	40 c	39 c	42 c	42 c	42 c	46 R	46 c	48 R	46 c	35 c	27 R	25 B	38 R	38 c	35 R	41 R	46
Cracow,	45 R	43 c	38 B	41 B	49 B	55 R	40 R	38 R	41 c	40 c	46 R	38 R	32 R	33 R	29 c	28 c	38 c	40 R	40 c	36 c	37 B	38 B	43 R	44 R	44 R	34 c	23 c	20 c	23 B	31 R	30 c	29 R	31
Vienna,	42 c	38 c	36 c	37 B	44 c	48 F	46 R	45 c	47 c	37 B	55 B	53 c	35 c	41 c	36 R	33 c	39 c	41 R	37 F	37 c	36 c	34 c	35 c	38 F	39 c	23 B	27 c	26 c	28 c	31 c	37 c	40 R	43
Kremsmunster,	40 B	42 c	46 c	45 c	47 c	40 c	39 R	38 c	42 R	50 c	43 R	36 R	36 B	38 R	36 c	38 c	39 c	41 c	39 c	36 c	41 c	40 R	41 c	42 R	46 R	30 c	27 B	28 B	35 c	38 c	38 c	45 R	46
Klagenfurt,	37	38	36	40	47	54	48 R	35 R	33	32	34	34	35	36	31 R	30 R	30 R	34 R	41	34	34	34	38	38	42 R	35	19 c	18 c	24 c	27 c	26 B	35 R	37
GERMANY.																																	
Leipzig,	30 B	33 B	33 B	39 B	50 R	36 R	45 B	53 B	45 B	45 R	54 c	41 B	35 B	33 c	36 R	27 B	34 c	37 c	35 B	31 B	38 B	41 c	37 c	44 c	41 c	26 B	22 B	31	37 B	38 c	42 R	39 R	44
Bamberg,	40	41	43	53	61	53	40	41	42	47	51	38	42	39	37	37	35	37	38	38	46	46	44	50	47	30	24	28	36	36	34	44	43
Munich,	42 B	42 R	47 R	57 c	58 c	56 R	37 R	39 R	37 R	43 R	48 R	41 c	37 R	43 R	40 R	35 c	37 R	34 R	34 c	34 c	43 B	49 B	49 R	49 R	48 R	31 c	24 c	30 R	37 B	31 R	31 R	41 c	40
SWITZERLAND.																																	
Aarau,	46 c	48 c	51 B	57 R	61 R	54 R	39 R	36 R	40 R	44 R	48 B	44 c	44 B	48 R	44 R	40 c	39 B	38 c	37 c	37 c	40 B	48 R	51 R	48 c	46 c	29 c	30 c	39 c	31 R	30 R	41 c	35 B	33
*Geneva,	49 c	50 c	51 R	57 R	62 c	60 R	39 R	40 R	39 R	42 R	43 B	40 B	42 c	46 c	44 R	41 c	38 R	38 c	37 c	34 c	45 c	53 R	49 R	46 c	46 c	39 c	30 c	37 R	42 R	32 R	31 B	34 B	33
NETHERLANDS.																																	
Groningen,	35 B	35 B	37 c	45 c	46 B	45 B	45 B	40 c	44 c	52 c	46 c	43 B	43 B	40 c	36 B	30 B	38 c	33 B	36 c	36 B	44 B	44 c	44 B	49 c	43 F	36 c	24 B	37 c	38 c	24	43 c	47 c	45
Helder,	39 B	46 B	43 c	47 c	50 B	48 c	48 R	42 R	49 c	54 c	51 B	45 B	45 c	43 R	41 B	35 B	43 R	39 c	41 F	35 B	47 B	47 c	46 c	50 R	46 F	32 c	28 B	38 R	32 R	22 c	46	47 c	45
Utrecht,	37 B	34 B	44 cr	45 Br	48 Br	44 Br	43 Br	43 cr	46 cr	54 cr	50 Br	43 Br	39 Br	42 cr	37 Br	29 Br	38 Br	31 B	35 B	36 Br	44 Br	43 c	41 Fr	50 c	46 R	24 B	25 cr	38 Br	42 Br	41 Br	43 Br	42 c	42
Flushing,	46 B	45 B	48 c	49 cr	49 Br	46 Br	45 Br	48 cr	47 Br	55 Br	51 cr	47 Br	42 Br	46 cr	42 Br	36 B	44 Br	43 B	42 B	43 B	46 Br	45 B	45 c	51 c	49 c	33 B	35 cr	46 Br	46 Br	42 cr	45 Br	47 B	44
BELGIUM.																																	
Brussels,	37 B	35 c	48 c	49 c	50 c	53 B	45 B	47 R	45 B	56 c	56 R	50 B	37 B	48 R	40 R	31 c	39 B	35 F	42 B	43 B	46 B	45 B	47 c	53 B	51 c	28 B	32 B	47 c	47 c	44 B	44 c	44 B	42
Luxemburg,	34 B	35 B	46 c	46 c	48 R	43 Br	43 c	39 Br	41 cr	51 R	51 R	50 c	31 Br	48 R	46 R	38 cr	38 R	35 c	35 c	30 B	45 R	43 cr	43 cr	47 Fr	48 cr	28 B	32	33 r	36 r	36	36	41	43
FRANCE.																																	
Dunkirk,	44 F	48 F	46 F	44 B	48 c	47 c	42 R	46 R	44 B	56 c	55 R	49 B	43 F	51 c	46 B	34 B	46 B	44 B	40 B	42 c	50 c	46 c	44 R	50 F	35 c	30 B	33 B	46 R	39 R	43 c	46 F	46 c	42
Mezières,	B	B	R	c	c	c	B	R	B	R	R	c	c	53 c	R	c	c	c	37 B	38 B	50 c	46 R	50 c	54 R	52 c	B	c	R	R	B	B	...	
Havre,	49 F	49 R	50 c	53 c	50 c	50 c	46 c	50 c	50 c	56 F	53 c	54 c	52 c	54 R	48 R	43 B	44 c	44 c	44 F	48 c	50 B	48 c	51 R	51 c	48 F	36 B	42 c	34 R	51 R	46 c	48 c	49 F	47
Cherbourg,	c	54 c	50 B	50 R	53 c	50 B	46 c	46 R	48 R	57 R	55 c	50 R	54 c	52 R	50 c	42 c	43 R	48 F	45 c	49 B	48 c	46 B	55 R	51 B	50 B	40 B	46 R	50 R	48 R	47 c	51 c	50 c	49
Paris,	40 c	43 F	44 c	47 c	47 c	44 c	44 B	44 R	43 B	54 c	54 c	51 R	46 c	50 c	40 c	38 c	36 c	41 c	34 B	36 B	48 c	47 R	48 B	49 c	46 F	32 B	34 c	48 R	51 R	39 B	39 c	41 B	[illegible]
Strasbourg,	38 B	41 F	44 F	49 R	54 R	50 c	47 c	43 B	46 B	48 R	53 c	51 R	41 B	48 R	48 R	40 c	41 R	37 c	35 F	35 F	...	49 R	46 F	47 B	48 F	31 B	34 F	32 c	42 c	39 B	37 c	...	[illegible]
Brest,	48 c	45 F	51 c	55 c	53 R	49 c	45 c	46 c	52 R	57 c	55 c	54 c	53 c	54 c	48 c	47 c	50 R	46 B	49 c	56 c	48 B	50 c	55 R	55 c	50 B	42 B	52 c	49 R	50 c	45 B	52 c	50 c	[illegible]
L'Orient,	49 F	42 B	45 c	55 c	55 R	50 c	44 c	46 R	49 c	56 c	53 c	53 c	54 B	53 c	46 B	38 c	47 R	43 B	45 F	54 c	44 B	47 F	54 c	52 F	48 B	47 B	52 R	50 R	55 R	38 B	50 c	48 c	[illegible]
Napoleon Vendée,	48 F	47 B	50 B	49 c	53 R	49 c	53 c	46 c	45 R	...	...	...	53 F	51 c	46 c	47 B	49 R	50 F	...	...	52	45 B	49 B	38 B	47 F	34 B	46 c	...	...	37 B	37 B	45 B	[illegible]
Besançon,	40 c	43 B	48 B	58 B	60 c	50 R	49 R	48 c	50 c	48 R	48 c	49 c	47 F	50 F	47 R	45 c	50 F	40 B	35 B	34 c	41 c	51 R	43 B	45 B	44 B	32 B	30 c	47 R	41 c	40 c	36 c	42 c	[illegible]
Lyons,	55 F	57 F	57 F	58 B	60 c	62 F	52 c	47 c	50 F	49 R	55 F	53 c	54 B	53 R	51 c	48 c	44 B	48 F	37 F	43 B	51 B	52 R	52 F	48 F	52 F	46 F	37	58 R	46 c	...	40 B	47 F	[illegible]
Limoges,	40 B	42 B	50 B	57 c	57 c	49 R	47 c	46 R	40 B	52 R	50 F	48 c	50 R	50 R	45 R	42 c	39 R	38 R	30 B	39 B	42 c	51 c	45 B	37 c	37 B	40 B	41 c	48 R	54 R	33 B	26 c	40 B	[illegible]
Rochfort,	48 F	45 F	54 R	52 R	55 R	54 c	50 B	50 R	48 R	54 c	52 F	52 c	...	...	48 B	43 B	48 c	...	...	48 c	52 F	55 c	52 R	48 B	46 B	45 B	46 c	54 c	58 B	43 B	39 B	46 F	[illegible]
Montauban,	49 B	49 B	53 B	59 B	55 c	57 B	52 R	46 B	49 B	50 R	50 B	44 F	46 R	52 F	48 c	46 c	44 R	43 R	37 B	41 B	49 B	48 c	48 B	46 F	44 F	47 c	46 B	50 R	52 B	41 B	37 B	45 c	[illegible]
Bayonne,	57 B	57 B	61 c	59 B	...	63 c	55 R	55 c	55 R	57 R	R	52 c	54 c	55 R	45 R	52 c	54 R	52 R	48 B	55 c	55 B	55 F	55 B	53 B	49 B	48 B	54 B	57 c	...	54 B	46 B	50 c	[illegible]

42 c	50 c	48 R	49 cr	44 Br	42	42 R	40 cr	57 cr	53 R	38 Br	45 c																							
51 c	55 c	54 R	54 cr	50 Br	...	48 R	46 cr	57 cr	56 c	46 Br	48 c	...	48 Br	44 B	46 cr	41 Br	52 c	...	...	50 cr	52 R													
48 c	53 c	51 R	53 cr	49 cr	47 B	47 cr	47 cr	55 cr	55 c	47 cr	49 c	54 R	46 cr	41 c	45 R	40 cr	49 c	52 c	49 B	49 cr	53 R	53 cr	51 c	45 c	49 c	49 cr	47 R							
54 c	54 c	52 R	52 R	50 cr	47 c	45 cr	48 R	56 cr	55 c	47 c	52 cr	54 c	43 Br	48 R	46 R	40 Br	49 c	53 c	50 c	52 cr	54 cr	54 c	53 c	45 c	50 R	51 cr	48 R	44 Br	51 c	49 c	49 c	51 R	45 cr	34 B
45 c	51 c	52 R	51 R	49 cr	...	45 cr	51 R	55 cr	55 c	51 R	56 cr	...	45 c	48 R	47 cr	44 B	51 c	...	...	52 cr	52 c	55 c	55 B	45 c	50 R	50 R	46 cr	44 c	51 c	51 c	52 c	50 c	45 B	37 B
50 cr	51 c	52 cr	51 R	46 cr	47 B	45 cr	51 R	54 cr	54 c	53 cr	53 cr	51	48 cr	48 cr	46 cr	45 cr	53 c	52	50	51 cr	48 c	54 c	53 cr	49 c	52 cr	45 c	45 R	46 cr	51 c	50 c	50 c	49 cr	46 cr	40 B
30 B	37 cr	42 c	40 c	41 cr	42 c	35 c	30 B	34 c	27 c	24 cr	27 c	26 c	13 B	11 Br	29 c	34 cr	23 c	22 cr	—3 B	9 c	—6 B	—11 B	31 cr	32 B	30 B	32 c	32 c	34 c	36 cr	10 B	7 B	1 c	22 cr	17 c
30 c	34 c	42 c	46 c	46 c	...	38 B	36 c	39 c	37 c	31 B	32 c	27 c	26 B	29 c	36 c	37 B	39 c	45 B	28 c	22 B	19 c	25 c	35 c	26 c	49 B	30 c	38 c	40 c	40 c	16 c	11 B	29 c	35 c	20 B
42	43	44	47	47	42	39	44	43	39	36	41	34	34	43	38	39	51	49	44	42	43	43	44	42	28	46	47	39	44	35	31	36	34	32
49	46	46	46	44	45	43	46	45	42	37	43	34	31	43	40	40	49	47	47	44	45	48	49	39	42	43	40	40	47	38	42	46	41	35
48	46	49	44	45	43	36	45	43	40	33	38	31	30	43	34	35	48	45	47	38	37	43	47	37	39	41	40	39	44	34	41	44	37	30
44	41	43	45	40	35	32	40	40	39	36	31	36	22	31	35	34	39	38	42	37	35	35	41	34	36	34	35	36	37	18	23	33	36	32
39 c	43 B	44 B	47 R	48 cr	45 Br	42 R	42 cr	41 cr	39 R	33 B	32 Br	30 B	24 B	32 cr	32 Br	33 c	47 c	46 c	40 cr	40 cr	31 c	38 cr	37 c	38 c	34 c	36 c	39 B	42 c	41 cr	19 B	22 B	32 cr	37 cr	24 B
35 F	35 F	35 F	32 F	26 F	34 F	32 R	34 B	25 F	36 F	33 R	32 R	24 F	25 R	15 R	4 B	31 R	26 F	22 F	4 B	1 B	—1 B	1 F	28 F	31 F	32 F	32 F	31 F	0 B	18 F	18 F	16 F	3 F	0 F	7 F
41 B	44 c	37 c	38 B	41 R	45 R	44 B	34 F	44 R	44 c	33 B	28 R	35 c	24 B	27 B	44 c	40 c	44 c	39 B	44 R	44 c	30 R	26 c	25 c	33 F	36 c	38 c	35 B	34 c	33 c	20 B	20 c	29 c	32 c	32 c
38 B	33 B	31 B	26 B	38 B	37 B	42 B	39 c	37 c	42 c	35 B	28 Br	30 R	28 B	24 B	32 B	33 B	40 R	42 B	36 R	35 c	37 c	35 cr	25 B	36 c	38 c	35 c	28 c	28 c	29 c	22 B	15 F	22 B	19 c	28 c
39	38	37	40	41	35	36	32	38	37	33	31	29	23	22	30	34	45	44	43	42	34	32	30	39	33	27	21	21	26	21	23	15	27	26
34 B	28 B	29 B	31 B	38 c	36 B	40 R	36 c	44 c	47 B	34 c	34 R	34 c	32 R	23 B	29 B	36 Fr	45 c	43 c	41 c	37 B	41 c	37 c	34 B	41 c	38 c	30 B	27 B	28 c	33 B	28 c	30 c	23 B	31 cr	34 c
37 c	34 B	32 B	40 B	46 R	48 c	47 B	41 B	48 B	48 B	43 B	40 B	35 c	36 Br	36 B	28 c	35 cr	37 c	45 c	45 cr	46 c	44 c	40 B	41 B	42 c	33 c	32 c	33 B	31 B	33 B	28 c	33 R	31 B	36 R	R
36 c	30	31 c	29 c	32 B	29 c	32 c	36 c	33 c	45 B	35 B	34 R	31 c	25 c	13 B	30 B	34 R	35 c	23 c	40 c	36 c	41 c	40 R	45 c	34 c	31 F	36 R	30 c	12	12 B	35 R	20	11 c	4 c	5 c
32 F	34 B	37 B	42 B	55 c	39 c	44 c	41 F	33 R	54 R	39 R	31 B	35 c	25 B	26 B	31 c	37 F	45 c	35 B	34 B	41 c	43 R	41 F	44 c	35 c	26 c	22 B	20 B	27 c	27 c	22 c	26 R	27 F	34 c	35 c
26 c	32 c	33 B	32 c	33 c	25 B	29 R	34 c	33 c	40 R	38 R	31 B	32 R	26 R	17 c	10 F	23 R	32 R	38 c	40 R	17 R	28 R	34 F	16 c	28 R	34 c	30 R	28 c	16 c	15 B	27 c	22 c	12 c	8 c	16 c
B	B	B	B	B	B	B	20 R	33 B	35 B	33 B	B	32 B	28 R	33 R	24 B	9 B	29 B	B	36 B	30 R	33 R	37 B	37 B	25 B	24 B	29 c	28 B	11 R	2 R	13 R	19 B	25 B	17 R	15 R
23 c	20 B	14 B	16 B	12 R	8 B	12 B	5 B	19 R	32 R	38 B	32 B	17 B	30 B	26 R	23 R	15 B	29 R	25 R	27 R	10 R	4 R	4 R	7 R	27 c	19 R	23 R	10 R	16 c	—36 B	6 c	10 B	12 B	—1 B	3 R
19 R	9 R	0 B	19 c	11 B	19 c	11 B	7 B	19 R	29 c	36 B	33 B	26 B	28 B	27 R	19 c	10 c	25 c	17 c	20 c	2 R	—3 c	—9 B	3 R	24 c	20 R	23 R	26 c	9 R	22 B	2 B	3 c	8 c	—9 B	—3 c
27 B	41 B	42 B	18 B	22 B	24 R	46 B	52 F	52 c	46 c	44 B	31 R	34 B	29 B	31 B	27 B	39 c	43 c	41 B	32 B	30 B	28 F	39 B	32 B	...	23 c	26 F	F	25 c	24 c	24 B	20 c	10 B	13 c	21 c
35 cr	24 Br	18 B	32 c	32 c	25 B	34 c	42 Br	40 c	45 c	46 B	49 R	31 B	33 c	29 cr	18 B	22 B	34 c	42 F	44 B	41 B	43 c	34 F	40 c	32 c	27 cr	18 F	23 F	23 c	21 c	33 cr	23 c	15 B	12 c	9 c
37 c	36 c	29 c	26 B	32 B	38 c	47 c	50 B	54 B	40 c	47 B	43 B	36 B	35 B	34 c	34 c	41 c	43 B	46 B	38 B	47 B	48 c	39 c	31 B	32 c	31 c	22 B	27 c	34 F	29 c	29 c	25 c	18 c	11 B	11 c
36 c	35 c	29 c	26 B	42 B	48 R	49 c	53 B	54 B	47 F	50 c	39	38 B	32 B	34 R	c	54 B	42 B	44 c	40 B	48 B	51 c	39 B	43 B	30 B	30 c	21 c	26 c	40 R	29 c	26 B	23 B	17 c	17 B	25 c
16 Br	16 B	15 B	12 B	22 cr	17 B	14 B	9 B	9 B	27 c	40 c	37 B	37 cr	37 c	26 B	24 cr	24 B	37 cr	36 c	28 c	17 B	24 B	19 B	9 B	—1 B	1 B	—1 B	—6 B	—3 B	...	4 B	16 c	37 cr	32 cr	31 cr
43 B	47 B	48 cr	45 B	41 c	44 B	41 B	46 Br	44 B	42 B	36 B	40 B	43 B	34 B	38 c	40 cr	39 c	47 c	44 c	45 B	45 B	41 F	46 F	46 F	39 c	43 c	42 B	42 cr	42 B	46 c	35 B	39 B	47 cr	37 B	36 B
45 c	46 c	44 c	50 R	44 c	45 R	42 B	45 R	44 R	45 B	38 c	41 c	30 B	32 B	34 c	34 c	39 B	43 c	43 c	44 c	46 c	44 c	45 cr	42 c	39 c	37 c	33 B	38 cr	41 R	34 c	31 B	28 c	43 c	37 B	32 B
44	45	50 R	53 R	51	39 R	39 R	40 R	47 R	43 R	38 R	40 R	40 R	36	34	37 R	38	42	41	48	49 R	49 R	46 R	45	33	28	29	37 R	37 R	41 R	44	46	44	40	37
44 c	43 c	48 R	50 c	46 R	...	...	45 R	51 cr	...	44 R	43 cr	...	39 B	36 B	40 c	41 cr	45 c	...	...	49 c	47 cr	47 cr	47 cr	...	33 c	40 c	...	...	...	45 c	46 c	44 c	42 R	38 B
39	38	42	50	51	42	30	40	42	45	35	35	33	32	30	31	33	42	39	40	45	45	46	44	42	34	32	34	35	36	32	34	35	38	38
47	47	51	58 R	51	39	38	38	44 R	41	36 R	38 R	36	33	31	34	36	40 R	37	41 R	46	46 R	43	42 R	35 R	28	26	34	36 R	34 R	37 R	42	42 R	37 R	33
49	51	58 R	60 R	54	42	41	41 R	50 R	46 R	37 R	37	37 R	35	33 R	38 R	33	47	42	50 R	45 R	49 R	53 R	51	32	23	35	38 R	37 R	41 R	44 R	44	43 R	39 R	36
42 B	45 c	46 B	53 R	54 R	42 c	41 c	45 R	43 B	49 c	36 R	33 R	36 c	30 c	27 c	39 c	37 c	42 B	42 B	43 B	45 R	47 c	47 R	46 R	34 c	24 B	25 B	31 R	37 B	36 c	33 B	41 R	41 c	39 c	36 c
43 c	46 c	51 B	55 R	57 R	41 R	41 B	42 c	45 B	52 c	42 R	36 B	40 R	35 R	33 R	40 c	39 c	42 c	42 c	42 c	46 R	46 c	48 R	46 c	35 c	27 R	25 B	38 R	38 c	35 R	41 R	46 c	42 c	40 c	38 c
43 c	38 B	41 B	49 B	55 R	40 R	38 R	41 c	40 c	46 R	38 R	32 R	33 R	29 c	28 c	38 c	40 R	40 c	36 c	37 B	38 B	43 R	44 R	44 R	34 c	23 c	20 c	23 B	31 R	30 c	29 R	31 R	31 c	30 B	35 c
38 c	36 c	37 B	44 c	48 F	46 R	45 c	47 c	37 B	55 B	53 c	35 c	41 c	36 R	33 c	39 c	41 R	37 F	37 c	36 c	34 c	35 c	38 F	39 c	23 B	27 c	26 c	28 c	31 c	37 c	40 R	43 c	40 B	34 c	36 c
42 c	46 c	45 c	47 c	40 c	39 R	38 c	42 R	50 c	43 R	36 R	36 B	38 R	36 c	38 c	39 c	41 c	39 c	36 c	41 c	40 R	41 c	42 R	46 R	30 c	27 B	28 B	35 c	38 c	38 c	45 R	46 c	40 c	37 c	40 c
38	36	40	47	54	48 R	35 R	33	32	34	34	35	36	31 R	30 R	30 R	34 R	41	34	34	34	38	38	42 R	35	19 c	18 c	24 c	27 c	26 B	35 R	37 B	33 B	33 R	36 c
33 B	33 B	39 B	50 R	36 R	45 B	53 B	45 B	45 R	54 c	41 B	35 B	33 c	36 R	27 B	34 c	37 c	35 B	31 B	38 B	41 c	37 c	44 c	41 c	26 B	22 B	31	37 B	38 c	42 R	39 R	44 c	42 c	38 c	36 B
41	43	53	61	53	40	41	42	47	51	38	42	39	37	37	35	37	38	38	46	46	44	50	47	30	24	28	36	36	34	44	43	40	38	34
42 R	47 R	57 c	58 c	56 R	37 R	39 R	37 R	43 R	48 R	41 c	37 R	43 R	40 R	35 c	37 R	34 R	34 c	34 c	43 B	49 B	49 R	49 R	48 R	31 c	24 c	30 R	37 B	31 R	31 R	41 c	40 R	35 B	34 c	35 R
48 c	51 B	57 R	61 R	54 R	39 R	36 R	40 R	44 R	48 B	44 c	44 B	48 R	44 R	40 c	39 B	38 c	37 c	37 c	40 B	48 R	51 R	48 c	46 c	29 c	30 c	39 c	31 R	30 R	41 c	35 B	33 B	35 B	37 B	34 B
50 c	51 R	57 R	62 c	60 R	39 R	40 R	39 R	42 R	43 B	40 B	42 c	46 c	44 R	41 c	38 R	38 c	37 c	34 c	45 c	53 R	49 R	46 c	46 c	39 c	30 c	37 R	42 R	32 R	31 B	34 B	33 c	33 c	35 R	37 c
35 B	37 c	45 c	46 B	45 B	45 B	40 c	44 c	52 c	46 c	43 B	43 B	40 c	36 B	30 B	38 c	33 B	36 c	36 B	44 B	44 c	44 B	49 c	43 F	36 c	24 B	37 c	38 c	24	43 c	47 c	45 c	45 c	42 c	37 c
46 B	43 c	47 c	50 B	48 c	48 R	42 R	49 c	54 c	51 B	45 B	45 c	43 R	41 B	35 B	43 R	39 c	41 F	35 B	47 B	47 c	46 c	50 R	46 F	32 c	28 B	38 R	32 R	22 c	46	47 c	45 B	48 B	45 R	43 c
34 B	44 cr	45 Br	48 Br	44 Br	43 Br	43 cr	46 cr	54 cr	50 Br	43 Br	39 Br	42 cr	37 Br	29 Br	38 Br	31 B	35 B	36 Br	44 Br	43 c	41 Fr	50 c	46 R	24 B	25 cr	38 Br	42 Br	41 Br	43 Br	42 c	42 B	43 Br	41 Br	38 B
45 B	48 c	49 cr	49 Br	46 Br	45 Br	48 cr	47 Br	55 Br	51 cr	47 Br	42 Br	46 cr	42 Br	36 B	44 Br	43 B	42 B	43 B	46 Br	45 B	45 c	51 c	49 c	33 B	35 cr	46 Br	46 Br	42 cr	45 Br	47 B	44 Br	44 B	45 Br	41 Br
35 c	48 c	49 c	50 c	53 B	45 B	47 R	45 B	56 c	56 R	50 B	37 B	48 R	40 R	31 c	39 B	35 F	42 B	43 B	46 B	45 B	47 c	53 B	51 c	28 B	32 B	47 c	47 c	44 B	44 c	44 B	42 c	43 c	41 R	38 B
35 B	46 c	46 c	48 R	43 Br	43 c	39 Br	41 cr	51 R	51 R	50 c	31 Br	48 R	46 R	38 cr	38 R	35 c	35 c	30 B	45 R	43 cr	43 cr	47 Fr	48 cr	28 B	32	33 r	36 r	36	36	41	43	39 r	37 r	36
48 F	46 F	44 B	48 c	47 c	42 R	46 R	44 B	56 c	55 R	49 B	43 F	51 c	46 B	34 B	46 B	44 B	40 B	42 c	50 c	46 c	44 R	50 F	35 c	30 B	33 B	46 R	39 R	43 c	46 F	46 c	42 F	68 F	44 R	42 c
B	R	c	c	c	B	R	B	R	R	c	c	53 c	R	c	c	c	37 B	38 B	50 c	46 R	50 c	54 R	52 c	B	c	R	R	B	B	...	c	c	c	B
49 R	50 c	53 c	50 c	50 c	46 c	50 c	50 c	56 F	53 c	54 c	52 c	54 R	48 R	43 B	44 c	44 c	44 F	48 c	50 B	48 c	51 R	51 c	48 F	36 B	42 c	34 R	51 R	46 c	48 c	49 F	47 B	34 c	50 R	44 B
54 c	50 B	50 R	53 c	50 B	46 c	46 R	48 R	57 R	55 c	50 R	54 c	52 R	50 c	42 c	43 R	48 F	45 c	49 B	48 c	46 B	55 R	51 B	50 B	40 B	46 R	50 R	48 R	47 c	51 c	50 c	49 c	50 B	48 B	43 B
43 F	44 c	47 c	47 c	44 c	44 B	44 R	43 B	54 c	54 c	51 R	46 c	50 c	40 c	38 c	36 c	41 c	34 B	36 B	48 c	47 R	48 B	49 c	46 F	32 B	34 c	48 R	51 R	39 B	39 c	41 B	42 c	40 c	39 c	37 B
41 F	44 F	49 R	54 R	50 c	47 c	43 B	46 B	48 R	53 c	51 R	41 B	48 R	48 R	40 c	41 R	37 c	35 F	35 F	...	49 R	46 F	47 B	48 F	31 B	34 F	32 c	42 c	39 B	37 c	...	42 c	35 F	39 R	40 c
45 F	51 c	55 c	53 R	49 c	45 c	46 c	52 R	57 c	55 c	54 c	53 c	54 c	48 c	47 c	50 R	46 B	49 c	56 c	48 B	50 c	55 R	55 c	50 B	42 B	52 c	49 R	50 c	45 B	52 c	50 c	48 c	50 F	46 c	43 c
42 B	45 c	55 c	55 R	50 c	44 c	46 R	49 c	56 c	53 c	53 c	54 B	53 c	46 B	38 c	47 R	43 B	45 F	54 c	44 B	47 F	54 c	52 F	48 B	47 B	52 R	50 R	55 R	38 B	50 c	48 c	39 F	42 F	40 B	42 B
47 B	50 B	49 c	53 R	49 c	53 c	46 c	45 R	...	...	...	53 F	51 c	46 c	47 B	49 R	50 F	...	...	52	45 B	49 B	38 B	47 F	34 B	46 c	...	...	37 B	37 B	45 B	40 c	...	39 B	37 B
43 B	48 B	58 B	60 c	50 R	49 R	48 c	50 c	48 R	48 c	49 c	47 F	50 F	47 R	45 c	50 F	40 B	35 B	34 c	41 c	51 R	43 B	45 B	44 B	32 B	30 c	47 R	41 c	40 c	36 c	42 c	38 F	35 c	40 R	39 c
57 F	57 F	58 B	60 c	62 F	52 c	47 c	50 F	49 R	55 F	53 c	54 B	53 R	51 c	48 c	44 B	48 F	37 F	43 B	51 B	52 R	52 F	48 F	52 F	46 F	37	58 R	46 c	...	40 B	47 F	45 F	42 F	39 R	42 F
42 B	50 B	57 c	57 c	49 R	47 c	46 R	40 B	52 R	50 F	48 c	50 R	50 R	45 R	42 c	39 R	38 R	30 B	39 B	42 c	51 c	45 B	37 c	37 B	40 B	41 c	48 R	54 R	33 B	26 c	40 B	35 B	46 F	36 c	37 c
45 F	54 R	52 R	55 R	54 c	50 B	50 R	48 R	54 c	52 F	52 c	...	...	48 B	43 B	48 c	...	...	48 c	52 F	55 c	52 R	48 B	46 B	45 B	46 c	54 c	58 B	43 B	39 B	46 F	39 B	39 c	39 B	39 B
49 B	53 B	59 B	55 c	57 B	52 R	46 B	49 B	50 R	50 B	44 F	46 R	52 F	48 c	46 c	44 R	43 R	37 B	41 B	49 B	48 c	48 B	46 F	44 F	47 c	46 B	50 R	52 B	41 B	37 B	45 c	42 c	39 F	42 R	41 B
57 B	61 c	59 B	...	63 c	55 R	55 c	55 R	57 R	B	52 c	54 c	55 R	45 R	52 c	54 R	52 R	48 B	55 c	55 B	55 F	55 B	53 B	49 B	48 B	54 B	57 c	...	54 B	46 B	50 c	50 c	48 B	50 c	48 c
						44 B	51 B	57 B	50 B	49 B	50 B	57 B	53 c	48 B	46 B	46 c	41 B	43 B	48 c	51 c	49 c	53 R	54 B	45 B	46	48 c	47 c	43 B	42 B	43 B	43 B	34 B	37 R	39 B
																		45 B	...	53 c	54 R	56 R	52 B	44 B	46 B	50 R	47 B	43 B	37 B	42 B	43 B	48 c	43 F	39 B
																												46 B	45 B	39 F	40 c	48 c	45 c	45 c

Valencia,																																				
Queenstown,	·09	·09	·42	·25	·37	...	...	·24	·30	·16	·36	·16	·20	...	...	...																				
SCOTLAND.																																				
East Yell,	...	...	·37	·92	·22	·19	·07	·10	·24	·10	·10	·26	·04	...	·14	·20	·36	·30	·10	·20	·03	...	...	...	...	·05	·35	·50	...	·20	·30	·56	·50	·40	·07	·18
Sandwick,	·11	·05	·08	·13	·82	·02	·06	·02	·03	·34	·13	·08	·11	·07	·01	·29	·41	·25	...	·07	·11	·01	...	·03	·04	...	...	·10	·32	·05	·22	·26	·03	·24	·11	·13
Stornoway,	...	·03	·42	·50	...	...	·18	...	...	·26	·23	·10	·05	·20	·06	·39	.25	·07	·08	·10	·93	·03	·10	...	·11	...	...	·43	·03	·17	·97	·03	·75	·06	·10	·03
Portree,	·47	·43	·10	·04	1·21	1·09	·10	·16	·05	1·00	·11	·04	...	·58	...	1·10	·66	·16	·28	·80	1·02	·10	·20	1·12	·20	...	·80	·64	·38	·38	12·48	·32	·48	1·20	·28	·32
New Pitsligo,	·16	·06	·83	·11	...	·01	...	·06	·08	·12	·10	...	·26	·29	...	·10	...	...	...	·05	·41	·08	...	...	...	...	·07	·45	·22	...	...	...	...	·12	·14	·02
Barry,	·06	·01	·46	·13	·02	·01	...	·01	·10	·06	...	...	·01	·48	...	·16	...	...	...	·12	·41	·02	·02	...	·02	...	·10	·61	·64	...	·10	...	...	·02	...	·02
Inveresk,	·01	·30	·23	·40	·09	...	...	·03	...	...	·01	...	·24	·50	·60	...	...	...	...	·45	...	...	·06	...	...	...	·10	·36	·67	...	·26	...	·15	...	...	...
South Cairn,	·30	·55	·20	·30	·30	·35	·10	...	·35	·15	...	·25	·75	·20	·35	·25	·35	·15	...	·30	·20	...	·10	·10	·10	...	·30	·98	·35	·30	·20	...	...	·15	...	...
ENGLAND.																																				
Silloth,	...	·21	1·25	...	·38	·38	...	·11	·08	·35	·12	...	·17	1·25	·05	·16	...	...	...	...	·64	...	·41	...	·05	...	...	·30	·29	...	·10	...	...	·05	·09	...
Shields,	...	·20	...	...	...	...	...	...	...	...	...	...	...	...	2·80	...	...	·25	...	...	·36	·43	...	...	...	...	...	·32	·70	·40	...	...	...	...	...	...
Scarborough,	...	...	...	...	...	·52	...	·70	1·40	·13	...	...	...	...	1·06	...	...	...	...	...	...	...	...	...	...	...	...	·60	...	1·02	...	...	...	...	·17	...
Liverpool,	..	...	·04	.03	·34	·68	...	1·02	·20	·26	·15	·15	·03	...	·59	·09	·31	·17	...	·20	·30	·23	·02	...	...	...	·01	·08	·30	·05	...	...	...	·06	·12	...
Holyhead,	...	·20	1·20	·15	·50	·12	·15	·50	·12	·30	·45	·18	·45	·03	·35	·35	...	...	...	...	...	·50	·35	...	·20	...	·09	·30	·42	...	·15	...	...	·07	...	...
Pembroke,	...	...	...	·46	·33	·45	...	·68	·35	·43	·23	·22	·18	...	·34	...	1·07	·13	·12	...	...	·75	·20	...	·30	...	·15	·50	·53	·11	·10	...	...	·13	...	...
London,	...	...	...	·11	·16	·33	...	·52	·11	·07	·18	·06	...	...	·42	...	·10	...	...	...	...	·16	·14	·09	...	...	...	·21	·63	...	·02	...	...	...	...	...
Dover,	...	...	...	·35	·35	·35	...	1·00	·50	·60	...	·25	...	...	1·00	...	·25	·50	...	...	...	·06	·07	·09	...	...	...	·50	1·02	...	...	...	...	...	·07	...
Portsmouth,	...	...	...	·29	·19	·43	...	·47	·10	·21	...	·07	...	...	·09	...	·28	·33	...	...	...	·25	·50	·19	...	...	...	·70	·60	...	·08	...	...	·14	...	...
Weymouth,	...	..	...	·35	·20	·32	·37	·42	·07	·18	...	...	·07	·02	·08	...	·50	·33	...	·18	·03	·26	·22	·17	...	...	·07	·69	·87	...	·07	...	...	...	·07	...
Plymouth,	...	...	...	...	·17	·80	...	·50	...	·60	...	...	·13	...	...	...	·60	...	...	...	...	·25	...	...	...	...	·21	·45	·25	...	...	...	...	...	...	...
Penzance,	...	...	·15	·24	·27	·52	...	·80	·27	·65	...	·25	·25	...	·60	·15	·55	·20	...	...	...	·45	...	...	·20	...	·15	·85	·55	·23	...	...	...	·15	·15	...
FRANCE.																																				
Brest,	...	...	...	·04	·24	·12	...	2·05	·71	·32	...	...	...	...	·75	...	1·18	·32	...	...	...	·51	·20	·04	...	...	·08	...	1·94	·16	...	...	...	...	...	...
L'Orient,	...	...	...	...	·16	·35	...	·87	·32	·20	·02	...	·08	...	·20	...	·87	·04	...	...	...	·28	·12	...	...	...	·04	·98	·35	·12	...	...	...	...	·08	...
Rochfort,	...	...	·04	·16	·20	·35	...	·48	·20	...	·12	...	...	...	·08	...	·28	·59	...	...	...	·04	...	...	...	...	...	·28	...	·12	...	...	...	...	·12	...
Dijon,	...	...	...	·14	·08	·30	·39	·10	...	·32	...	...	...	...	·26	...	·08	·28	...	...	...	·16	·04	...	...	...	...	·49	·39	...	...	...	...	...	·14	...
BELGIUM.																																				
Brussels,	...	...	·01	·19	·09	·27	·10	·18	·11	·32	·06	·14	...	·41	·55	...	·06	...	...	...	...	...	...	·01	...	...	...	·18	·55	...	...	...	...	·24	·04	...
Luxemburg,	...	...	...	...	·41	1·16	...	·36	.39	·07	·01	...	·11	·20	...	·07	·01	...	...	...	·05	·48	·01	·08	·01	...	...	·56	·96	...	...	...	...	·21	·18	...
NETHERLANDS.																																				
Groningen,	...	...	·10	·17	...	·21	·08	·10	·26	·22	·07	·11	...	·24	·04	·04	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
Helder,	...	...	·10	·06	·22	·02	·20	·42	·30	·01	·09	...	·05	·52	...	·21	·05	·05	·02	...	...	·03	·28	...	...	...	...	·11	·70	·02	...	...	·15	·03	·04	...
Utrecht,	...	...	·01	·26	·14	·01	...	·06	·16	·09	·43	...	·01	·42	·07	·01	·03	...	...	·01	·01	...	·07	...	·01	...	·01	·03	·72	·44	·01	...	...	·90	·18	...
Flushing,	...	...	...	·14	·02	·07	·04	·10	·27	·16	·06	·23	·02	·14	·32	...	·10	...	...	·01	...	...	...	...	...	...	·04	·22	·12	·02	...	...	·01	...	·04	·01
DENMARK.																																				
Kiel,	...	...	·06	·19	...	·08	·27	·02	·06	·04	...	...	...	·02	·15	...	·13	...	...	...	...	·30	·03	...	...	...	...	·38	·20	·12	...	·09	...	·21	...	...
Heligoland (England),	...	...	...	·10	...	·15	...	...	·05	·25	...	...	·09	...	·03	...	...	·30	...	...	...	·08	·05	·05	·04	...	...	...	...	...	...	...	...	...	...	...
SWEDEN.																																				
Haparanda,	·15	...	·09	...	...	·70	...	...	...	...	...	·17	...	...	...	·09	...	·32	...	·23	...	...	...	...	·03	...	...	...	...	...	·42	...	...	...	·39	...
Stockholm,	...	...	...	...	·06	·70	·05	·02	·29	·10	·27	...	·01	...	...	·02	·18	...	...	...	·01	·01	...	·002	...	...	...	...	...	...	·16	...	...	·11	·06	...
RUSSIA.																																				
Kostroma,	...	...	...	...	...	...	...	·02	...	...	·38	·23	...	·34	·23	...	·09	·04	·06	...	·02	·08	·12	·06	...	·02	...	·02	...	...	...	...	...	...	...	...
Lougan,	·62	·60	·14	...	...	...	...	...	·01	...	...	...	...	...	...	·01	...	...	...	...	...	...	...	...	...	...	·01	...	...	...	...	...	...	...	...	...
Orenbourg,	...	...	...	...	...	...	...	...	·44	...	...	...	...	...	·06	·20	...	...	...	...	...	·06	·04	...	...	...	...	...	...	·14	·04	·16	...	...	·18	·26
Slatoust,	...	...	...	...	...	·01	...	...	...	·04	·01	...	...	...	...	·14	·04	...	·02	·57	·51	·01	·02	·07	·03	...	·05	·11	·20	...	...	...	...	...	·04	...
Catherinenbourg,	...	·10	·09	...	...	...	...	...	...	·05	...	...	...	...	...	·06	...	...	...	...	...	·07	...	...	·02	...	·01	...	...	...	...	·01	...	...	...	...
PRUSSIA.																																				
Putbus,	...	...	...	...	·32	...	...	...	...	·59	...	·25	·17	...	...	...	...	...	·01	...	·06	...	·07	...	·08	·03	...	...	...	·08	·37	·11	·03	·11	...	...
Paderborn,	...	...	...	·20	·35	...	...	...	·07	·28	·92	·01	...	·43	...	·04	·01	...	...	...	·01	·18	·11	·06	...	...	...	...	·43	·08	·01	·10	·09	·02	...	...
Breslau,	...	...	...	·24	·20	...	...	...	·02	...	...	·04	·52	...	...	...	...	...	...	...	...	·34	...	·14	·04	...	...	...	·35	...	...	·62	...	...	...	...
AUSTRIAN EMPIRE.																																				
Cracow,	·01	...	...	...	...	·04	·08	·31	...	...	·02	·45	·01	·03	...	...	...	·62	...	...	...	...	·04	·13	·04	...	...	...	...	·15	...	·09	·31	...	...	...
Prague,	...	...	...	...	11	·07	·04	...	...	...	...	·04	...	·13	·04	·02	...	...	...	...	...	·11	...	·19	...	...	...	...	·02	...	·41	·06	...	...	...	...
Kremsmunster,	...	...	...	...	...	...	·19	...	·18	...	·10	·15	...	1·15	...	...	...	...	...	...	...	·15	...	·16	·06	...	...	...	...	...	...	·28	...	...	...	...
Klagenfurt,	...	...	...	...	...	...	1·07	·86	...	...	...	...	...	...	·60	·34	·20	·02	...	...	...	...	...	...	·10	...	...	...	...	...	...	...	...	...	·20	...
GERMAN STATES.																																				
Munich,	...	·01	·01	...	...	·09	·45	·20	·11	·02	·18	...	·13	·16	·21	...	·17	·04	...	...	...	...	·13	·01	·04	...	...	·01	...	·08	...	·01	...	·03	...	...
Dresden,	...	...	...	...	·03	·27	...	...	...	...	·11	...	·11	...	·03	...	...	...	...	...	...	·62	...	·51	·08	...	...	...	·04	·16	·28	·41	·16	...	...	...
SWITZERLAND.																																				
Aarau,	...	...	...	·06	·08	·80	·50	·04	·17	·79	...	...	...	·56	·05	...	...	...	...	...	...	·23	·02	...	...	...	...	·30	·04	...	...	...	...	...	...	...
Geneva,	...	...	·06	·19	...	·08	·27	·02	·06	·04	...	...	...	·02	·15	...	·13	...	...	...	...	·30	·03	...	...	...	...	·38	·20	·12	...	...	...	·21	...	...
SPAIN AND PORTUGAL.																																				
Lisbon,	·20	...	...	...	·05	·01	...	·06	...	...	...	...	...	...	...	...	·04	...	...	·04	...	·28	...	...	...	...	...	...	...	...	...	...	...	...	...	...
Madrid,	·37	·06	...	...	...	...	·08	...	...	...	...	...	...	...	...	...	...	...	...	...	·02	...	...	...	...	·03	·03	...	...	...	...	...	...	...	...	...

TABLE V.—Showing the Direction and Force of the Wind in the Morning at 126 places in Europe, from 26th October to 12th November, from 20th to 26th November, from 3
14th to 18th December 1863. Light air to a moderate breeze printed thus, WNW ; a fresh breeze to a fresh gale, WSW ; a strong gale or hurric

	OCTOBER						NOVEMBER.																					
	26.	27.	28.	29.	30.	31.	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	20.	21.	22.	23.	24.	25.	26.	30.	1.	2
Ireland.																												
Greencastle,	NNW	W	NW	SE	**WSW**	NW	SW	N	SSW	W	W	NW	E	W	NE	NW	NW	S	SW	SSW	NW	SW	NW	SW	SSW	SSE	**S**	W
*Belfast,	NE	NE	W	SW	SW	W	W	N	S	W	SW	SW	SW	NW	SW	SW	SW	SW	SW	SW	SW	SW	SW	SW	SW	SE	E	W
Galway,	ENE	SSW	WSW	SSW	SW	**W**	NW	NW	**W**	SW	WNW	ENE	SW	NW	NW	SW	NW	NW	SSE	SW	W	SSW	SSE	SSE	S	ESE	SSW	WS
Dublin,	N	WSW	WNW	**SW**	SW	**SW**	WSW	**N**	SW	WSW	calm	calm	W	WNW	calm	WSW	WNW	WNW	S	SSE	SW	SW	SW	SW	SE	SE	SW	WN
Valencia,	calm	SE	NNE	**NW**	**WNW**	**WNW**	SSW	**N**	W	WSW	SSW	SE	W	N	calm	NW	WSW	NNW	SSW	SW	SW	S	S	SSW	SSW	S	W	N
Queenstown,	SE	SW	NW	SW	SW	W	...	NW	W	W	SSW	E	WSW	...	NE	W	WNW	NW	S	W	...	W	SW	SW	SW	SE	SW	WN
Cork,	SE	SW	NW	SW	NW	SW	SW	**NW**	SW	SW	SW	SW	NW	NW	NW	NW	NW	NW	S	SW	W	SW	SE	S	S	SE	SW	W
Scotland.																												
East Yell,	SW	WSW	SW	WSW	**WSW**	E	WNW	E	WNW	WNW	W	W	N	NNE	SW	WNW	WNW	W	SW	SW	ESE	W	W	SSE	SSW	SSW	**S**	SSW
Sandwich,	SW	SW	calm	SE	SW	NW	NW	SE	W	NW	NW	W	N	NE	calm	W	W	NW	S	SW	N	SW	W	SE	SE	SE	SE	calm
Stornoway,	SW	SW	SW	SE	W	NW	SW	SW	SW	NW	NW	SW	NE	NE	SW	SW	W	N	SW	SW	NE	SW	SW	S	SW	S	SE	SW
Portree,	SE	calm	W	SSE	SW	W	SE	calm	SW	W	W	WSW	E	E	**E**	SW	W	N	S	S	W	E	SSW	S	**S**	SE	SSE	SSE
Dunrobin,	W	W	W	SE	SW	W	W	NW	NW	NW	NW	NW	N	SE	NE	SW	W	SW	SW	S	NW	E	SW	S	SW	S	S	N
Elgin,	SW	W	W	SE	**SW**	SW	W	SW	SW	W	SW	W	N	N	SW	SW	W	NW	S	SW	W	SW	SW	SE	SW	SE	**SE**	E
*New Pitsligo,	W	W	N	S	S	SW	W	W	W	NW	SW	SW	N	W	W	W	W	NW	S	S	W	SW	W	S	S	S	S	calm
Aberdeen,	N	SW	NW	S	SW	W	W	N	W	NW	SW	W	N	E	W	W	W	W	SW	SW	W	var.	NW	S	S	S	S	SE
Oban,	calm	calm	NNW	SE	**WSW**	W	SW	N	SE	WNW	NW	SW	S	E	S	W	WNW	NNE	S	SSW	W	SSW	calm	**SSW**	S	SE	S	calm
Barry,	NE	S	E	SE	S	SW	calm	N	calm	SW	NW	W	calm	N	calm	calm	calm	N	SW	S	S	S	calm	SE	S	S	SE	NW
Baillieston,	W	SW	SW	E	W	SW	NE	NE	SW	W	SW	NE	NE	SE	NE	W	W	N	NE	S	SW	SW	NE	SW	SW	NE	E	E
Inveresk,	NW	S	W	SE	SW	SW	SW	SE	SW	**SW**	SW	SW	SW	N	W	W	WNW	N	SW	SW	**W**	W	SW	**SW**	SW	SW	ESE	SE
Thurston,	N	S	NW	**SE**	S	SW	S	NE	S	**SW**	S	calm	S	**NE**	N	S	S	N	**SW**	**SE**	**N**	calm	S	SW	SW	S	SE	SE
Cargen,	SW	W	NW	E	W	...	NW	NE	NW	W	NW	N	N	NE	NE	NW	NW	N	SW	S	**W**	SW	SW	S	S	NW	SE	W
South Cairn,	N	W	WSW	S	W	NW	NNW	SSE	S	W	N	SE	S	NE	E	W	W	N	S	S	**W**	W	calm	**S**	S	SSE	**S**	S
England.																												
Silloth,	E	calm	calm	SE	SW	SW	W	NE	W	SW	W	calm	E	NE	N	W	W	N	E	S	**SW**	SW	SW	SW	SW	E	SE	E
Shields,	W	W	WNW	WNW	SSE	NE	NE	SE	NW	...	WNW	NW	WNW	N	NE	WSW	WNW	NE	WSW	SW	SW	W	SW	S	W	SW	S	SSE
Scarborough,	WNW	SSW	SSW	SE	SSW	SW	W	S	WNW	NE	W	NNE	SW	E	NE	SW	WSW	N	SSW	W	W	WSW	S	SSW	SSW	S	S	S
Leeds,	NW	SE	SW	SE	SW	SW	SW	NE	SW	NW	SW	SW	NE	NW	NE	NW	SW	NW	SE	SE	SW	SW	SW	SE	SE	SE	SE	SE
Liverpool,	SSE	SE	S	SE	SSE	SW	SSW	N	S	WSW	SSW	SE	WSW	**W**	S	W	SE	N	SE	SE	WSW	SSE	SSE	ESE	SE	ESE	ESE	ESE
Holyhead,	calm	N	W	SE	SW	SW	WSW	SE	SW	SW	W	ESE	SW	NW	NNE	W	NNW	NW	SSE	SSE	WSW	SW	SW	SSE	SSE	SSE	SSE	ESE
*Nottingham,	SE	SE	SW	W	SW	SW	SW	S	NW	SW	W	W	NNW	NW	N	W	NW	NNW	SW	SW	SW	SW	SW	SSW	S	ESE	SSE	SSE
Yarmouth,	SE	NNE	SE	S	SSW	**WSW**	WSW	S	WNW	W	NW	N	NW	ENE	ENE	SSW	W	W	SSW	S	**WNW**	SW	S	S	S	SSE	SSE	SE
Banbury,	SE	N	S	S	S	**SW**	S	S	SW	W	NE	N	S	W	N	S	S	NW	S	S	S	S	S	S	S	SE	S	SE
Pembroke,	NNE	SW	WNW	**SSW**	**SW**	**W**	W	NW	WSW	W	W	E	NW	N	NNE	NW	NNE	NNW	S	SW	WSW	SW	SSW	S	SE	SE	SSE	**NW**
Clifton,	NE	E	WSW	S	S	WSW	SW	W	S	WSW	WSW	E	WSW	W	N	WSW	NE	NNW	SE	S	WSW	S	E	S	SE	E	SE	WSW
London,	ENE	calm	WSW	SSW	**SSW**	**WSW**	W	NNW	WNW	W	W	N	NW	SW	NE	SW	SE	NNW	SSW	S	SW	WSW	SSW	SW	S	EE	SSE	S
Dover,	N	SE	SW	SW	**SW**	**W**	...	**WSW**	**W**	WSW	W	NW	NW	...	**NE**	W	SSE	N	...	...	...	W	SSW	SSW	SSW	SE	S	**S**
Portsmouth,	NE	NE	WSW	SW	**SW**	**WSW**	W	W	WNW	WSW	W	NE	NW	W	NNE	S	S	N	SW	S	W	SW	SW	SSW	SE	SE	SSW	SW
Weymouth,	NE	calm	SW	SSW	SSW	W	W	WNW	W	W	W	NE	calm	WNW	NNE	W	SE	NW	SE	SSE	W	W	SW	S	ESE	ESE	S	W
Plymouth,	WNW	...	W	SW	**WSW**	**W**	...	W	**WSW**	W	W	E	W	...	N	WSW	N	**NNW**	ESE	...	...	**SW**	calm	ESE	ESE	**NE**	SSE	**SW**
Penzance,	calm	calm	SW	SW	SSW	W	W	NW	WSW	W	W	calm	NW	W	NE	NNW	**N**	NNE	S	S	W	WSW	calm	SSW	S	SE	SW	**NW**
Norway and Sweden.																												
Haparanda,	S	W	SW	S	S	SE	**S**	calm	calm	calm	NE	calm	N	N	NW	W	S	S	E	NE	N	N	N	calm	S	SW	SW	N
Hernösand,	SW	calm	S	SW	**S**	ESE	S	WNW	N	calm	N	W	N	N	N	SSW	S	calm	calm	W	E	calm	calm	ENE	S	calm	NW	WNW
Christiansund,	WNW	calm	SSE	SW	S	ESE	**SW**	SE	SE	calm	**W**	**WNW**	NNE	NE	WNW	SSW	SE	SW	SW	WSW	calm	calm	calm	calm	SE	calm	calm	SE
Skudesnos,	S	S	SE	NW	SW	W	W	SE	SE	SE	NW	**N**	**NW**	NE	NE	**SW**	SE	NW	SW	S	**SE**	NW	SE	S	S	S	S	**SE**
Mandal,	SSW	SSW	SE	SW	**SW**	**WSW**	W	ENE	ESE	ENE	WNW	WNW	WNW	NE	ENE	**SW**	calm	ENE	SW	SSW	SSW	calm	ENE	NE	calm	E	E	SSE
Christiania,	calm	SW	S	SSE	S	NNW	calm	calm	NE	S	SSW	SSW	calm	N	NNE	N	SSW	W	calm	calm	E	NNE	NNW	ENE	W	WSW	SSW	SE
Stockholm,	WSW	WSW	SSE	SSE	S	**S**	**SSW**	SW	ESE	SW	W	WNW	WNW	NNW	WNW	SSW	SSW	NNE	WSW	WSW	WSW	SW	N	NW	SSE	W	WSW	S
Russia.																												
Archangel,	S	S	SE	S	S	SE	SE	SE	W	E	S	N	NW	N	N	S	E	S	W	NE	SW	S	NE	S	W	S	SW	W
Helsingfors,	calm	W	S	SE	SSE	**SSE**	SE	SW	calm	SE	WSW	WSW	NW	NNE	WNW	W	SE	SSW	W	...	W	W	NW	calm	...	W	W	WSW
St Petersburg,	S	W	S	S	SE	S	S	S	SE	S	S	NW	S	...	N	W	SW	S	W	NW	NE	E	WNW	E	S	NW	W	NW
Dorpat,	NE	SW	**SW**	S	W	S	**S**	SW	S	**SW**	SW	SW	NW	NW	NW	SW	SE	S	SW	SW	SW	SW	NW	W	E	SW	W	SW
Riga,	W	SE	SE	SE	SE	SE	SE	S	SE	N	SW	S	SW	SW	W	S	SE	SE	SW	SW	S	SE	S	SW	E	...	SW	S
Libau,	E	SE	SSE	SSE	SSE	S	SE	WSW	SE	WSW	WSW	WNW	NW	W	S	SSE	SE	SSE	W	W	SSW	SW	NW	SSE	ESE	WNW	S	S
Warsaw,	**N**	N	S	SE	SE	**SE**	SW	SW	S	SW	**SW**	W	**W**	W	W	W	S	SE	SW	SW	SW	S	W	W	N	NE	SE	...
Moscow,	calm	SE	calm	calm	calm	E	S	SSW	S	...	S	...	SE	...	NW	N	SW	S	W	W	W	S	N	SW	N	...	W	W
*Kostroma,	NW	calm	W	NW	SE	S	S	S	S	SE	S	SW	SSE	SW	NW	NW	SE	S	W	WNW	W	SE	SE	NW	NNE	NW	WNW	N
Catherinenbourg,	calm	calm	NW	calm	N	W	W	calm	calm	W	SW	**SW**	SW	calm	SW	calm	W	calm	calm	calm	SE	NW	calm	NE	SE	calm	W	**SW**
Slatoust,	SE	calm	N	calm	calm	NW	calm	calm	SE	NW	NW	NW	calm	SE	calm	NW	calm	NW	NW	NW	calm	NW	**SE**	**NW**	NW	NW	**NW**	W
Orenbourg,	**N**	NNE	S	NNE	ENE	**ESE**	ENE	E	**ESE**	N	SSE	S	S	S	WSW	S	S	N	**W**	S	WSW	WSW	WSW	W	WSW	**WNW**	...	...
Kiev,	NE	NE	NW	NE	NE	NE	S	S	S	S	SW	NW	NW	SW	W	W	SW	SW	NW	W	NE	SE	SW	NW	NW	...	N	N

Wind in the Morning at 126 places in Europe, from 26th October to 12th November, from 20th to 26th November, from 30th November to 5th December, and from er 1863. Light air to a moderate breeze printed thus, wnw ; a fresh breeze to a fresh gale, WSW ; a strong gale or hurricane, **NNE.**

NOVEMBER.																			DECEMBER.									
2.	**3.**	**4.**	**5.**	**6.**	**7.**	**8.**	**9.**	**10.**	**11.**	**12.**	**20.**	**21.**	**22.**	**23.**	**24.**	**25.**	**26.**	**30.**	**1.**	**2.**	**3.**	**4.**	**5.**	**14.**	**15.**	**16.**	**17.**	**18.**
n	ssw	w	w	nw	e	w	ne	nw	nw	s	SW	SSW	nw	sw	nw	sw	SSW	SSE	**S**	w	...	WSW	WSW	wsw	wsw	N	WNW	wsw
n	s	w	sw	sw	sw	nw	sw	sw	sw	sw	sw	sw	sw	sw	sw	sw	sw	se	e	w	nw	sw	sw	sw	sw	w	nw	sw
NW	**W**	SW	wnw	ene	SW	nw	nw	sw	NW	nw	SSE	SW	w	SSW	sse	SSE	S	ese	SSW	wsw	**WNW**	e	sw	w	sw	sw	WSW	w
N	sw	WSW	calm	calm	w	wnw	calm	wsw	wnw	wnw	s	sse	SW	sw	sw	SW	se	se	SW	wnw	W	SW	**SW**	calm	sw	wsw	wnw	wsw
N	W	wsw	SSW	se	w	N	calm	NW	WSW	nnw	SSW	sw	SW	s	s	SSW	SSW	s	W	NW	**NNW**	SW	W	s	s	N	NW	calm
nw	W	W	SSW	e	wsw	...	ne	W	wnw	nw	s	w	...	w	sw	SW	sw	SE	SW	wnw	...	SW	SW	sw	sw	NW	NW	nw
NW	SW	SW	sw	sw	nw	NW	nw	NW	NW	nw	S	SW	w	sw	se	S	s	SE	sw	W	NW	sw	**SW**	sw	s	W	n	n
e	wnw	wnw	w	W	N	nne	sw	wnw	wnw	W	sw	sw	ese	w	w	SSE	SSW	SSW	**S**	ssw	sw	WSW	WSW	w	**WSW**	**SW**	n	wnw
se	w	NW	NW	W	n	ne	calm	w	w	nw	S	sw	n	sw	w	SE	SE	SE	SE	calm	calm	W	W	W	sw	**W**	N	W
sw	sw	nw	NW	sw	ne	NE	sw	sw	w	n	SW	SW	ne	sw	sw	S	SW	S	SE	sw	w	SW	SW	W	SW	W	nw	SW
calm	sw	w	w	WSW	e	e	**E**	SW	w	n	S	S	W	e	ssw	S	**S**	SE	SSE	sse	nw	SW	SW	wsw	SSW	W	N	W
NW	NW	NW	nw	nw	n	SE	ne	sw	w	sw	SW	s	NW	e	sw	S	sw	s	S	n	nw	sw	W	W	sw	NW	n	W
sw	sw	w	SW	w	n	n	sw	SW	w	nw	S	sw	W	SW	sw	SE	SW	SE	**SE**	e	w	sw	sw	W	w	**NW**	**NW**	NW
w	w	nw	sw	sw	n	w	w	w	w	nw	s	s	w	sw	w	s	s	s	s	calm	nw	sw	sw	w	sw	w	nw	w
N	w	nw	sw	w	n	e	w	w	w	W	sw	sw	w	var.	nw	S	s	S	S	se	n	sw	SW	sw	sw	w	N	w
n	se	wnw	nw	sw	s	e	s	w	wnw	nne	S	SSW	w	ssw	calm	**SSW**	S	se	S	calm	NNW	SW	W	w	wsw	NW	N	w
n	calm	sw	nw	w	calm	n	calm	calm	calm	n	sw	s	s	s	calm	se	s	s	se	nw	nw	w	S	calm	sw	sw	n	sw
ne	sw	W	sw	ne	ne	SE	ne	W	W	n	ne	s	SW	sw	ne	SW	sw	ne	e	e	NE	SW	W	W	sw	W	NW	w
SE	sw	**SW**	sw	sw	sw	n	w	w	wnw	n	sw	sw	**W**	w	sw	**SW**	SW	sw	ESE	se	SW	**SW**	**SSW**	W	w	SW	n	SW
ne	s	**SW**	s	calm	s	**NE**	n	s	s	n	**SW**	**SE**	**N**	calm	s	sw	sw	s	se	se	**N**	**SW**	**SW**	**W**	sw	sw	**NW**	**S**
NE	nw	W	NW	n	n	ne	ne	nw	nw	n	SW	S	**W**	sw	sw	S	S	nw	SE	w	n	**N**	**SW**	w	w	**NW**	**N**	NW
SSE	s	W	n	se	s	ne	e	W	w	n	S	S	**W**	w	calm	**S**	S	SSE	**S**	s	NW	SW	S	w	WNW	W	N	nw
NE	w	SW	w	calm	e	ne	n	w	w	n	e	s	**SW**	sw	sw	SW	sw	e	SE	e	NW	W	**SW**	w	w	e	W	w
SE	NW	...	wnw	nw	wnw	n	ne	wsw	wnw	ne	wsw	sw	sw	w	sw	s	w	sw	S	sse	SSW	nw	WSW	nw	nw	NW	**NE**	nw
S	wnw	NE	w	nne	sw	E	ne	sw	wsw	n	ssw	W	W	wsw	s	ssw	SSW	s	S	S	**SSE**	w	SW	wnw	w	W	**NNE**	wsw
ne	SW	NW	SW	SW	ne	NW	NE	NW	sw	NW	se	SE	SW	SW	SW	SE	SE	SE	SE	SE	SW	SW	SW	NW	SW	NW	SW	SW
n	s	WSW	ssw	se	wsw	**W**	s	W	se	n	se	SE	WSW	sse	sse	ese	se	ese	ESE	ese	**W**	wsw	SSW	w	wsw	wsw	**NW**	ssw
SE	SW	SW	w	ese	sw	NW	nne	W	NNW	NW	SSE	SSE	WSW	sw	sw	SSE	SSE	sse	SSE	ESE	**WNW**	WSW	SSW	wsw	sw	wsw	**NNW**	w
s	nw	sw	w	w	nnw	nw	n	w	nw	nnw	sw	sw	sw	sw	sw	ssw	s	ese	sse	sse	wnw	nw	sw	wsw	sw	n	wsw	sw
S	WNW	W	NW	N	NW	ene	ENE	SSW	W	W	SSW	S	**WNW**	SW	S	s	S	SSE	SSE	SE	**SSW**	WNW	SW	NW	wnw	WSW	**NNW**	nne
S	sw	W	NE	n	s	w	N	S	s	nw	s	S	S	S	s	S	S	se	s	se	**W**	W	SW	NW	w	NW	SW	n
NW	WSW	W	W	e	nw	N	nne	NW	nne	nnw	S	SW	WSW	sw	ssw	S	se	se	sse	**NW**	**NW**	WNW	**SSW**	wnw	sw	NW	N	calm
w	s	wsw	wsw	e	wsw	w	n	wsw	ne	nnw	se	s	wsw	s	e	s	se	e	se	wsw	WNW	wnw	sw	wnw	wnw	nw	nw	nnw
NNW	wnw	W	W	n	nw	sw	NE	sw	se	nnw	ssw	S	SW	wsw	ssw	sw	s	ee	sse	S	**W**	W	WSW	w	w	WSW	NW	ne
WSW	**W**	wsw	w	nw	nw	...	**NE**	w	sse	n	...	...	...	w	ssw	ssw	ssw	se	S	**S**	**W**	...	W	w	w	W	**NNW**	n
W	wnw	wsw	W	ne	nw	W	NNE	s	S	n	sw	S	W	SW	sw	ssw	se	se	ssw	SW	**WNW**	WNW	SW	nw	w	WSW	NNW	nne
wnw	w	w	w	ne	calm	WNW	nne	w	SE	NW	se	SSE	W	w	sw	S	ESE	ESE	S	w	**NW**	wnw	W	wnw	calm	W	NW	nnw
W	**WSW**	W	W	E	w	...	N	WSW	N	**NNW**	ESE	...	...	**SW**	calm	ESE	ESE	**NE**	sse	**SW**	**W**	w	**SSW**	calm	calm	SSW	**NW**	nnw
NW	WSW	W	w	calm	nw	W	NE	nnw	**N**	NNE	S	S	W	wsw	calm	SSW	s	SE	sw	**NW**	NW	wnw	SW	calm	calm	NW	N	calm
calm	calm	calm	ne	calm	n	n	NW	w	s	S	e	ne	n	n	n	calm	s	sw	sw	n	S	S	**S**	w	calm	w	S	n
wnw	n	calm	n	W	n	n	N	ssw	s	calm	calm	W	e	calm	calm	ene	S	calm	nw	wnw	S	S	SW	calm	calm	s	sw	nw
se	se	calm	**W**	**WNW**	NNE	ne	wnw	ssw	SE	sw	SW	WSW	calm	calm	calm	calm	se	calm	calm	SE	SSE	sw	SSE	SW	calm	calm	**N**	s
SE	SE	SE	NW	**N**	**NW**	ne	ne	**SW**	se	nw	sw	S	**SE**	nw	se	s	S	S	S	**SE**	sw	N	**S**	se	s	SW	N	se
ene	ese	ENE	wnw	wnw	WNW	ne	ene	**SW**	calm	ENE	SW	ssw	SSW	calm	ene	ne	calm	e	E	sse	se	nnw	S	calm	calm	SW	w	calm
calm	NE	s	ssw	ssw	calm	n	nne	n	ssw	w	calm	calm	e	nne	nnw	ene	w	wsw	ssw	se	ese	NE	s	calm	calm	nw	nw	calm
sw	ese	sw	W	wnw	WNW	nnw	wnw	ssw	ssw	nne	wsw	wsw	wsw	sw	n	nw	sse	W	WSW	s	SSE	SSE	SSE	calm	nnw	S	ssw	nw
SE	W	E	s	n	NW	N	N	S	e	S	W	ne	sw	S	NE	S	W	S	SW	W	N	E	SE	e	se	s	e	SE
sw	calm	se	wsw	wsw	nw	nne	wnw	w	SE	ssw	w	...	w	w	NW	calm	...	w	w	wsw	se	e	s	wnw	ne	sse	sse	W
S	se	S	S	NW	s	...	n	W	SW	S	w	nw	ne	e	wnw	e	S	NW	w	nw	nw	**S**	S	w	sw	sw	S	SW
sw	S	**SW**	sw	sw	nw	nw	NW	SW	se	s	sw	sw	sw	SW	nw	w	E	SW	W	sw	se	**SE**	S	nw	s	ssw	s	w
S	se	N	SW	s	SW	sw	w	s	SE	se	sw	sw	s	se	s	sw	E	...	sw	s	se	**SE**	SE	NW	SW	s	s	sw
WSW	SE	wsw	WSW	WNW	NW	W	s	sse	SE	sse	W	W	ssw	sw	nw	sse	ESE	wnw	s	s	sse	SSE	sse	ne	ene	sse	S	...
sw	s	sw	**SW**	W	**W**	W	w	w	S	se	sw	sw	SW	S	W	W	n	ne	se	...	se	S	SE	NW	SE	sw	se	sw
SSW	s	...	s	...	se	...	NW	N	sw	s	w	w	w	s	n	sw	N	...	w	W	n	W	se	n	n	N	N	S
s	s	se	s	sw	sse	sw	nw	nw	se	s	w	wnw	w	se	se	nw	nne	nw	wnw	n	n	se	s	w	n	n	n	se
calm	calm	w	sw	**SW**	SW	calm	sw	calm	w	calm	calm	calm	se	nw	calm	ne	se	calm	W	**SW**	w	n	nw	se	se	SE	e	ne
calm	se	NW	nw	nw	calm	se	calm	nw	calm	nw	nw	NW	calm	NW	**SE**	**NW**	nw	nw	**NW**	w	nw	nw	se	SE	se	SE	**SE**	SE
E	**ESE**	N	SSE	S	s	S	WSW	S	S	N	**W**	S	WSW	wsw	wsw	W	WSW	**WNW**	...	...	...	...	...	...	...	...	...	...
s	s	s	sw	nw	nw	sw	w	w	sw	SW	nw	w	ne	se	sw	nw	nw	...	n	n	calm	se	e	nw	nw	nw	nw	s
					S	calm	sw	calm	calm	calm	calm	calm	calm	SW	sw	calm	W	W	n	calm	calm	e	E	S	w	w	W	w
																nw	n	N	n	n	N	NE	NE	**NW**	NW	N	calm	calm

ENGLAND.																												
Silloth,	E	calm	calm	SE	SW	SW	W	NE	W	SW	W	calm	E	NE	N	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
Shields,	W	W	WNW	WNW	SSE	NE	NE	SE	NW	...	WNW	NW	WNW	N	NE	WSW	WNW	NE	WSW	SW	SW	W	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
Scarborough,	WNW	SSW	SSW	SE	SSW	SW	W	S	WNW	NE	W	NNE	SW	E	NE	SW	WSW	N	SSW	W	W	WSW	S	SSW	**SSW**	S	S	[illegible]
Leeds,	NW	SE	SW	SE	SW	SW	SW	NE	SW	NW	SW	SW	NE	NW	NE	NW	SW	NW	SE	SE	SW	SW	SW	SE	SE	SE	SE	[illegible]
Liverpool,	SSE	SE	S	SE	SSE	SW	SSW	N	S	WSW	SSW	SE	WSW	**W**	S	W	SE	N	SE	SE	WSW	SSE	SSE	ESE	SE	ESE	ESE	[illegible]
Holyhead,	calm	N	W	SE	SW	SW	WSW	SE	SW	SW	W	ESE	SW	NW	NNE	W	NNW	NW	SSE	SSE	WSW	SW	SW	ESE	SSE	SSE	SSE	[illegible]
*Nottingham,	SE	SE	SW	W	SW	SW	SW	S	NW	SW	W	W	NNW	NW	N	W	NW	NNW	SW	SW	SW	SW	SW	SSW	S	ESE	SSE	[illegible]
Yarmouth,	SE	NNE	SE	S	SSW	**WSW**	WSW	S	WNW	W	NW	N	NW	ENE	ENE	SSW	W	W	SSW	S	**WNW**	SW	S	S	S	SSE	SSE	[illegible]
Banbury,	SE	N	S	S	S	**SW**	S	S	SW	W	NE	N	S	W	N	S	S	NW	S	S	S	S	S	S	S	SE	S	[illegible]
Pembroke,	NNE	SW	WNW	**SSW**	**SW**	**W**	W	NW	WSW	W	W	E	NW	N	NNE	NW	NNE	NNW	S	SW	WSW	SW	SSW	S	SE	SE	SSE	[illegible]
Clifton,	NE	E	WSW	S	S	WSW	SW	W	S	WSW	WSW	E	WSW	W	N	WSW	NE	NNW	SE	S	WSW	S	E	S	SE	E	SE	[illegible]
London,	ENE	calm	WSW	SSW	**SSW**	**WSW**	W	NNW	WNW	W	W	N	NW	SW	NE	SW	SE	NNW	SSW	S	SW	WSW	SSW	SW	S	EE	SSE	[illegible]
Dover,	N	SE	SW	SW	**SW**	**W**	...	**WSW**	**W**	WSW	W	NW	NW	...	**NE**	W	SSE	N	...	...	...	W	SSW	SSW	SSW	SE	S	[illegible]
Portsmouth,	NE	NE	WSW	SW	**SW**	**WSW**	W	W	WNW	WSW	W	NE	NW	W	NNE	S	S	N	SW	S	W	SW	SW	SSW	SE	SE	SSW	[illegible]
Weymouth,	NE	calm	SW	SSW	SSW	W	W	WNW	W	W	W	NE	calm	WNW	NNE	W	SE	NW	SE	SSE	W	W	SW	S	ESE	ESE	S	[illegible]
Plymouth,	WNW	...	W	SW	**WSW**	**W**	...	W	**WSW**	W	W	E	W	...	N	WSW	N	**NNW**	ESE	...	...	**SW**	calm	ESE	ESE	**NE**	SSE	[illegible]
Penzance,	calm	calm	SW	SW	SSW	W	W	NW	WSW	W	W	calm	NW	W	NE	NNW	**N**	NNE	S	S	W	WSW	calm	SSW	S	SE	SW	[illegible]
NORWAY AND SWEDEN.																												
Haparanda,	S	W	SW	S	S	SE	**S**	calm	calm	calm	NE	calm	N	N	NW	W	S	S	E	NE	N	N	N	calm	S	SW	SW	N
Hernösand,	SW	calm	S	SW	**S**	ESE	S	WNW	N	calm	N	W	N	N	N	SSW	S	calm	calm	W	E	calm	calm	ENE	S	calm	NW	[illegible]
Christiansund,	WNW	calm	SSE	SW	S	ESE	**SW**	SE	SE	calm	**W**	**WNW**	NNE	NE	WNW	SSW	SE	SW	SW	WSW	calm	calm	calm	calm	SE	calm	calm	SE
Skudesnos,	S	S	SE	NW	SW	W	W	SE	SE	SE	NW	**N**	**NW**	NE	NE	**SW**	SE	NW	SW	S	**SE**	NW	SE	S	S	S	S	**SE**
Mandal,	SSW	SSW	SE	SW	**SW**	**WSW**	W	ENE	ESE	ENE	WNW	WNW	WNW	NE	ENE	**SW**	calm	ENE	SW	SSW	SSW	calm	ENE	NE	calm	E	E	SSE
Christiania,	calm	SW	S	SSE	S	NNW	calm	calm	NE	S	SSW	SSW	calm	N	NNE	N	SSW	W	calm	calm	E	NNE	NNW	ENE	W	WSW	SSW	SE
Stockholm,	WSW	WSW	SSE	SSE	S	**S**	**SSW**	SW	ESE	SW	W	WNW	WNW	NNW	WNW	SSW	SSW	NNE	WSW	WSW	WSW	SW	N	NW	SSE	W	WSW	S
RUSSIA.																												
Archangel,	S	S	SE	S	S	SE	SE	SE	W	E	S	N	NW	N	N	S	E	S	W	NE	SW	S	NE	S	W	S	SW	W
Helsingfors,	calm	W	S	SE	SSE	**SSE**	SE	SW	calm	SE	WSW	WSW	NW	NNE	WNW	W	SE	SSW	W	...	W	W	NW	calm	...	W	W	WSW
St Petersburg,	S	W	S	S	SE	S	S	S	SE	S	S	NW	S	...	N	W	SW	S	W	NW	NE	E	WNW	E	S	NW	W	NW
Dorpat,	NE	SW	**SW**	S	W	S	**S**	SW	S	**SW**	SW	SW	NW	NW	NW	SW	SE	S	SW	SW	SW	SW	NW	W	E	SW	W	SW
Riga,	W	SE	SE	SE	SE	SE	SE	S	SE	N	SW	S	SW	SW	W	S	SE	SE	SW	SW	S	SE	S	SW	E	...	SW	S
Libau,	E	SE	SSE	SSE	SSE	S	SE	WSW	SE	WSW	WSW	WNW	NW	W	S	SSE	SE	SSE	W	W	SSW	SW	NW	SSE	ESE	WNW	S	S
Warsaw,	**N**	N	S	SE	SE	**SE**	SW	SW	S	SW	**SW**	W	**W**	W	W	W	S	SE	SW	SW	SW	S	W	W	N	NE	SE	...
Moscow,	calm	SE	calm	calm	calm	E	S	SSW	S	...	S	...	SE	...	NW	N	SW	S	W	W	W	S	N	SW	N	...	W	W
*Kostroma,	NW	calm	W	NW	SE	S	S	S	S	SE	S	SW	SSE	SW	NW	NW	SE	S	W	WNW	W	SE	SE	NW	NNE	NW	WNW	N
Catherinenbourg,	calm	calm	NW	calm	N	W	W	calm	calm	W	SW	**SW**	SW	calm	SW	calm	W	calm	calm	calm	SE	NW	calm	NE	SE	calm	W	**SW**
Slatoust,	SE	calm	N	calm	calm	NW	calm	calm	SE	NW	NW	NW	calm	SE	calm	NW	calm	NW	NW	NW	calm	NW	**SE**	**NW**	NW	NW	**NW**	W
Orenbourg,	**N**	NNE	S	NNE	ENE	**ESE**	ENE	E	**ESE**	N	SSE	S	S	S	WSW	S	S	N	**W**	S	WSW	WSW	WSW	W	WSW	**WNW**	...	...
Kiev,	NE	NE	NW	NE	NE	NE	S	S	S	S	SW	NW	NW	SW	W	W	SW	SW	NW	W	NE	SE	SW	NW	NW	...	N	N
Lougan,	NE	NE	NE	calm	calm	calm	NE	S	calm	calm	calm	calm	S	calm	SW	calm	calm	calm	calm	calm	calm	SW	SW	calm	W	W	N	calm
Nicolaiev,	**NE**	**NNW**	N	NE	calm	NE	SE	SE	SSE	S	W	calm	NW	calm	calm	calm	W	SE	W	W	SW	S	SE	NW	N	N	N	N
Odessa,	**NE**	**NE**	**NE**	NE	NE	E	**SW**	**SW**	NE	S	calm	calm	W	NW	W	NE	NE	S	NW	NW	SW	SW	W	N	N	N	NE	NW
Tiflis,	NW	calm	SW	calm	calm	NW	calm	SE	calm	calm	NW	NW	calm	calm	NW	NW	SE	calm	calm	calm	calm	calm	calm	**NW**	**NW**	calm	calm	N
DENMARK.																												
Iviglut (Greenland),	NW	N	N	calm	NE	calm	calm	calm	calm	calm	calm	E	calm	**SE**	**NW**	N	calm	N	**E**	**E**	calm	calm	calm	NW	NE	SE	NE	NE
Skagen,	SSW	SW	S	S	W	**W**	W	WSW	S	S	NW	NW	NW	NE	N	W	SSW	W	W	SW	SW	W	W	SW	S	SE	SSE	S
Copenhagen,	SE	SSE	SE	S	SSW	SW	SW	SSW	S	SSW	W	NW	WNW	calm	NE	SW	SW	WSW	WNW	SW	SW	SSW	W	SW	SSW	SSE	S	SSE
*Kiel,	SE	E	S	SW	SW	SW	SW	SW	SE	S	W	W	NW	W	NE	SW	SW	SW	SW	W	SW	W	SW	SW	S	E	E	SSE
Heligoland (England),	SE	SE	SE	SW	WNW	**WSW**	...	...	SSE	WSW	**WNW**	**NNW**	**NNW**	...	E	S	SW	W	W	...	...	**W**	WSW	WSW	SSW	...	SE	S
PRUSSIA.																												
*Königsberg,	NE	SW	E	SE	SE	SE	SE	SE	SE	SW	W	SW	NW	NW	N	W	SE	W	W	W	W	SW	SW	W	NE	S	S	SE
Putbus,	SW	SW	SW	SW	S	**W**	**SW**	SW	SE	S	**NW**	**NW**	**NW**	NW	N	S	SW	W	W	S	SW	W	NW	SW	N	W	SE	SE
Paderborn,	SE	S	SSE	SW	**SW**	SW	SW	SE	SW	SW	W	W	NW	SW	NNE	E	SE	SE	SE	SE	SW	S	S	SW	SW	SE	SE	ESE
Breslau,	SE	SE	SE	SE	S	SE	SW	SE	W	SE	SW	W	**W**	W	E	ESE	ESE	NW	SE	SE	SE	S	W	ESE	NW	SE	SE	SE
AUSTRIAN EMPIRE.																												
Cracow,	N	NE	NE	E	NE	NE	W	W	E	W	W	**W**	SW	SW	NW	NE	E	SW	W	W	W	W	W	SW	W	NE	E	N
Vienna,	N	calm	ESE	WSW	calm	calm	calm	calm	WNW	calm	NW	**W**	calm	calm	W	calm	SE	calm	calm	SSE	S	SSE	calm	calm	...	calm	SW	SE
Kremsmunster,	NE	NE	NE	NE	NE	W	NE	NE	W	W	**W**	**W**	W	W	NE	**E**	E	E	W	W	W	W	E	E	E	E	E	**E**
Klagenfurt,	SW	NW	W	SW	W	SW	**SW**	**E**	SE	NW	W	W	W	NE	**E**	E	SE	W	S	S	SW	W	W	SE	W	NE	E	E
GERMAN STATES.																												
Leipzig,	SE	SE	SSE	SSE	S	SSW	SSE	S	S	SSW	WSW	WSW	SW	SSW	ENE	ENE	S	SW	SSE	SSW	SSW	S	SW	S	...	ENE	SW	SE
*Bamberg,	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	SE	SE
Munich,	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	E	E
NETHERLANDS.																												
Groningen,	E	ESE	E	S	S	S	S	S	S	SW	SSW	NW	W	SW	NE	SE	S	S	S	S	S	SSW	S	WSW	S	E	E	SE
Leeuwarden,	ESE	ESE	SE	SSE	S	SW	**SSW**	**SE**	W	SW	W	NW	SW	SSW	NE	SE	SW	SW	SSE	SSE	SSW	SSW	SSW	SSW	S	E	ESE	SE
Helder,	SSE	NNE	SSW	S	SW	**WSW**	SW	SSE	**WSW**	WSW	WNW	W	WNW	E	ENE	S	WNW	S	S	S	SW	WSW	SSW	SSW	S	ESE	SSE	SSE
*Amsterdam,	SSE	SSE	S	SW	SSW	SW	SE	ESE	S	SSW	WSW	N	S	SSE	NNE	SSE	SSW	SW	E	S	S	S	S	S	S	ESE	SSE	E
Utrecht,	SE	SE	SSW	SW	SSW	SW	SW	SE	**NW**	SW	W	NNE	SW	SSW	NE	SE	SW	SW	SSW	S	SW	SW	SSW	SSW	SSE	ENE	SE	SE
Hellevoetsluis,	SE	E	S	S	SSW	**SW**	SW	SE	**NW**	SW	SW	NNW	NNW	SE	NNE	SSE	SSW	N	S	S	SW	SSW	S	SW	SE	E	ESE	SE
Breda,	E	SE	SW	SW	SW	SW	WSW	SSW	W	W	WNW	NW	N	SW	NE	SE	SW	SSW	SSW	SW	WSW	WSW	SW	SSW	SSE	E	SSE	SSE
Flushing,	SE	SE	SW	SW	SW	SW	SW	**S**	**NW**	W	NE	NE	W	S	E	WSW	SW	E	SW	S	SW	SW	SW	SW	SE	SE	SSE	SSE
BELGIUM.																												
Brussels,	NNE	S	SSW	S	SSW	SW	SSE	SSE	SSW	WSW	SSW	WNW	SSW	S	NNE	SE	SE	N	S	SSE	SSW	SSE	SE	SSW	SE	NE	SE	SE
[illegible]	[illegible]	[illegible]	WNW	S	SW	SW	SW	S	SW	SW	SW	NW	S	N	N	E	SW	N	S	SSW	SW	S	SSW	SW	ESE	N	S	S

w	NE	w	SW	w	calm	e	ne	n	w	w	n	e	s	**SW**	sw	sw	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
ne	SE	NW	...	wnw	nw	wnw	n	ne	wsw	wnw	ne	wsw	sw	sw	w	sw	s	w	sw	**S**	**SSE**	**SSW**	nw	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
w	S	wnw	NE	w	nne	sw	E	ne	sw	wsw	n	ssw	W	W	wsw	s	ssw	SSW	s	S	S	**SSE**	w	**SW**	**WNW**	**W**	**W**	[illegible]	**WSW**
SW	ne	SW	NW	SW	SW	ne	NW	NE	NW	sw	NW	se	SE	SW	SW	SW	SE	SE	SE	SE	SE	SW	SW	SW	NW	SW	NW	SW	**SW**
SSW	n	s	WSW	ssw	se	wsw	**W**	s	W	se	n	se	SE	WSW	sse	sse	ese	se	ESE	ESE	ese	**W**	wsw	SSW	w	wsw	wsw	**NW**	ssw
WSW	SE	**SW**	SW	w	ese	sw	NW	nne	W	NNW	NW	SSE	SSE	WSW	sw	sw	SSE	SSE	sse	SSE	ESE	**WNW**	WSW	SSW	wsw	sw	wsw	**NNW**	w
sw	s	nw	sw	w	w	nnw	nw	n	w	nw	nnw	sw	sw	sw	sw	sw	ssw	s	ese	sse	sse	wnw	nw	sw	wsw	sw	n	wsw	sw
WSW	S	WNW	W	NW	N	NW	ene	ENE	SSW	W	W	SSW	S	**WNW**	SW	S	s	S	SSE	SSE	SE	**SSW**	WNW	SW	NW	wnw	WSW	**NNW**	nne
S	S	sw	W	NE	n	s	w	N	S	s	nw	s	S	S	S	s	S	S	se	s	se	**W**	W	SW	NW	w	NW	SW	n
w	NW	WSW	W	W	e	nw	N	nne	NW	nne	nnw	S	SW	WSW	sw	ssw	S	se	se	sse	**NW**	**NW**	WNW	**SSW**	wnw	sw	NW	N	calm
sw	w	s	wsw	wsw	e	wsw	w	n	wsw	ne	nnw	se	s	wsw	s	e	s	se	e	se	wsw	WNW	wnw	sw	wnw	wnw	nw	nw	nnw
W	NNW	wnw	W	W	n	nw	sw	NE	sw	se	nnw	ssw	S	SW	wsw	ssw	sw	s	ee	sse	S	**W**	W	WSW	w	w	WSW	NW	ne
...	**WSW**	**W**	wsw	w	nw	nw	...	**NE**	w	sse	n	...	...	...	w	ssw	ssw	ssw	se	S	**S**	**W**	...	W	w	w	W	**NNW**	n
W	W	wnw	wsw	W	ne	nw	W	NNE	s	S	n	sw	S	W	SW	sw	ssw	se	se	ssw	SW	**WNW**	WNW	SW	nw	w	WSW	NNW	nne
W	wnw	w	w	w	ne	calm	WNW	nne	w	SE	NW	se	SSE	W	w	sw	S	ESE	ESE	S	w	**NW**	wnw	W	wnw	calm	W	NW	nnw
...	W	**WSW**	W	W	E	w	...	N	WSW	N	**NNW**	ESE	...	...	**SW**	calm	ESE	ESE	**NE**	sse	**SW**	**W**	w	**SSW**	calm	calm	SSW	**NW**	nnw
W	NW	WSW	W	w	calm	nw	W	NE	nnw	**N**	NNE	S	S	W	wsw	calm	SSW	s	SE	sw	**NW**	NW	wnw	SW	calm	calm	NW	N	calm
S	calm	calm	calm	ne	calm	n	n	NW	w	s	S	e	ne	n	n	n	calm	s	sw	sw	n	S	S	**S**	w	calm	w	S	n
s	wnw	n	calm	n	W	n	n	N	ssw	s	calm	calm	W	e	calm	calm	ene	S	calm	nw	wnw	S	S	SW	calm	calm	s	sw	nw
SW	se	se	calm	**W**	**WNW**	NNE	ne	wnw	ssw	SE	sw	SW	WSW	calm	calm	calm	calm	se	calm	calm	SE	SSE	sw	SSE	SW	calm	calm	**N**	s
W	SE	SE	SE	NW	**N**	**NW**	ne	ne	**SW**	se	nw	sw	S	**SE**	nw	se	s	S	S	S	**SE**	sw	N	**S**	se	s	SW	N	se
W	ene	ese	ENE	wnw	wnw	WNW	ne	ene	**SW**	calm	ene	SW	ssw	SSW	calm	ene	ne	calm	e	E	sse	se	nnw	S	calm	calm	SW	w	calm
calm	calm	NE	s	ssw	ssw	calm	n	nne	n	ssw	w	calm	calm	e	nne	nnw	ene	w	wsw	ssw	se	ese	NE	s	calm	calm	nw	nw	calm
SSW	sw	ese	sw	W	wnw	WNW	nnw	wnw	ssw	ssw	nne	wsw	wsw	wsw	sw	n	nw	sse	W	WSW	s	SSE	SSE	SSE	calm	nnw	S	ssw	nw
SE	SE	W	E	s	n	NW	N	N	S	e	S	W	ne	sw	S	NE	S	W	S	SW	W	N	E	SE	e	se	s	e	SE
SE	sw	calm	se	wsw	wsw	nw	nne	wnw	w	SE	ssw	w	...	w	w	NW	calm	...	w	w	wsw	se	e	s	wnw	ne	sse	sse	W
S	S	se	S	S	NW	s	...	n	W	SW	S	w	nw	ne	e	wnw	e	S	NW	w	nw	nw	**S**	S	w	sw	sw	S	SW
S	sw	S	**SW**	sw	sw	nw	nw	NW	SW	se	s	sw	sw	sw	SW	nw	w	E	SW	W	sw	se	**SE**	S	nw	s	ssw	s	w
SE	S	se	N	SW	s	SW	sw	w	s	SE	se	sw	sw	s	se	s	sw	E	...	sw	s	se	**SE**	SE	NW	SW	s	s	sw
se	WSW	SE	wsw	WSW	WNW	NW	W	s	sse	SE	sse	W	W	ssw	sw	nw	sse	ESE	wnw	s	s	sse	SSE	sse	ne	ene	sse	S	...
sw	sw	s	sw	**SW**	W	**W**	W	w	w	S	se	sw	sw	SW	S	W	W	n	ne	se	...	se	S	SE	NW	SE	sw	se	sw
s	SSW	s	...	s	...	se	...	NW	N	sw	s	w	w	w	s	n	sw	N	...	w	W	n	W	se	n	n	N	N	S
s	s	s	se	s	sw	sse	sw	nw	nw	se	s	w	wnw	w	se	se	nw	nne	nw	wnw	n	n	se	s	w	n	n	n	se
w	calm	calm	w	sw	**SW**	SW	calm	sw	calm	w	calm	calm	calm	se	nw	calm	ne	se	calm	W	**SW**	w	n	nw	se	se	SE	e	ne
calm	calm	se	NW	nw	nw	calm	se	calm	nw	calm	nw	nw	NW	calm	NW	**SE**	**NW**	nw	nw	**NW**	w	nw	nw	se	SE	se	SE	**SE**	SE
ENE	E	**ESE**	N	SSE	S	s	S	WSW	S	S	N	**W**	S	WSW	wsw	wsw	W	WSW	**WNW**	...	...	...	...	...	...	...	...	...	...
S	s	s	s	sw	nw	nw	sw	w	w	sw	SW	nw	w	ne	se	sw	nw	nw	...	n	n	calm	se	e	nw	nw	nw	nw	s
ne	s	calm	calm	calm	calm	S	calm	sw	calm	calm	calm	calm	calm	calm	SW	sw	calm	W	W	n	calm	calm	e	E	S	w	w	W	w
SE	se	SSE	s	w	calm	NW	calm	calm	calm	w	SE	w	w	sw	s	se	nw	n	N	n	n	N	NE	NE	**NW**	NW	N	calm	calm
SW	**SW**	NE	S	calm	calm	W	nw	w	ne	NE	S	nw	nw	sw	sw	W	n	n	N	NE	nw	**NE**	SE	ne	**N**	N	NE	nw	calm
calm	se	calm	calm	NW	nw	calm	calm	nw	NW	se	calm	calm	calm	calm	calm	calm	**NW**	**NW**	calm	calm	n	calm	calm	se	calm	**NW**	nw	calm	nw
calm	calm	calm	calm	calm	E	calm	**SE**	**NW**	n	calm	n	**E**	**E**	calm	calm	calm	NW	NE	se	NE	NE	se	NE	...	calm	calm	**E**	e	calm
w	WSW	s	S	NW	NW	nw	ne	n	W	ssw	w	w	sw	SW	w	w	sw	s	se	SSE	S	s	e	SW	n	e	sw	w	n
SW	ssw	s	ssw	w	nw	wnw	calm	ne	sw	sw	wsw	wnw	sw	sw	SSW	w	sw	ssw	sse	s	sse	s	sse	wnw	ne	nw	sw	sw	NE
sw	sw	se	s	w	w	nw	w	ne	sw	sw	sw	sw	w	sw	w	sw	sw	s	e	e	sse	sw	w	sw	w	w	w	sw	nw
...	...	SSE	wsw	**WNW**	**NNW**	**NNW**	...	E	S	SW	w	w	...	...	**W**	WSW	wsw	ssw	...	SE	S	...	...	...	...	...	...	...	...
se	se	se	sw	w	sw	nw	nw	n	w	se	w	w	w	w	sw	sw	w	ne	s	s	se	se	se	se	nw	sw	se	sw	w
SW	SW	SE	S	**NW**	**NW**	**NW**	nw	n	s	sw	w	w	s	sw	W	nw	sw	n	w	SE	se	s	**SW**	w	n	**N**	w	w	N
sw	se	SW	SW	W	w	nw	sw	nne	e	se	se	se	se	sw	s	s	sw	sw	se	se	ESE	s	**N**	sw	n	w	sw	sw	n
sw	se	w	SE	SW	w	**W**	W	e	ese	ese	nw	se	se	se	S	w	ese	nw	se	SE	se	s	s	SW	W	**W**	s	s	s
w	w	e	w	w	**W**	SW	sw	nw	ne	e	sw	w	w	w	w	w	sw	w	ne	e	n	w	n	ne	sw	ssw	w	e	nw
calm	calm	wnw	calm	nw	**W**	calm	calm	w	calm	se	calm	calm	sse	s	sse	calm	calm	...	calm	sw	se	se	calm	WNW	WNW	wnw	nw	calm	calm
ne	ne	w	w	**W**	**W**	w	W	ne	**E**	e	e	w	w	w	w	e	e	e	e	E	**E**	ne	w	w	**W**	W	w	w	w
SW	**E**	se	nw	w	w	w	ne	**E**	e	se	w	s	s	sw	w	w	se	w	ne	e	E	sw	w	se	e	s	nw	w	sw
SSE	S	S	SSW	WSW	wsw	SW	ssw	ENE	ENE	s	sw	SSE	ssw	SSW	s	sw	s	...	ENE	sw	SE	SSW	WSW	**SW**	sw	wsw	sw	ssw	NW
...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	se	se	sse	w	s	nw	nw	se	s	nw
...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	e	e	w	**W**	sw	**W**	W	s	s	W
S	S	s	sw	ssw	nw	w	sw	ne	se	s	s	s	s	s	ssw	s	wsw	s	e	e	se	s	**W**	sw	w	sw	s	s	nw
[illegible]	**SE**	W	SW	w	nw	sw	ssw	ne	se	sw	sw	sse	sse	ssw	ssw	ssw	ssw	s	e	ese	SE	sse	**WNW**	ssw	w	sw	ssw	SW	n
[illegible]	sse	**WSW**	WSW	wnw	W	WNW	e	ene	s	wnw	s	s	s	sw	wsw	ssw	ssw	s	ese	sse	sse	s	nw	sw	nw	w	sw	W	n
[illegible]	ESE	s	ssw	wsw	N	s	sse	nne	sse	ssw	sw	e	s	s	s	s	s	s	ese	sse	e	se	sw	se	w	s	se	nnw	sw
[illegible]	SE	**NW**	SW	w	nne	sw	ssw	NE	se	sw	sw	ssw	s	sw	sw	ssw	ssw	sse	ene	se	SE	S	NW	SW	w	w	SW	SW	**N**
[illegible]	SE	**NW**	SW	sw	nnw	nnw	se	nne	sse	ssw	n	s	s	sw	ssw	s	sw	se	e	ese	SE	SSE	NW	SSW	w	sw	SSW	NW	n
[illegible]	ssw	w	w	wnw	nw	n	sw	ne	se	sw	ssw	ssw	sw	wsw	wsw	sw	ssw	sse	e	sse	sse	ssw	wnw	sw	wnw	wnw	w	w	nne
[illegible]	**S**	**NW**	W	ne	ne	w	s	E	wsw	sw	e	sw	S	sw	sw	sw	sw	se	se	sse	SSE	**SSW**	nnw	SW	wnw	w	SW	NW	NE
[illegible]	SSE	SSW	WSW	ssw	wnw	SSW	s	NNE	se	se	n	s	sse	ssw	sse	se	ssw	se	ne	se	SE	SE	SW	SSW	ssw	ssw	ssw	sw	n
[illegible]	S	SW	sw	sw	nw	s	n	N	e	sw	n	s	ssw	sw	s	ssw	sw	ese	n	s	s	SW	WNW	sw	sw	sw	ssw	sw	wsw
[illegible]	[illegible]	sw	sw	sw	sw	n	sw	ssw	nw	nw	ssw	ne	ne	se	nnw	nnw	w	w	sw	sw	sw	ssw	sw	w	w	w	w	sw	nw

Utrecht, ...	SE	SE	SSW	SW	SSW	SW	SW	SE	**NW**	SW	W	NNE	SW	SSW	NE	SE	SW	SW	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
Hellevoetsluis, . .	SE	E	S	S	SSW	**SW**	SW	SE	**NW**	SW	SW	NNW	NNW	SE	NNE	SSE	SSW	N	S	S	SW	SSW	S	SW	SE	E
Breda, ...	E	SE	SW	SW	SW	SW	WSW	SSW	W	W	WNW	NW	N	SW	NE	SE	SW	SSW	SSW	SW	WSW	WSW	SW	SSW	SSE	E
Flushing, ...	SE	SE	SW	SW	SW	SW	SW	**S**	**NW**	W	NE	NE	W	S	E	WSW	SW	E	SW	S	SW	SW	SW	SW	SE	SE
BELGIUM.																										
Brussels, ...	NNE	S	SSW	S	SSW	SW	SSE	SSE	SSW	WSW	SSW	WNW	SSW	S	NNE	SE	SE	N	S	SSE	SSW	SSE	SE	SSW	SE	NE
Maastricht, ...	S	S	WNW	S	SW	SW	SW	S	SW	SW	SW	NW	S	N	N	E	SW	N	S	SSW	SW	S	SSW	SW	ESE	N
Luxemburg, . . .	E	E	E	SW	SW	SSW	SW	SSE	SSW	SW	SW	SW	N	SW	SSW	NW	NW	SSW	NE	NE	SE	NNW	NNW	W	W	SW
FRANCE.																										
Dunkirk, ...	SSE	calm	NW	SW	SW	WSW	SW	S	NW	W	WNW	NE	W	WNW	**ENE**	S	S	NNE	S	S	SW	SW	S	S	SE	SE
Mezières, ...	E	ESE	E	SE	W	W	SW	S	NW	WNW	WNW	N	N	NW	NE	E	W	NNE	E	E	S	E	E	SE	E	E
Havre, ...	SE	SE	SW	SW	WSW	W	**WSW**	**WSW**	NW	NNW	W	NE	SW	NW	**NE**	WNW	SSW	NNE	E	SE	WSW	W	SW	ESE	ESE	E
Cherbourg, ...	NNE	NNW	S	WSW	SSW	WSW	WSW	WSW	WSW	WSW	SW	E	S	SW	NNW	SW	S	N	SSE	S	WSW	SSW	S	S	SE	SE
Paris, ...	ESE	ESE	SSW	S	SSW	SW	SSW	**SW**	WSW	WSW	WSW	WSW	calm	WSW	NNW	WNW	SSW	NNW	SE	SE	SW	W	S	ESE	ESE	ESE
Strasbourg, . . .	NE	N	N	S	SW	SW	S	SW	W	SW	SSW	SW	SW	N	S	N	S	N	N	SE	...	S	SW	S	S	N
Brest, ...	NE	N	SSW	SW	SSW	W	WSW	**WSW**	W	W	WSW	W	NW	NNW	N	W	NW	NNW	SE	SW	WNW	SW	SSW	SSE	ESE	SE
L'Orient, ...	NE	NE	ESE	SSW	SW	W	W	WSW	**WSW**	WSW	W	WSW	NW	W	N	E	W	NNW	ESE	S	WSW	SE	S	ESE	E	ENE
Napoleon Vendée, .	NE	E	E	S	WSW	WNW	W	W	NNW	...	...	...	NW	NNW	N	N	W	...	...	SE	W	W	E	ESE	SE	E
Besançon, ...	N	NE	S	SW	SW	W	W	SW	WSW	SW	ESE	SW	W	SW	W	SW	SE	E	N	NE	WSW	SW	NE	E	E	SE
Lyons, ...	E	S	S	S	S	...	NE	SW	SW	SW	SW	NW	NW	S	W	NW	SE	SW	SE	NE	S	S	S	S	SE	SE
Limoges, ...	E	SE	S	W	S	NW	NW	SW	NW	NW	NW	NE	S	SW	SW	SE	NE	N	SW	SW	S	W	SW	NW	SW	S
Rochfort, ...	N	NE	SE	S	SW	W	W	**WNW**	W	W	W	NW	W	...	NW	NW	NW	N	S	SE	SSW	WSW	S	SE	SE	SE
Bordeaux, ...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	SE	SE	SW	SW	SSE	S	S	E
Montauban, . . .	E	E	E	S	S	S	W	S	SW	SW	SW	N	W	W	W	SW	SW	NW	SE	SE	SE	S	SE	S	SE	SE
Bayonne, ...	SE	S	E	E	...	N	WNW	SW	NW	W	E	E	E	S	WNW	E	NW	N	SE	S	SE	SE	S	SE	SE	SE
Montpellier, . . .	N	NE	NE	N	N	NNE	NE	N	NW	NW	NW	N	NW	NW	NW	N	W	NW	N	NNW	NNE	N	NNW	NNE	N	NNE
Marseilles, ...	E	E	ESE	SE	ESE	E	NE	NE	NW	WNW	N	N	N	NW	NW	N	NW	NE	NE	NE	...	E	W	NE	NE	NE
Toulon, ...	N	E	E	ENE	ENE	SW	E	WNW	NW	NW	ENE	NNE	...	W	NW	ENE	NW	NW	ENE	ENE	...	E	NNE	NE	ENE	NE
SPAIN and PORTUGAL.																										
Bilbao, ...	SE	SE	SE	E	SE	NW	NNW	NW	...	SE	SE	SE	SE	SE	NW	NW	...	NW	SE	...	SW	SE	SE	SE	SE	SE
Barcelona, ...	NE	N	N	N	SW	NW	NE	W	W	NW	W	N	W	N	NW	W	NW	**NW**	N	W	N	N	S	N	N	N
Oporto, ...	N	SSE	N	NNW	...	S	NNW	NW	NE	NE	E	E	E	E	N	N	...	N	E	S	S	SE	SSE	...	SE	E
Lisbon, ...	E	ESE	...	N	SE	SSW	NNE	W	NE	NE	NE	NE	NE	NE	NNE	NE	...	N	E	ENE	NNE	NE	ENE	...	ESE	E
Madrid, ...	E	SE	SSE	NNW	S	NW	SE	SW	NE	E	E	E	E	NNE	ENE	N	S	NW	E	E	...	E	N	N	E	...
Alicante, ...	NE	calm	calm	calm	calm	S	S	NNE	...	calm	calm	calm	...	NE	SW	NW	...	NNE	SE	SE	SE	SE	ENE	...	N	NE
San Fernando, . .	SSE	SSE	SSW	ENE	SE	NE	N	NNE	NE	ESE	ESE	SSE	ENE	SE	E	NNE	NE	WNW	SSE	SE	ESE	ESE	SE	ESE	ESE	SE
Palma (Majorca), . .	NNW	NE	...	calm	calm	ENE	S	SW	N	E	NNW	NW	N	WSW	NW	ENE	NNW	NNW	N	N	N	N	N	N	...	ENE
AFRICA.																										
Algiers, ...	E	W	ESE	E	W	E	...	W	W	E	SW	N	SE	W	SW	W	NW	...	E	SW	NE	E	...	...	...	W
ITALY.																										
Turin, ...	...	...	NW	W	E	NE	N	NW	N	NW	NW	E	W	...	...	W	S	SW	E	E	W	W	W	W	W	W
Leghorn, ...	E	E	ENE	ENE	SSE	S	S	WSW	WSW	ENE	NE	NE	NE	NE	WSW	E	E	E	ENE	ENE	NE	ENE	NE	NE	...	...
Florence, ...	NE	SE	E	...	SE	S	...	...	S	NE	...	...	S	...	NW	NE	...	SE	NE	S	SW	...	...	NW	SE	NE
Ancona, ...	...	...	NW	SE	W	SW	SSW	ESE	NW	N	W	W	WNW	...	SSE	ENE	E	SSE	W	E	NNW	NNW	NE	NW	...	W
Rome, ...	N	...	N	E	E	S	SE	S	SW	N	N	N	N	N	SE	NE	S	SE	N	N	NW	N	N	...	N	N
Naples, ...	NE	NE	E	W	SW	SW	SW	SW	SSW	E	NE	N	NE	N	SW	SW	S	N	NE	N	...	NNE	NNE	E	NE	NE
Palermo (Sicily), . .	WSW	WSW	WSW	WSW	SW	SW	WSW	SW	WNW	NW	W	WSW	WSW	SW	WNW	NNW	W	SW	NNE	WSW	WSW	WNW	WSW	WSW	WSW	WSW
TURKEY.																										
Constantinople, . .	N	N	N	W	N	...	S	S	S	S	...	...	N	...	S	N	...	...	...	NE	...	...	calm	N	...	N

SW	SE	**NW**	SW	W	NNE	SW	SSW	NE	SE	SW	SW	[illegible]	[illegible]	[illegible]															
SW	SE	**NW**	SW	SW	NNW	NNW	SE	NNE	SSE	SSW	N	S	S	SW	SSW	S	SW	SE	E	ESE	SE	SSE	NW	SSW	W	SW	SSW	NW	N
WSW	SSW	W	W	WNW	NW	N	SW	NE	SE	SW	SSW	SSW	SW	WSW	WSW	SW	SSW	SSE	E	SSE	SSE	SSW	WNW	SW	WNW	WNW	W	W	NNE
SW	**S**	**NW**	W	NE	NE	W	S	E	WSW	SW	E	SW	S	SW	SW	SW	SW	SE	SE	SSE	SSE	**SSW**	NNW	SW	WNW	W	SW	NW	NE
SSE	SSE	SSW	WSW	SSW	WNW	SSW	S	NNE	SE	SE	N	S	SSE	SSW	SSE	SE	SSW	SE	NE	SE	SE	SE	SW	SSW	SSW	SSW	SSW	SW	N
SW	S	SW	SW	SW	NW	S	N	N	E	SW	N	S	SSW	SW	S	SSW	SW	ESE	N	S	S	SW	WNW	SW	SW	SW	SSW	SW	WSW
SW	SSE	SSW	SW	SW	SW	N	SW	SSW	NW	NW	SSW	NE	NE	SE	NNW	NNW	W	W	SW	SW	SW	SSW	SW	W	W	W	W	SW	NW
SW	S	NW	W	WNW	NE	W	WNW	**ENE**	S	S	NNE	S	S	SW	SW	S	S	SE	SE	SSE	SW	SW	**NNW**	SW	W	WNW	SW	NNW	NE
SW	S	NW	WNW	WNW	N	N	NW	NE	E	W	NNE	E	E	S	E	E	SE	E	E	SE	SSE	S	NW	SSW	…	NNW	W	NNW	NE
WSW	**WSW**	NW	NNW	W	NE	SW	NW	**NE**	WNW	SSW	NNE	E	SE	WSW	W	SW	ESE	ESE	E	ESE	WSW	WSW	**NNW**	WSW	NNW	SE	SW	**NNW**	ENE
WSW	WSW	WSW	WSW	SW	E	S	SW	NNW	SW	S	N	SSE	S	WSW	SSW	S	S	SE	SE	S	SW	WSW	N	SW	WSW	SSW	SW	NW	NNE
SSW	**SW**	WSW	WSW	WSW	WSW	calm	WSW	NNW	WNW	SSW	NNW	SE	SE	SW	W	S	S	ESE	ESE	ESE	SE	S	SSW	W	S	calm	calm	S	W	NNW
S	SW	W	SW	SSW	SW	SW	N	S	N	S	N	N	SE	…	S	SW	S	S	N	N	N	SSE	SW	N	…	SSW	N	WSW	N
WSW	**WSW**	W	W	WSW	W	NW	NNW	N	W	NW	NNW	SE	SW	WNW	SW	SSW	SSE	ESE	SE	W	WNW	WNW	W	SW	E	ESE	W	NNW	NNE
W	WSW	**WSW**	WSW	W	WSW	NW	W	N	E	W	NNW	ESE	S	WSW	SE	S	ESE	E	ENE	S	W	SW	NW	SW	NE	ENE	SSE	WNW	N
W	W	NNW	…	…	…	NW	NNW	N	N	W	…	…	SE	W	W	E	ESE	SE	E	SW	**NW**	**NW**	NNW	S	NE	E	…	N	NNE
W	SW	WSW	SW	ESE	SW	W	SW	W	SW	SE	E	N	NE	WSW	SW	NE	E	E	SE	NE	SW	SE	SW	NW	NW	W	W	NW	N
NE	SW	SW	SW	SW	NW	NW	S	W	NW	SE	SW	SE	NE	S	S	S	S	SE	SE	SE	S	W	…	NW	N	W	N	S	N
NW	SW	NW	NW	NW	NE	S	SW	SW	SE	NE.	N	SW	SW	S	W	SW	NW	SW	S	NW	SW	SW	NW	N	ENE	W	WNW	N	NE
W	**WNW**	W	W	W	NW	W	…	NW	NW	NW	N	S	SE	SSW	WSW	S	SE	SE	SE	S	W	**WSW**	NW	ESE	NE	NE	NW	NW	N
…	…	…	…	…	…	…	…	…	…	…	…	SE	SE	SW	SW	SSE	S	S	E	SE	**S**	S	N	SE	E	E	E	N	N
W	S	SW	SW	SW	N	W	W	W	SW	SW	NW	SE	SE	SE	S	SE	S	SE	SE	SW	E	SW	SW	SW	SE	SW	SW	SW	NW
WNW	SW	NW	W	E	E	E	S	WNW	E	NW	N	SE	S	SE	SE	SE	S	SE	SE	SE	S	S	…	E	E	E	E	SE	E
NE	N	NW	NW	NW	N	NW	NW	NW	N	W	NW	N	NNW	NNE	N	NNW	NNE	N	NNE	NNE	NW	N	NW	WNW	NE	N	NW	NW	N
NE	NE	NW	WNW	N	N	N	NW	NW	N	NW	NE	NE	NE	…	E	W	NE	NE	NE	ENE	SE	NNW	…	N	NE	NE	E	NE	N
E	WNW	NW	NW	ENE	NNE	…	W	NW	ENE	NW	NW	ENE	ENE	…	E	NNE	NE	ENE	NE	NNE	NE	…	NW	NW	ENE	ENE	ENE	NW	E
NNW	NW	…	SE	SE	SE	SE	SE	NW	NW	…	NW	SE	…	SW	SE	SE	SE	SE	SE	N	WSW	…	…	SE	SE	SE	SE	NW	NNW
NE	W	W	NW	W	N	W	N	NW	W	NW	**NW**	N	W	N	N	S	N	N	N	W	W	W	W	W	N	N	W	NW	W
NNW	NW	NE	NE	E	E	E	E	N	N	…	N	E	S	S	SE	SSE	…	SE	E	…	SW	…	…	E	E	SSE	N	NW	NNE
NNE	W	NE	NE	NE	NE	NE	NE	NNE	NE	…	N	E	ENE	NNE	NE	ENE	…	ESE	E	NE	SW	S	…	NNE	calm	NNE	N	NE	NNE
SE	SW	NE	E	E	E	E	NNE	ENE	N	S	NW	E	E	…	E	N	N	E	…	SE	W	W	N	ENE	NE	NE	E	NE	N
S	NNE	…	calm	calm	calm	…	NE	SW	NW	…	NNE	SE	SE	SE	SE	ENE	…	N	NE	ESE	ENE	NW	N	ENE	SW	NW	NW	WNW	NW
N	NNE	NE	ESE	ESE	SSE	ENE	SE	E	NNE	NE	WNW	SSE	SE	ESE	ESE	SE	ESE	ESE	SE	ESE	NW	NNW	N	NNW	SE	SE	SE	SSE	NNE
S	SW	N	E	NNW	NW	N	WSW	NW	ENE	NNW	NNW	N	N	N	N	N	N	…	ENE	N	N	N	NNE	NNE	NNW	N	NNW	NE	NE
…	W	W	E	SW	N	SE	W	SW	W	NW	…	E	SW	NE	E	…	…	…	W	E	SW	SW	NW	N	W	W	N	W	N
N	NW	N	NW	NW	E	W	…	…	W	S	SW	E	E	W	W	W	W	W	W	W	N	NW	E	W	W	W	NW	N	NW
S	WSW	WSW	ENE	NE	NE	NE	NE	WSW	E	E	E	ENE	ENE	NE	ENE	NE	NE	…	…	NE	ENE	ENE	E	NE	NE	NE	NE	S	NE
…	…	S	NE	…	…	S	…	NW	NE	…	SE	NE	S	SW	…	…	NW	SE	NE	NE	SE	NE	…	NE	SE	SE	E	S	S
SSW	ESE	NW	N	W	W	WNW	…	SSE	ENE	E	SSE	W	E	NNW	NNW	NE	NW	…	W	W	W	NE	WNW	ENE	SE	SSW	NNW	NNW	ESE
SE	S	SW	N	N	N	N	N	SE	NE	S	SE	N	N	NW	N	N	…	N	N	…	N	…	…	…	N	N	…	N	E
SW	SW	SSW	E	NE	N	NE	N	SW	SW	S	N	NE	N	…	NNE	NNE	E	NE	NE	N	NNE	ENE	SW	E	N	W	NNE	NNW	WSW
WSW	SW	WNW	NW	W	WSW	WSW	SW	WNW	NNW	W	SW	NNE	WSW	WSW	WNW	WSW	WSW	WSW	WSW	WSW	WSW	…	SW	…	WSW	SW	WSW	WSW	WSW
S	S	S	S	…	…	N	…	S	N	…	…	…	NE	…	…	calm	N	…	N	N	N	…	…	N	…	N	…	S	S

XVII.—*On the Celtic Topography of Scotland, and the Dialectic Differences indicated by it.* By W. F. Skene, Esq.

(Read 17th April 1865.)

The etymology of the names of places in a country is either a very important element in fixing the ethnology of its inhabitants, or it is a snare and a delusion, just according as the subject is treated. When such names are analysed according to fixed laws, based upon sound philological principles, and a comprehensive observation of facts, they afford results both important and trustworthy; but if treated empirically, and based upon resemblance of sounds alone, they become a mere field for wild conjectures and fanciful etymologies, leading to no certain results. The latter is the ordinary process to which they are subjected. The natural tendency of the human mind is to a mere phonetic etymology of names, both of persons and of places. It is this tendency which has given rise to what may be called punning etymologies, in which the King of Scotland plays so facetious a part, when the first Guthrie had that name fixed upon by the king, from his proposing when asked, how many fish should be prepared, to *gut three;* and when Rosemarkie received its name because the king, on asking what land he neared, was answered, *Ross mark ye.* This illustrates the natural tendency to suggest a mere phonetic etymology, in which the sounds of the name of the place appear to resemble the sounds in certain words of a certain language, the language from which the etymology is derived being selected upon no sound philological grounds, but from arbitrary considerations merely.

Unhappily, an etymology founded upon mere resemblance of sounds has hitherto characterised all systematic attempts to analyse the topography of Scotland, and to deduce ethnologic results from it. Prior to the publication of the "Statistical Account of Scotland" in 1792, it may be said that no general attempt had been made to explain the meaning of the names of places in Scotland, or to indicate the language from which they were derived. We find occasionally, in old lives of the saints, and in charters connected with church lands, that names of places occurring in them are explained; and these interpretations are very valuable, as indicating what may be termed the common tradition of their meaning and derivation at an early period. Of very different value are a few similar derivations in the fabulous histories of Boece, Buchanan, and John Major, which are usually mere fanciful conjectures of pedantry.

The first impetus to anything like a general etymologising of Scottish topography was given when Sir John Sinclair projected the "Statistical Account of

Scotland." In the schedule of questions which he issued in 1790 to the clergy of the Church of Scotland, the first two questions were as follows:—

1. What is the ancient and modern name of the parish?
2. What is the origin and etymology of the name?

This set every minister thinking what was the meaning of the name of his parish. The publication of the "Poems of Ossian," and the controversy which followed, had tended greatly to identify national feeling and the history of the country with Gaelic literature and language, and, with few exceptions, the etymology was sought for in that language. The usual formula of reply was, "the name of this parish is derived from the Gaelic," and then followed a Gaelic sentence resembling in sound the name of the parish, and supposed admirably to express its characteristics, though the unfortunate minister is often obliged to confess that the parish is remarkably free from the characteristics expressed by the Gaelic derivation of its name. These etymologies are usually suggested irrespective entirely of any known facts as to the history or population of the parish, and are purely phonetic.

Thus the writer of the account of Elie, in the New Statistical, observes:—"The writer of the former Statistical Account has, according to the fashion which seems to have prevailed in his day, as well as now, had recourse to Gaelic, the mother as it should seem of languages, and tells us that the parish received its name from '*A Liche*,' signifying 'out of the sea.' We are disposed to doubt its soundness, for the village is not further out of the sea than any other part of the coast, nay, it extends further into it. We should rather be inclined to consider Elie as having sprung from the Greek word *elos*, a marsh."

Both etymologies are entirely irrespective of the fact, that the old form of the word was "chellin."

After the publication of the Statistical Account, Gaelic was in the ascendant as the source of all Scottish etymologies, till the publication of CHALMERS' "Caledonia" in 1807. JOHN PINKERTON had indeed tried to direct the current of popular etymology into a Teutonic channel, but his attempts to find a meaning in Gothic dialects for words plainly Celtic were so unsuccessful, that he failed even to gain a hearing. CHALMERS was more fortunate. His theory was, that a large proportion of the names of places in Scotland are to be derived from the Welsh, and indicate an original Welsh population. And this he has worked out with much labour and pains. In doing so, he was the first to attempt to show evidence of the dialectic difference between Welsh and Gaelic pervading the names of places, and to discriminate between them; but for almost all the names of places in the Lowlands of Scotland he furnishes a Welsh etymology, which, like his predecessors the Scottish clergy, he supposes to be expressive of the characteristics of the locality. His theory has, in the main, commanded the assent of subsequent writers, and is usually assumed to be, on the whole, a correct representation of the state of the fact. Yet his system was as purely one of a phonetic etymology, founded upon mere resemblance of sounds, as those of his predecessors. The MSS. left by GEORGE CHALMERS show how he set about preparing his etymologies, and we now know the process he went through. He had himself no knowledge of either branch of the Celtic language, but he sent his list of names to Dr OWEN PUGHE; and that most ingenious of all Welsh lexicographers, who was capable of reducing every word in every known language in the world to a Welsh original, sent him a list of Welsh renderings of each word, varying from twelve to eighteen in number, out of which CHALMERS selected the one which seemed to him most promising.

As an instance, we may refer to a pet etymology of CHALMERS, on which he has built as historical fact, and which has been followed by all subsequent writers. He interprets Kilspindy, the name of a place in Aberlady Bay, which belonged to the bishop of Dunkeld, as signifying in Welsh *Cill ys pendu*, which he renders "the Cell of the Black Heads," and supposed that it indicated a settlement of the Culdees. We have no reason to suppose that the Culdees were distinguished by having black head-dresses; but the etymology is philologically false, for *Cill* is Gaelic and not Welsh. *Ys* is no known form of the article in Welsh, and *pen du* means black head in the singular. In the plural, it would be *penau duon*. The old form of the word puts the etymology to rout, for it was originally written "Kinespinedin." His other etymologies are equally founded on a mere resemblance of sounds between the modern form of the word and the modern Welsh, as those of the clergy in the Statistical Account were between the modern form of the word and the modern Gaelic.

That system of interpreting the names of places, which I have called phonetic etymology, is, however, utterly unsound. It can lead only to fanciful renderings, and is incapable of yielding any results that are either certain or important.

Names of places are, in fact, sentences or combinations of words originally expressive of the characteristics of the place named, and applied to it by the people who then occupied the country, in the language spoken by them at the time, and are necessarily subject to the same philological laws which governed that spoken language. The same rules must be applied in interpreting a local name as in rendering a sentence of the language.

That system, therefore, of phonetic etymology which seeks for the interpretation of a name in mere resemblance of sound to words in an existing language, overlooks entirely the fact that such names were fixed to certain localities at a much earlier period, when the language spoken by those who applied the name must have differed greatly from any spoken language of the present day.

Since the local names were deposited in the country, the language itself from which they were derived has gone through a process of change, corruption, and decay. Words have altered their forms—sounds have varied—forms have become obsolete, and new forms have arisen—and the language in its present state no longer represents that form of it which existed when the local nomenclature was formed. The topographical expressions, too, go through a process of change and corruption till they diverge still further from the spoken form of the language as it now exists.

This process of change and corruption in the local names varies according to the change in the population. When the population has remained unchanged, and the language in which the names were applied is still the spoken language of the district, the names either remain in their original shape, in which case they represent an older form of the same language, or else they undergo a change analogous to that of the spoken language. Obsolete names disappear as obsolete words drop out of the language, and are replaced by more modern vocables. Where there has been a change in the population, and the older race are replaced by a people speaking a kindred dialect, the names of places are subjected to the dialectic change which characterises the language. There are some striking instances of this where a British form has been superseded by a Gaelic form, as, for instance, Kirkintulloch, the old name of which, NENNIUS informs us, was Cærpentaloch, *kin* being the Gaelic equivalent of the Welsh *pen;* Penicuik, the old name of which was Penjacop; Kincaid, the old name of which was Pencoed.

When, however, the new language introduced by the change of population is one of a different family entirely, then the old name is stereotyped in the shape in which it was when the one language superseded the other, becomes unintelligible to the people, and undergoes a process of change and corruption of a purely phonetic character, which often entirely alters the aspect of the name. In the former cases it is chiefly necessary to apply the philologic laws of the language to its analysis. In the latter, which is the case with the Celtic topography of the low country, it is necessary, before attempting to analyse the name, to ascertain its most ancient form, which often differs greatly from its more modern aspect.

It is with this class of names we have mainly to do, as presenting the phenomena I am anxious to investigate.

When the topography of a country is examined, its local names will be found, as a general rule, to consist of what may be called generic terms and specific terms. What I mean by generic terms are those parts of the name which are common to a large number of them, and are descriptive of the general character of the place named; and by specific terms, those other parts of the name which have been added to distinguish one place from another. The generic terms are usually general words for river, mountain, valley, plain, &c.; the specific terms, those words added to distinguish one river or mountain from another. Thus, in the Gaelic name Glenmore, *glen* is the generic term, and is found in a numerous class of words—*more*, great, the specific, a distinguishing term, to distinguish it from another called Glenbeg. In the Saxon term Oakfield, field is the generic term, and oak the specific, to distinguish it from Broomfield, &c.

When the names of places are applied to purely natural objects, such as rivers, mountains, &c., which remain unchanged by the hand of man, the names applied by the original inhabitants are usually adopted by their successors, though speaking a different language; but the generic term frequently undergoes a phonetic corruption, as in the Lowlands, where Aber has in many cases become Ar in Arbroath, Arbuthnot; Ballin has become Ban, as in Bandoch; Pettin has become Pen as Pendriech; Pol has become Pow; and Traver has become Tar and Tra.

On the other hand, where the districts have been occupied by different branches of the same race, speaking different dialects, the generic terms exhibit the dialectic differences when the sounds of the word are such as to require the dialectic change; thus in Welsh and Gaelic:—

Pen and Kin—a head,
Gwyn and Fionn—white,

shows the phonetic difference between these dialects.

The comparison of the generic terms which pervade the topography of a country affords a very important means of indicating the race of its early inhabitants, and discriminating between the different branches of the race to which the respective portions of it belong.

Between the Celtic and Teutonic races the generic terms afford this great leading distinction, that in Celtic names they are invariably found at the beginning of the word; in Teutonic names, at the end of the word. Thus, Glenesk in Celtic is Eskdale in Teutonic; Dunedin is Edinburgh; Auchindarroch is Oakfield, and so forth. In the one, the generic term, at the beginning of the word; in the other, at the end.

It was early observed that there existed in the Celtic generic terms a difference which seemed to indicate dialectic distinction. Even in the Old Statistical Account, the minister of the parish of Kirkcâldy remarks,—"To the Gaelic language a great proportion of the names of places in the neighbourhood, and indeed through the whole of Fife, may unquestionably be traced. All names of places beginning with Bal, Col or Cul, Dal, Drum, Dun, Inch, Inver, Auchter, Kil, Kin, Glen, Mon, and Strath, are of Gaelic origin. Those beginning with Aber and Pit are supposed to be Pictish names, and do not occur beyond the territory which the Picts are thought to have inhabited."

CHALMERS states it still more broadly and minutely. He says,—"Of those words which form the chief compounds in many of the Celtic names of places in the Lowlands, some are exclusively British, as Aber, Llan, Caer, Pen, Cors, and others; some are common to both British and Irish, as Carn, Craig, Crom, Bre, Dal, Eaglis, Glas, Inis, Rinn, Ros, Strath, Tor, Tom, Glen; and many more are significant only in the Scoto-Irish or Gaelic, as Ach, Ald, Ard, Aird, Auchter, Bar, Blair, Ben, Bog, Clach, Corry, Cul, Dun, Drum, Fin, Glac, Inver, Kin, Kil, Knoc, Larg, Lurg, Lag, Logie, Lead, Letter, Lon, Loch, Meal, Pit, Pol, Stron, Tullach, Tullie, and others."

This attempt at classification is, however, exceedingly inaccurate. Two of the words in the first class, Llan and Caer, are common to both British and Irish;

and a large portion of the third class are significant in pure Irish, as well as in the Scoto-Irish or Gaelic. No attempt is made to show, by the geographical distribution of these words, in what parts of the country the respective elements prevail.

In a recent work, however, of some pretension, by an eminent Gaelic scholar, this attempt is made; and I refer to it to show how very loosely popular ideas on this subject are taken up. He says, "The Blackadder and Whiteadder contain distinctly the British Dwfr or Dwr, water." The two names are Teutonic, and have obviously no Celtic form. "In East Lothian, Yester is the old British word Ystrad, a valley." This is correct, but it is on British ground. "Tranent and Traquair have the British Tre, a town." The old form is Travernent and Traverquair, and Traver is unknown in Welsh topography. "On crossing the Forth, British names still appear nowhere more clearly than in the name of the Ochil Hills, where the British Uchel (high), cannot be mistaken." This is phonetic etymology, and, as we shall see, it has been mistaken. "In Fife we find several Abers, Pits, and Pittens, indicating the existence of a British population; and again the Pits and Pittens of Forfarshire are numerous." Of the Abers we shall talk presently; but if the Pits and Pittens indicate a British population, how comes it that they are unknown in Wales, and are not to be found in Welsh topography. "We have," says he, "Pens and Abers and Pits in abundance on through Kincardine and Aberdeenshire." Abers and Pits certainly, but no Pens except one solitary instance, which is doubtful. I need not proceed. The statement goes on in the same strain, at equal variance with topographical and philological facts.

The most popular view of the subject, and that which has recently been most insisted in, is the line of demarcation between a Kymric and a Gaelic population, supposed to be indicated by the occurrence of the words Aber and Inver.

This view has been urged with great force by KEMBLE, in his Anglo-Saxons; but I may quote the recent work by Mr ISAAC TAYLOR, on words and places, as containing a fair statement of the popular view of the subject:—

"To establish the point that the Picts or the nation, whatever was its name, that held central Scotland, was Cymric, not Gaelic, we may refer to the distinction already mentioned between *Ben* and *Pen*. *Ben* is confined to the west and north; *Pen* to the east and south. *Inver* and *Aber* are also useful test words in discriminating between the two branches of the Celts. The difference between the two words is dialectic only; the etymology and the meaning is the same—a confluence of waters, either of two rivers or of a river with the sea. Aber occurs repeatedly in Brittany, and is found in about fifty Welsh names, as Aberdare, Abergavenny, Abergele, Aberystwith, and Barmouth, a corruption of Abermaw. In England we find *Aber*ford in Yorkshire, and *Ber*wick in Northumberland and Sussex; and it has been thought that the name of the Humber is a corruption of the same root. *Inver*, the Erse and Gaelic forms, is common in Ireland, where *Aber* is unknown. Thus, we find places called Inver in Antrim, Donegal, Mayo, and Invermore in Galway and in Mayo. In Scotland the *Invers* and *Abers* are distributed in a curious and instructive manner. If we draw a line across the map from a point a little south of Inveraray to one a little north of Aberdeen, we shall find that (with very few exceptions) the *Invers* lie to the north of the line and the *Abers* to the south of it. This line nearly coincides with the present southern limit of the Gaelic tongue, and probably also with the ancient division between the Picts and the Scots."

Nothing can be more inaccurate than this statement. Ben is by no means confined to the west and north; and as examples of Pen, he refers, among others, to the Pentland Hills, Pentland being a Saxon word, and corrupted from Pectland; and to Pendriech in Perthshire, which is a corruption from Pettindriech.

So far from Inver being common in Ireland, it is very rare. The *Index locorum* of the Annals of the Four Masters shows only six instances. On the other hand, Aber is not unknown in Ireland. It certainly existed formerly, to some extent, in the north of Ireland; and Dr Reeves produces four instances near Ballyshannon.

The statement with regard to the distribution of Aber and Inver in Scotland here is, that there is a line of demarcation which separates the two words—that, with few exceptions, there is nothing but Invers on one side of this line, nothing but Abers on the other; and that this line extends from a point a little south of Inveraray to a point a little north of Aberdeen. This is the mode in which the distribution of these two words is usually represented; but nothing can be more perfectly at variance with the real state of the case. South of this line there are as many Invers as Abers. In Perthshire, south of the Highland line, there are nine Abers and eight Invers; in Fifeshire, four Abers and nine Invers; in Forfar, eight Abers and eight Invers; in Aberdeenshire, thirteen Abers and twenty-six Invers. Again, on the north side of this supposed line of demarcation, where it is said that Invers alone should be found, there are twelve Abers, extending across to the west coast, till they terminate with Abercrossan, now Applecross, in Ross-shire. In Argyleshire alone there are no Abers. The true picture of the distribution of these two words is—in Argyllshire, Invers alone; in Inverness and Ross shires, Invers and Abers in the proportion of three to one and two to one; and on the south side of the supposed line, Abers and Invers in about equal proportions.

Again he says, quoting Chalmers, "The process of change is shown by an old charter, in which King David grants to the monks of May, 'Inverin qui fuit Aberin.' So Abernethy became Invernethy, although the old name is now restored." In order to produce the antithesis of Inverin and Aberin, one letter in this charter has been altered. The charter is a grant of "Petneweme et Inverin quæ fuit Averin;" and I have the authority of the first charter antiquary in Scotland for saying that this construction is impossible; quæ fuit does not, in charter Latin, mean "which was," but "which belonged to," and Averin was the name of the previous proprietor of the lands. Abernethy and Invernethy are not the same place, and the former never lost its name. Invernethy is at the junction of the Nethy with the Earn, and Abernethy is a mile further up the river.

When we examine these Abers and Invers more closely, we find that in some parts of the country they appear to alternate, as in Fife—Inverkeithing, Aberdour, Inveryne, Abercrombie, Inverlevin, and so forth. *2d*, That some of the Invers and Abers have the same specific terms attached to them, as Abernethy and Invernethy, Aberuchill and Inveruchil, Abercrumbye and Invercrumbye, Abergeldie and Invergeldy; and, *3d*, That the Invers are always at the mouth of the river, close to its junction with another river, or with the sea; and the Abers usually a little distance up the river where there is a ford. Thus, Invernethy is at the mouth of the Nethy; Abernethy a mile or two above. These and other facts lead to the conclusion that they are part of the same nomenclature, and belong to the same period and to the same people.

When we look to the south of the Forth, however, we find this remarkable circumstance, that in Ayrshire, Renfrew, and Lanarkshire, which formed the possessions of the Strathclyde Britons, and was occupied by a British people till as late a period as the more northern districts were occupied by the Picts, there are no Abers at all.

What we have, therefore, is the Scots of Argyle with nothing but Invers, the Picts with Abers and Invers together, and the Strathclyde Britons with no Abers. As a mark of discrimination between races this criterion plainly breaks down, and the words themselves contain no sounds which, from the different phonetic laws of the languages, could afford an indication of a dialectic difference. The truth is, that there were three words expressive of the junction of one stream with another, and all formed from an old Celtic word, Ber, signifying water. These were Aber, Inver, and Conber (pronounced in Welsh *cummer*, in Gaelic *cumber*.) These three words were originally common to both branches of the Celtic as derivations from one common word. In old Welsh poems we find not only Aber as a living word in Welsh, but Ynver likewise,* and Dr REEVES notices an Irish document in which Applecross or Appurcrossan is called Conber Crossan. Ynver, however, became obsolete in Welsh, just as Cummer or Cumber and Aber became obsolete in Irish; but we have no reason to know that it did so in Pictish. In the Pictish districts, therefore, the Abers and Invers were deposited when both were living words in the language. When the Scots settled in Argyle, Aber had become obsolete in their language, and Inver was alone deposited, and in Strathclyde both words seem to have gone into desuetude.

In the same manner Dwfr or Dwr is quoted as a word for water, peculiar to the Welsh form of Celtic, and an invariable mark of the presence of a

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* *Ynver* occurs twice in the Book of Taliessin.

British people, but the old form of this word in Scotland was Doboir, as appears from the Book of Deer, where Aberdour is written Abber-doboir, and in CORMAC's Glossary of the old Irish, Doboir is given as an old Irish word for water. In another old Irish glossary we have this couplet :—

> "Bior and An and Dobar,
> The three names of the water of the world."

These words, therefore, form no criterion of difference of race, and to judge by them is to fall into the mistake of the phonetic etymologists, viz., to apply to old names, as the key, the present spoken language, which does not contain words which yet existed in it in its older form.

In order to make generic terms a test of dialect they must be words which contain sounds affected differently by the different phonetic laws of such dialects, —such as Pen, Gwastad, Gwern, and Gwydd, which all enter copiously into Welsh topography, and the equivalents of which in the Gaelic dialects are Ken, Fearn, and Fiodh—Gwastad having no equivalent.

Such generic terms afford a test by which we can at once determine whether the Celtic topography of a country partakes most of the Kymric or the Gaelic character. The earliest collection of names in North Britain is to be found in Ptolemy's Geography in the second century, but we know too little of the origin of his names, whether they were native terms, or names applied by the invaders, to obtain from them any certain result. After Ptolemy, the largest collection of names in Great Britain is in the work of the anonymous geographer of Ravenna, a work of the seventh century. The exact localities are not given, but the names are grouped according to the part of Britain to which they belong. Those which commence the topography of Scotland are placed under this title :— "Iterum sunt civitates in ipsa Britannia quæ recto tramite de una parte in alia id est de oceano in oceano existunt, ac dividunt in tertia portione ipsam Britanniam." They commence with the stations on the Roman wall between the Tyne and the Solway, and then proceed northwards. Among these we find two names together, Tadoriton and Maporiton, and as Tad and Map are Kymric forms for father and son, we have no doubt that here we are on the traces of a Kymric population. The next group is arranged under this head :—" Iterum sunt civitates in ipsa Britannia, recto tramite una alteri conexæ, ubi et ipsa Britannia plus angustissima, de oceano in oceano esse dinoscitur." This part of Britain, which is *plus angustissima*, is the isthmus between the Forth and the Clyde, and in proceeding with the names northwards we come to one called Cindocellum. The Ocelli Montes were the Ochills, and here the Gaelic form of Kin is equally unmistakeable.

In the twelfth century, the Chartularies have preserved charters which contain the names of places, accompanied by an interpretation of the meaning of them. One bears upon the topography of Moray. It is a charter by Alexander II. to the monks of Kinloss of the lands of Burgyn, now Burgy, and has attached to it an old interpretation. Rune Pictorum is glossed the Pechts' fields, and Raoin is Gaelic for field. Tuber na crumkel, ane well with ane thrawn mouth—Tobar is well in Gaelic; Crom, crooked; and Beul, for which Kell is probably written by mistake, is mouth. Tuber na fein—of the Grett or Kempis men called Fenis, ane well.

In a perambulation of the marches of Monymusk by Malcolm IV., we have several such interpretations. Coritobrich is glossed Vallis fontis—Corre is Gaelic for valley, and Tobar, well. Scleuemingorn, Mora caprarum—Sliabh, Gaelic for moor; and Gabhar, goat. Alde clothi, rivulus petrosus—Ault, Gaelic for a stream; Clachach, stony. Breacachach, campus distinctis coloribus—Breacach, striped; Ach, field.

In a perambulation of the marches of Kingoldrum in 1256, we have names which are also glossed in a subsequent charter. Invercrumbyn is said to be the Concursus duorum amnium, Melgour et Crumbyn. Monybrech, Murrais of the quhilk runs ane strype—Monadh, a moss; Breac, striped. Pool of Monbuy, yellow pool—Buidh, yellow. Athyncroich, Gallow burne, from Ald, burn; Croich, gallows.

Thus on three points in the north-eastern lowlands, in Morayshire, in Aberdeenshire, and in Forfarshire, we find, as early as the thirteenth century, the local names interpreted in Gaelic. The names themselves are, too, in the Scotch Gaelic, not in the Irish form, and in most cases we find the dental substituted for the guttural, as *clothi* for *clachach.* When we apply to the present topography the testing words Pen, Gwynn, and Gwydd, the Gaelic equivalents of which are Kin, Fearn, and Fiodh, we find that with one exception, Pen, though frequent south of the Forth, where there was a British population, does not occur north of the Forth, while it is full of Kins, and Gwern and Gwydd occur only in their Gaelic equivalents.

Such then being the aspect in which the question really presents itself, it becomes important, with a view to ethnological results, to ascertain more closely the geographical distribution of the generic terms over Scotland, and in order to show this I have prepared a table of such distribution. The generic terms are taken from the index to the Record of Retours; and as this record relates to properties, and not to mere natural objects, the generic terms they contain are to a great extent confined to names of places connected with their possession by man, and more readily affected by changes in the population. For the purposes of comparison I have framed a list of generic terms contained in Irish topography from the index to the Annals of the Four Masters, and of those in Welsh topography from a list in the Cambrian Register. I have divided Scotland into thirteen districts, so as to show the local character of the topography of each part of Scotland, and opposite each generic term in Scotch topography is marked, 1*st,* if it occurs in Ireland, and how often; 2*d,* if it occurs in Wales; and 3*d,* I have marked the number of times it occurs in each district of Scotland from the Index of Retours.

On examining this table it will be seen that there are five terms peculiar to the districts occupied by the Picts. These are Auchter, Pit, Pitten, For, and Fin. Now none of these five terms are to be found in Welsh topography at all, and For and Fin are obviously Gaelic forms.

It is necessary, however, in examining these terms, which may be called Pictish, to ascertain their old form. Auchter appears to be the Gaelic Uachter, upper; and as such we have it in Ireland, and in the same form, as in Scotland Ochtertire, in Ireland Uachtertire. It does not occur in Wales.

The old form of Pit and Pitten, as appears from the Book of Deer, is Pette, and it seems to mean a portion of land, as it is conjoined with proper names, as Pette MacGarnait, Pette Malduib. But it also appears connected with Gaelic specific terms, as Pette an Mulenn, the Pette of the Mill, and in a charter of the Chartulary of St Andrews, of the church of Migvie, the terra ecclesiæ is said to be vocatus Pettentaggart—"an tagart" being the Gaelic form of the expression "of the priest."

The old forms of For and Fin are Fothuir and Fothen. The old form of Forteviot is Fothuir-tabaicht, and of Finhaven is Fothen-evin.

The first of these words, however, discloses a very remarkable dialectic difference. Fothuir becomes For, as Fothuir-tabacht is Forteviot; Fothuirduin is Fordun, but Fothuir likewise passes into Fetter, as Fothuiresach becomes Fetteresso; and these two forms are found side by side, Fordun and Fetteresso being adjacent parishes. The form of For extends from the Forth to the Moray Firth—that of Fetter from the Esk, which separates Forfar and Kincardine, to the Moray Firth.

An examination of some other generic terms will disclose a perfectly analogous process of change. The name for a river is Amhuin. The word is the same as the Latin Amnis. The old Gaelic form is Amuin, and the *m*, by aspiration, becomes *mh*, whence Amhuin, pronounced Avon. In the oldest forms of the language the consonants are not aspirated, but we have these two forms, both the old unaspirated form and the more recent aspirated form, in our topography, lying side by side in the two parallel rivers which bound Linlithgowshire—the Amond and the Avon. There is also the Amond in Perthshire. We know from the Pictish Chronicle that the old name was Aman, and the Avon, with its aspirated *m*, is mentioned in the Saxon Chronicle. It is a further proof that Inver is as old as Aber in the eastern districts, that we find Aman in its old form conjoined with Inver in the Pictish Chronicle in the name Inveraman.

In Dumbartonshire we find the names Lomond and Leven together. We have Loch Lomond and Ben Lomond, with the river Leven flowing out of the loch through Strathleven; but we have the same names in connection in Fifeshire, where we have Loch Leven with the two Lomonds on the side of it, and the river Leven flowing from it through Strathleven. This recurrence of the same words in connection would be unaccountable were it not an example of the same thing. Leven comes from the Gaelic Leamhan, signifying an elm tree, but the old form is Leoman, and the *m* becomes aspirated in a later stage of the language and forms Leamhan, pronounced Leven. Here the old form adheres to the mountain, while the river adopts the more modern.

A curious illustration of two different terms lying side by side, which are derived from the same word undergoing different changes, will be found in Forfarshire, where the term Llan for a church appears, as in Lantrethin. It is a phonetic law between Latin and Celtic, that words beginning in the former with *pl* are in the latter *ll.* The word Planum, in Latin signifying any cultivated spot, in contradistinction from a desert spot, and which, according to DUCANGE, came to signify Cimiterium, becomes in Celtic Llan, the old meaning of which was a fertile spot, as well as a church. In the inquisition, in the reign of David I., into the possessions of the See of Glasgow, we find the word in its oldest form in the name Planmichael, now Carmichael; and as we find Ballin corrupted into Ban, as Ballindoch becomes Bandoch, so Plan becomes corrupted into Pan, and we find it in this form likewise, in Forfarshire, in Panmure and Panbride. In the Lothians and the Merse this word has become Long, as in Longnewton and Longniddrie.

The Celtic topography of Scotland thus resembles a palimpsest, in which an older form is found behind the more modern writing. I shall not detain the Society further by going through other examples. The existence of the phenomenon is sufficiently indicated by those I have brought forward, and I shall conclude by stating shortly the results of this investigation.

1*st,* In order to draw a correct inference from the names of places as to the ethnological character of the people who imposed them, it is necessary to obtain the old form of the name before it became corrupted, and to analyse it according to the philological laws of the language to which it belongs.

2*d,* A comparison of the generic terms affords the best test for discriminating between the different dialects to which they belong, and for this comparison it is necessary to have a correct table of their geographical distribution.

3*d,* Difference between the generic terms in different parts of the country may arise from their belonging to a different stage of the same language, or from a capricious selection of different synonyms by different tribes.

4*th,* In order to afford a test for discriminating between dialects, the generic terms must contain within them those sounds which are differently affected by the phonetic laws of each dialect.

5*th,* Applying this test, the generic terms do not show the existence of a Kymric language north of the Forth.

6th, We find in the topography of the north-east of Scotland traces of an older and of a more recent form of Gaelic—the one preferring labials and dentals, and the other gutturals; the one hardening the consonants into tenues—the other softening them by aspiration; the one having Abers and Invers—and the other having Invers alone; the one a low Gaelic dialect—the other a high Gaelic dialect; the one I conceive the language of the Picts—the other that of the Scots.

Generic Terms.	Ireland.	Wales.	Scotland.												
			Angli.		Britones.				Picti.						Scoti.
			Berwick, Roxburgh, Haddington.	Mid-Lothian, Linlithgow.	Selkirk, Peebles.	Dumfries.	Ayr, Renfrew, Lanark.	Stirling, Dumbarton.	Perth.	Fife, Kinross.	Forfar.	Kincardine, Aberdeen, Banff.	Elgin and Nairn, Inverness, Ross, and Sutherland.	Kirkcudbright, Wigton.	Argyle, Bute.
Aber, . .	...	W	3	3	...	4	...	...	12	4	7	18	6	...	...
Ard, . .	66	...	...	...	...	...	...	16	34	6	14	66	51	5	93
Arn, . .	...	...	...	...	...	...	...	4	15	5	...	...	...	...	...
Ar, . . .	...	W	...	...	...	...	...	...	...	...	...	...	...	15	...
Auch, . .	25	...	...	...	...	...	25	...	24	6	27	162	153	12	107
Auchin, .	..	...	...	4	...	23	88	34	30	...	...	22	8	25	...
Auchter, .	...	...	...	...	...	...	...	...	6	10	6	12	4	...	...
Auld, . .	...	...	...	...	...	...	...	...	...	...	...	33	9	...	...
Bal, . . .	...	...	...	...	...	...	36	63	90	88	127	67	59	56	39
Balna, . .	...	...	...	...	...	...	...	...	...	...	...	10	...	...	...
Ballie, . .	104	...	...	...	...	...	...	...	...	...	...	...	...	...	...
Ballin, . .	...	...	3	...	...	...	...	...	...	...	...	3	...	...	...
Belloch, .	36	W	...	...	...	...	9	...	...	...	...	...	...	...	...
Bellie, . .	...	...	...	...	...	...	...	...	...	...	...	...	...	...	14
Ban, . .	...	...	...	...	...	...	...	...	...	16	...	...	...	...	...
Bar, . .	...	...	...	...	...	27	66	6	...	...	11	...	...	90	19
Barn, . .	...	...	...	...	...	...	...	...	...	...	...	...	...	6	...
Blair, . .	...	...	...	...	...	...	16	51	29	8	...	11	8	...	...
Bo, . . .	...	...	...	...	...	...	...	...	5	...	...	...	10	...	...
Carn, . .	28	...	...	...	...	...	11	...	13	8	8	54	15	4	...
Car, . . .	...	W	8	6	...	12	36	12	7	18	10	18	5	15	...
Col, . . .	...	...	...	...	...	...	...	...	7	...	...	17	...	...	...
Corrie, . .	...	...	...	...	...	...	...	...	9	...	...	...	8	...	...
Cambus, .	...	...	...	...	..	...	...	...	12	...	...	...	...	...	...
Clon, . .	93	...	...	...	...	8	13	...	...	...	...	...	...	7	...
Craig, . .	16	W	...	19	...	21	42	21	43	25	12	46	8	31	19
Cors, . .	...	W	...	...	...	14	...	...	...	...	...	...	...	9	...
Cul, . . .	39	...	...	...	...	...	47	...	25	11	...	22	22	7	...
Cumber, .	...	...	...	...	...	6	...	4	...	...	...	...	...	...	...
Cult, . .	...	...	...	...	...	...	...	...	10	...	...	...	...	...	...
Dal, . .	10	W	...	...	...	20	82	8	52	...	...	...	...	24	...
Drum, . .	64	...	...	4	...	30	50	26	51	33	25	56	36	57	25
Dun, . .	95	...	3	6	...	14	16	17	26	11	17	...	20	...	14
Fetter, . .	...	...	...	...	...	...	...	...	...	...	...	4	...	...	...
For, . .	...	...	...	...	...	...	...	...	13	9	11	22	...	...	...

Generic Terms.	Ireland.	Wales.	Scotland.												
			Angli.		Britones.				Picti.						Scoti.
			Berwick, Roxburgh, Haddington.	Mid-Lothian, Linlithgow.	Selkirk, Peebles.	Dumfries.	Ayr, Renfrew, Lanark.	Stirling, Dumbarton.	Perth.	Fife, Kinross.	Forfar.	Kincardine, Aberdeen, Banff.	Elgin and Nairn, Inverness, Ross, and Sutherland.	Kirkcudbright, Wigton.	Argyle, Bute.
Fin,	...	...	...	...	...	...	...	...	14	6	4	...	3	...	...
Glen,	35	W	5	...	17	42	44	...	56	...	23	52	54	61	52
Gar,	...	...	...	...	...	...	34	...	...	...	...	17	...	23	...
Garth,	...	W	...	...	...	...	10	23	13	...	...	...	...	...	10
Inch,	90	...	...	...	...	...	...	18	30	25	10	17	31	...	...
Iron,	...	...	...	...	...	...	...	...	...	...	...	...	...	15	...
Inver,	6	...	...	2	...	...	...	5	32	10	16	37	69	...	24
Kin,	30	...	...	3	...	...	...	6	43	34	52	88	57	...	7
Knock,	29	...	...	...	...	5	64	...	6	...	...	32	30	37	31
Larg,	...	...	...	...	...	...	...	...	...	...	...	...	...	13	...
Lin,	...	W	...	...	...	...	...	...	...	5	8	...	...	...	...
Lan,	3	W	...	...	...	...	...	...	...	6	...	...	...	...	6
Lath,	...	...	...	...	...	...	...	...	...	9	...	...	...	...	...
Loch,	100	...	...	7	...	14	34	16	30	18	...	15	19	26	...
Locher,	...	...	...	...	...	5	...	...	...	...	...	...	...	...	...
Led,	...	...	...	...	...	...	...	6	6	...	...	...	...	...	...
Mon,	14	...	...	...	...	7	...	...	11	13	13	31	...	...	...
Mul,	15	...	...	...	...	...	...	...	...	...	...	...	8	...	...
Pen,	...	W	9	3	...	5	7	...	...	...	...	...	...	2	...
Penny,	...	...	...	...	...	...	7	...	...	...	...	...	...	...	11
Pet,	...	...	...	...	...	...	...	...	...	...	...	30	5	...	...
Pit,	...	...	...	...	...	...	...	...	75	52	38	69	30	...	...
Pitten,	...	...	...	...	...	...	...	...	7	9	...	...	...	...	...
Pol,	...	...	...	...	...	13	7	6	9	...	...	...	17	9	...
Port,	22	...	...	...	...	...	...	3	...	...	...	...	...	8	...
Ra,	63	...	...	...	...	...	...	...	17	6	...	...	...	...	...
Strath,	...	...	...	...	...	...	...	...	19	13	...	27	35	...	13
Stron,	...	...	...	...	...	...	...	...	...	...	...	...	...	...	17
Stuck,	...	...	...	...	..	...	...	8	...	...	...	...	...	...	6
Tar,	...	...	...	...	...	...	...	...	...	...	...	14	...	...	...
Tra,	...	...	...	...	...	...	...	...	...	...	...	...	...	5	...
Tom,	...	...	...	...	...	...	...	11	...	...	...	...	...	...	...
Tor,	...	...	...	11	...	11	...	...	...	9	...	21	22	10	19
Tullie,	...	...	...	...	...	...	...	...	...	...	...	38	...	...	...
Tulli,	...	...	...	...	...	...	...	7	25	...	11	42	7	...	...
Tulloch,	17	...	...	...	...	...	...	5	...	...	...	10	...	...	...

Fig. 1

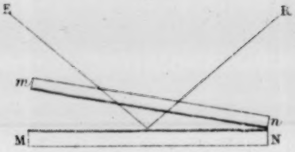


Fig. 2

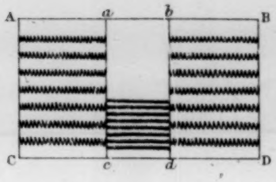


Fig. 3

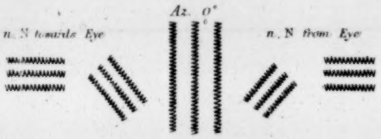


Fig. 4

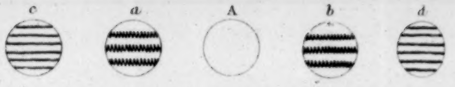


Fig. 5

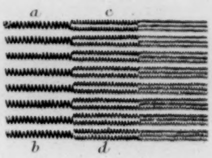
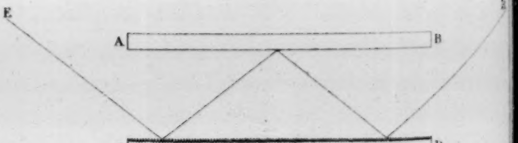


Fig. 6



XVIII.—*On the Bands formed by the Superposition of Paragenic Spectra produced by the Grooved Surfaces of Glass and Steel.*[*] PART I. By Sir DAVID BREWSTER, K.H., F.R.S. Lond. and Edin. (Plate XXII.)

(Read 7th March 1864.)

In examining the colours produced by thin laminæ of the crystalline lens of fishes, I observed a series of rectilineal serrated fringes perpendicular to the direction of the fibres, and produced by inclining the laminæ in a plane cutting these fibres at right angles. I was thus led to imitate these fringes or bands by combining grooves or striæ cut upon glass or steel surfaces, or grooves taken from these surfaces upon isinglass or gums.

In my first experiments I combined a system of grooves on glass, executed for me by Mr DOLLOND, with a similar system on steel executed by Sir JOHN BARTON, both of them containing 2000 divisions in an inch. The plate of glass was placed above the plate of steel, and slightly inclined to it, as shown in Plate XXII. figs. 1 and 2. The glass plate ABCD, fig. 2, was covered with grooves, but the steel plate below it was grooved only on the shaded portion *abcd*, the parts A*a*C*c*, B*bd*D being polished so as to reflect to the eye at E (fig. 1), the grooves on the glass when illuminated by rays, R*r*, proceeding from the first pair of the paragenic spectra produced by the grooves.

When the direction of the grooves *ac* is nearly parallel to the plane of reflexion, and to one another, a series of minute serrated bands is seen on the space *abcd*, where the light has been transmitted twice through the grooves on glass, and reflected once from those on steel; but no bands are seen upon A*ac*C, B*bd*D, where the steel was only polished.

When the grooves were slightly inclined to the plane of reflexion, large serrated bands appeared upon the spaces A*ac*C, B*bd*D, and when this inclination was increased, these large bands became smaller and more numerous, crowding towards C*c* and *d*D. On the other hand, they become larger and larger as the direction of the grooves returned into the plane of reflexion. In the azimuth of 0° they become straight, and by increasing the azimuth, they pass, as it were, to the right hand, as shown in fig. 3.

When the direction of the grooves is inclined to the plane of reflexion, the minute serrated bands upon *abcd* become smaller and less serrated.

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[*] In a very interesting paper on the Spectra produced by Gratings or Grooved Surfaces, M. BABINET has given them the appropriate name of *Paragenic*, in order to distinguish the Spectra produced by refraction from those produced by the lateral propagation of light. "*Sur la Paragenie ou propagation laterale de la lumiere.*" Paris, 1864. Extrait du COSMOS.

When the inclination *mn*NM of the grooved plates is increased, the large bands become smaller and smaller, and when it is diminished, they become larger and larger, getting inclined as in fig. 3, and becoming parallel at 0° of inclination.

Having been provided, by the kindness of Sir JOHN BARTON, with two grooved plates of glass containing 500 divisions in an inch, I was enabled to examine the fringes on the paragenic spectra under different circumstances.

When the grooved surfaces of the plates were placed in contact, and the grooves formed a small angle with one another, the middle or principal image, A (fig. 4), when observed with a lens whose anterior focus coincided with the grooves, had no bands, but the paragenic spectra *a*, *c*, *b*, *d*, on each side had numerous serrated bands or fringes perpendicular to the direction of the grooves, the number on the first spectra *a*, *b*, being at the rate of 19 in an inch of the luminous disc, and increasing in arithmetical progression.

When the luminous object is rectangular, and the rectangular paragenic spectra are brought nearly into contact, as at *ab* and *cd* (fig. 5), the bands, as seen at nearly a perpendicular incidence, are shown in this figure.

When the incident light is inclined to the direction of the grooves, the bands suffer no change, and appear immoveable on the surface of the glass plates.

When the ray of light is perpendicular to the direction of the grooves, and the surface of the glass on which they are cut is inclined to the ray of light, the bands all descend from *a* to *b* (fig. 5), moving off, as it were, at *b*, and *d*, and succeeded by others when the angle of incidence increases, while they ascend from *b* to *a*, and from *d* to *c*; moving off at *a* and *c*, when the angle of incidence diminishes. In this case, the grooves of the plate next the eye are turned to the left, the opposite motions taking place when they are turned to the right.*

The bands correspond to the intersection of the one set of grooves with the other set, and consequently they diminish in number, and recede from one another when the inclination of the one set of grooves to the other diminishes, becoming parallel to the grooves when the grooves on both plates are parallel.

Interference bands, parallel to the grooves, may be seen by transmitted light upon the paragenic spectra, when two systems of grooves are placed parallel to each other, and when the grooves in the one system are parallel to those in the other. They are seen both at a perpendicular incidence and when the plates are inclined in a plane parallel to the grooves.

These bands become narrow as the distance of the two grooved surfaces is increased, and they are seen at all angles of incidence, and in all planes of reflexion from the grooved surfaces.

I have observed those bands, which are generally more or less serrated, in combinations of 1000 with 1000, 1000 with 2000, 1000 with 500, 2000 with 500, and in the combination of four surfaces of 2000, 1000, 100, and 500.

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* This motion of the bands is not seen when the grooved surfaces are perfectly parallel.

In the combination of 1000 and 500, and in no other, a very peculiar system of bands is seen with a lens. They are not serrated, and not perpendicular to the grooves. The system consists of two sets equally inclined to the direction of the grooves, when the grooves in one plate are slightly inclined to those in the other. By diminishing the inclination of the grooves, the inclination of the bands to the direction of the grooves diminishes, and when the grooves become parallel, the bands become parallel and disappear.

These bands must have a different origin from those previously described, as they are similar in number upon all the prismatic images.

In these experiments the duplication of the bands on the second spectrum, and their increase in arithmetical progression on the other spectra, is a remarkable fact which it is difficult to explain. The second spectrum differs only from the first, and the third from the second, only in their length; and we can hardly suppose that they have a property in a direction perpendicular to their length, or to FRAUNHOFER's lines, which would increase the number of their bands.

The bands which we have described are more distinct when the spectra are pure or formed from a narrow line or bar of light; but when we wish to see the bands on the bar of light or the central image *O* (fig. 4), the spectra must be formed from wide spaces which gave impure spectra.

In order to examine the interference bands under different conditions, I placed (as in fig. 6) a plate of polished steel at different distances from another plate of steel, containing six systems of grooves executed by Sir JOHN BARTON, varying from 312·5 divisions in an inch to 10,000. When the light was reflected twice from the grooved surface and once from the plain steel surface, the bands which covered the colourless image and the paragenic spectra were splendid beyond description, and unlike anything of the kind that I had previously seen.

1. The bands were parallel to the grooves, or to the lines in the spectra.
2. They are smaller and more numerous when the grooves are wider or fewer in an inch.
3. They become smaller and more numerous when the distance of the plates is increased.
4. They are smaller and more numerous when the angle of incidence is increased.
5. They become more numerous by increasing the number of reflexions.
6. They appear like minute black lines upon the colourless image, but when their magnitude is increased, they appear like blue or pink bands on a ground of a different colour, which is generally white or whitish blue.

These bands were visible on the systems of grooves, 312·5, 625, 1250, and 2500 in an inch, but not on the systems of 5000 or 10,000 in an inch.

When the spectra had suffered three, four, five, and six reflexions, the central and other images were covered with the same number of bands, as with two reflexions from the grooved steel; but another series of wider bands was superposed.

The following results were obtained with grooved surfaces having 1250 divisions in an inch:—

Distance of plates, . .	0·11 inch.	Angular breadth of each, . .	7° 50′
Distance of circular disc, .	115·5 inches.	Distance of plates, . . .	0·22 inch.
Diameter of disc, . . .	1·317 inch.	Angle of incidence, . . .	63° 30′
Angle of incidence, . .	63° 30′	Number of fringes on the disc, and on the first spectrum, .	10
Angular diameter of disc, .	39° 30′	Angular breadth of each, . .	3° 55′
Number of fringes on disc, and on the first spectrum,	5		

In order to observe the effect produced by varying the angle of incidence, I placed a luminous disc three inches and six-tenths in diameter* at the distance of nine feet six inches from the grating, and obtained the following results:—

Angle of Incidence.	No. of Bands on the Disc.	Angle of Incidence.	No. of Bands on the Disc.
70	29	50	17
60	21	40	14

The bands were seen at an incidence of 87¼°, when the plates were nearly in contact.

The following were the colours seen on the two spectra on one side of the colourless image; but I have not measured the precise angle of incidence at which they were seen, nor mentioned in my journal whether they were seen with the 625 or the 1250 grating:—

	First Spectrum.		Second Spectrum.
Great Incidences	White.	Great Incidences	Blue.
	Pale Red.		Bluish.
	Red.		Less Blue.
	Purple.		Bluish White.
	Blue.		White.
	Bluish.		Pale Red.
	Less Blue.		Red.
Lesser Incidences	White.	Lesser Incidences	Purple. Blue.

At small angles of incidence, about 42°, the bands become less distinct, and paler in colour, the white becoming yellow and the blue brownish.

In the systems of grooves, whether on glass or on steel, employed in the preceding experiments, the part of the original surface not removed by the grooves bears a very considerable proportion to the part removed; but when the grooves occupy a large part of the surface, and the intermediate parts a very small one, a new set of phenomena are produced, which must change in a remarkable manner all the bands of interference. The execution, however, of such systems of grooves is very difficult. Sir JOHN BARTON, with all his experience, failed in producing good specimens; but even with those which he executed for me, phenomena of a remarkable kind were exhibited, not only on the middle or colourless image, but upon all the paragenic spectra, varying with the number of grooves, but still more remarkably with the angle of incidence.*

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* This disc included part of the spectrum on each side of the bright image.

P.S.—The preceding experiments were made in 1823 and 1827, and those described in p. 223, were repeated in 1838. Having lost or mislaid the glass gratings which I then employed, I am not able to compare the bands which they produced with a more remarkable series which I have recently obtained with new gratings, and which will be the subject of another communication

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* See Phil. Trans. 1829, p. 301.

Fig. 1

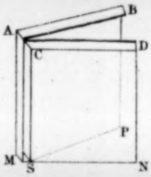


Fig. 2

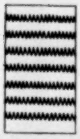


Fig. 3

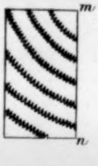


Fig. 4

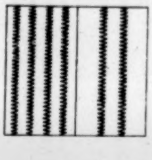
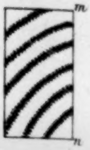


Fig. 5



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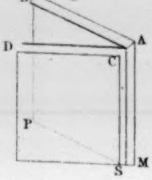

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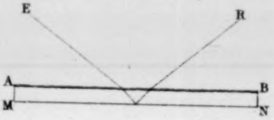

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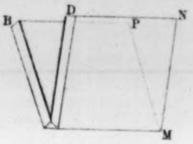
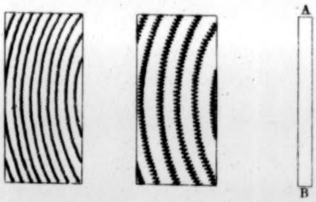


Fig. 9



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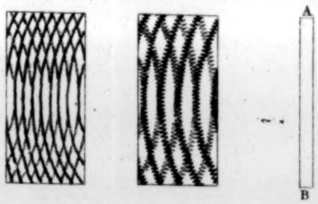
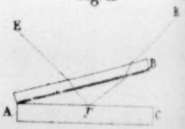


Fig. 11



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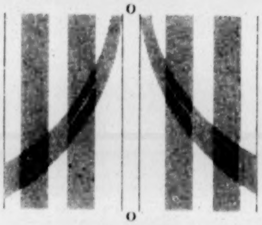

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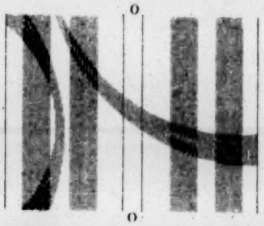

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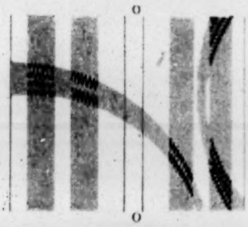

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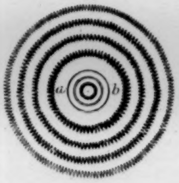

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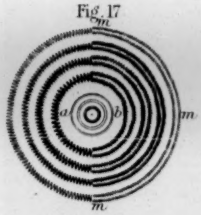

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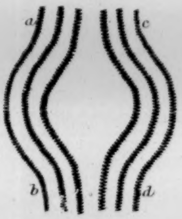
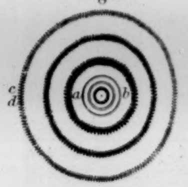


Fig. 19



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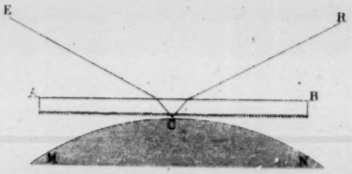

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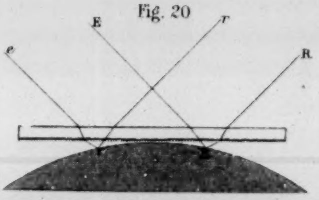

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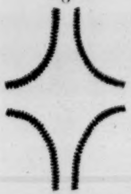

XIX.—*On the Bands formed by the Superposition of Paragenic Spectra produced by the Grooved Surfaces of Glass and Steel.* Part II. By Sir DAVID BREWSTER, K.H., F.R.S. Lond. and Edin. (Plate XXIII.)

(Read 17th April 1865.)

In the preceding paper I have described the bands produced by gratings or grooved surfaces with 500 divisions in an inch, when the two grooved surfaces are in contact, and the grooves in the one slightly inclined to those in the other.

The following results were obtained with two gratings, one of which had 2000 and the other 1000 divisions in an inch.

1. When the surfaces are in perfect contact, and the grooves parallel, very irregular bands are seen on the united surfaces, either with a lens or by ordinary vision, and are parallel to the grooves. They are seen only on the 2d, 4th, 6th, &c., spectra on each side of the luminous bar or disc.

By turning the nearest grating slightly to the right from the azimuth 0°, the bands fall back to the left, increasing in number, and descending with their concave sides downwards into distinct serrated black and white bands, nearly perpendicular to the grooves. When the nearest grating is turned to the left, the bands descend towards the right, with their concave sides upwards, till they become nearly perpendicular to the grooves. In all these positions, the bands are twice as numerous on the fourth spectrum as on the second, and thrice as numerous on the sixth as on the second; and when the grooved surfaces are perfectly parallel, the bands are immoveable on the grooved surfaces at all angles of incidence.

2. When the grooved surfaces are separated by the thickness of one or both of the plates of glass, the bands are very indistinctly seen, and they seem to diminish in size with the distance of the grooved surfaces; but this is not certain, owing to the difficulty of fixing the plates with the grooves at the same inclination to each other.

Similar bands were seen on the united surfaces of gratings of 2000 and 2000, 1000 and 1000, 500 and 500, 1000 and 500, and 2000 and 500 divisions in an inch, but always less distinctly when the grooved surfaces are separated by the thickness of one or both of the plates.

The beauty and distinctness of these bands depend upon the skill with which the gratings are ruled. In several of the gratings which I possess, the phenomena I have described can hardly be recognised.

When the combined gratings have the same number of divisions, such as 1000 and 1000, the bands are seen upon all the spectra, and sometimes very faintly on the luminous disc, but when one of the gratings has twice the number of divisions as the other, such as 2000 and 1000, the bands appear as already mentioned, only on the 2d, 4th, 6th, &c., spectra. In such combinations, the 1st, 3d, 5th, 7th, &c., spectra of the 1000 grating have no corresponding spectra in the 2000 grating, with which they can interfere, whereas, when the divisions in both are the same, all the spectra of the one are superposed upon all the spectra of the other and, therefore, bands are produced upon each of them.

In like manner, if the number of divisions in the one grating is to those on the other as n to 1, n being a whole number, the bands will appear only on the spectra n, $2n$, $3n$, $4n$, &c.

When a grating of 1000 is placed above one of 2000, I have observed faint bands upon the spectra, 1, 3, 5, &c., of the 1000 grating, though none of the spectra of the 2000 grating could interfere with them. These bands are more numerous than those between which they lie, and are doubtless produced by the interference of spectra reflected from the plane surfaces of the glass plates with those seen by transmitted light.

When the gratings of 1000 and 2000 are placed at a small angle, as in Plate XXIII., fig. 1, the grooves being parallel to AM, and the light incident perpendicularly, the bands on the left-hand spectra are parallel and rectilineal, and highly *purple* and *green*, as in fig. 2.

By turning the gratings round AM as an axis, in the direction from D to B, the bands descend from m, as in fig. 3, till they become parallel vertical lines, increasing in number and less coloured, as in fig. 4, the number of bands on the second left-hand spectrum being double those on the first.

When the rotation is in the opposite direction from B to D, fig. 1, the bands rise from n, fig. 5, till they become parallel and vertical as before.

The opposite effects take place when the gratings are placed as in fig. 6, AM and CS being coincident, and when we observe the spectra on the right hand of the luminous disc. The bands now descend and ascend from the same points m, n, now on the outer side of the spectra.

When the two edges, AM, CS of the gratings are not parallel, as they are in fig. 1, but inclined at a small angle, AMSC, fig. 6, then if, when the fringes are parallel at a perpendicular incidence, we turn the gratings round AM as an axis from B to D, the fringes descend from m, becoming smaller and smaller, till they are parallel and vertical, but when the gratings revolve from D to B, the fringes become larger and larger, less numerous, and more coloured, till they are finally parallel to AM, the fringes being twice as numerous on the second spectrum as on the first.

When the grooves are perpendicular to AM, as in fig. 7, the bands are faint and indistinct. The light being incident perpendicularly, and the gratings turned round AM on a plane perpendicular to AM, the fringes do not increase in number or greatly change, if the motion is accurately in a plane perpendicular to AM.

When the gratings are turned in the plane of the horizon passing through AM, the side NM approaching the eye, the fringes on the left-hand spectra descend, increasing rapidly in number, and when the side MN recedes from the eye, the fringes ascend, increasing in magnitude and diminishing in number, and are highly coloured. At a certain angle, they become parallel to the grooves, when by continuing the rotation they move downwards increasing in number, and becoming parallel to the grooves.

In the preceding experiments, the bands are seen on the surface of the gratings, but when the grooved surfaces are in contact, and the grooves parallel, bands of an entirely different kind are seen, not on the surface of the gratings, but by rays diverging from the luminous disc. If we use a long and narrow bar of light, such as the opening between the window-shutters, then, when the grooves are parallel to the bar, and the grooved surfaces perpendicular to the plane of incidence, the bands are parallel to the bar and its spectra. By inclining the grooves to the luminous bar, the bands are inclined to the spectra, dividing each of them into a great number of spectra, and at an azimuth of 45° the bands become perpendicular to the spectra. At all these inclinations the bands on the second spectrum are double those on the first, the number increasing in arithmetical progression on succeeding spectra.

When the angle of incidence is increased, the bands increase in number, but very slightly with gratings of 1250 divisions in an inch.

By increasing the distance between the gratings, the bands also increase in number.

Bands similar to those now described are produced with interesting phenomena by a single grating placed as in fig. 8, so that the image of the grooved surface AB, reflected from MN, the lower surface of the glass is superposed as it were upon the grooved surface itself.

1. When the plane of reflexion is perpendicular to the grooved surface, and the grooves in the same plane, the bands on the spectra are parallel to the bar of light AB, those on the second spectrum being double those on the first. They are seen at all angles of incidence, and are larger and more distinct at small angles.

When the grating is turned round in its own plane, at any angle of incidence, so that the grooves form different angles with the bar of light, the bands cross the spectra and become perpendicular to them in the azimuth of 45°. The paragenic spectra are thus divided into a great number of spectra, the number increasing as formerly on each succeeding spectrum.

2. When the grooves are parallel to the bar of light, and the plane of reflexion perpendicular to the grooves, the bands are apparently segments of concentric

circles at great angles of incidence, the radius of which increases as the angle of incidence diminishes, so that they become straight lines at a perpendicular incidence. The bands are smaller at their upper and lower ends, and those on the second spectrum are, as before, double those on the first, as shown in fig. 9.

In the spectra on the left hand of the bar of light, the concave side of the circular bands is towards the bar; and in the spectra on the right hand of the bar of light the convex side of the circular bands is toward the bar. The bands on the right-hand spectra are smaller and more numerous than those on the left-hand spectra; and yet, by increasing the angle of incidence, the bands on all the spectra increase in size and diminish in number.

If at any particular incidence we turn the grating in its own plane, the bands cross the spectra at angles increasing with the degree of rotation, and becoming smaller and more numerous. When the end of the grating nearest the eye (A, fig. 8) ascends, the fringes, great and small, diminish and become more distinct, and the centres of the circles descend. When the grating is turned in the opposite direction, the centres of the circles ascend.

In the principal gratings which I possess, when upon thin glass,* including those of 1000 and 2000 in an inch, these circular bands are accompanied by another system of circular bands, convex to the luminous bar when seen on the left-hand spectra, and concave to it when seen on the right-hand spectra; but, what is remarkable, they are smaller and more numerous on the first spectrum than on the second, as shown in fig. 10. They are best seen when the principal circular bands cross the spectra obliquely.

In the preceding experiments with one grating, the grooves of the reflected image are necessarily parallel to those of the real grating, owing to the parallelism of the surfaces of the plate of glass, and therefore they cannot exhibit the result of superposing two systems of grooves inclined to each other. This condition, however, may be obtained by drawing the grooves on the faces of a prism with a small angle, or by placing a fluid prism between an ordinary grating and a plate of thin parallel glass, which would enable us to vary the inclination of the two sets of grooves. A better arrangement, however, is to place the grating AB (fig. 11) upon a polished metallic surface, MN. A ray from the luminous bar at R, incident on AC at r, reaches the eye at E, after reflexion from the steel surface MN, so that the reflected image of the grating, AB, is superposed as it were on the direct image.

When the grating, AB, of 1000 grooves in an inch is laid upon a steel surface, MN, and the grooves are in the plane of incidence, the paragenic spectra of a luminous bar are covered with bands, not serrated, parallel to the spectra, exhibiting all the phenomena already described as seen by reflexion from a single grating.

---

* These bands are not seen on a beautiful Munich grating, kindly lent me by Professor STOKES, having 3750 divisions in an inch. As the bands become smaller with the thickness of the glass, their absence in this grating arises doubtless from its great thickness, which is 0·158 of an inch, the thickness of the gratings upon which they appear being about 0·04.

The bands are of the same size as with a single grating when the grooved surface is uppermost, but they are very much larger when the grooved surface is in contact with the steel.

When the grooved surface is slightly inclined to the steel surface, as in fig. 11, and the grooves parallel to the plane of reflexion, a double system of hyperbolic bands is seen, as in fig. 12, having one asymptote coincident with the bar of light and the other at right angles to it. One of the systems of hyperbolas is on one side of the bar and the other system on the other side, the number of bands on the second spectrum being double those on the first.

When the grooves are inclined to the plane of reflexion by turning them to the left or to the right, the double system of hyperbolas moves to the left or to the right, the curves of each system crossing the spectra, as in fig. 13, and being, as before, twice as numerous on the second as on the first spectrum on both sides of the bar. By increasing the inclination of the grooves to the plane of incidence, the system of hyperbolas moves farther to the left or to the right.

When the bar of light is placed at E and the eye at R, fig. 11, the system of hyperbolas is inverted, as in fig. 14.

It is curious to observe the passage of the parallel rectilineal bands into hyperbolas, when the inclination of the grooved to the steel surface commences. The parallel bands open at their lower end, as in fig. 13, or at their upper end, as in fig. 14, and change into hyperbolas. When the light was strong, I observed a second but fainter system of hyperbolas lying between the principal system and the luminous bar, and caused probably by reflexion from the second surface of the grating. The effect produced by the crossing of the bands arising from these two systems of hyperbolas was remarkable, and similar to what I had observed in combining two gratings of 500 divisions in an inch. This second system of hyperbolas was most distinct when the plane of reflexion from the surface of the steel was coincident with the plane of reflexion from the glass; and the double system was seen with grooved surfaces of 500, 1000, and 2000 divisions in an inch.

In using accidentally a steel surface that was not perfectly flat, I was surprised to observe that the bands were not hyperbolas, but circular rings varying in form and size with the angle which the grooves formed with the plane of reflexion. In order to examine this new and beautiful phenomenon, I placed the grooved surface of the grating, AB, upon a convex surface of steel, MN, as in fig. 15, so that the rays from the luminous body might reach the eye at E, after reflexion from the convex surface, MN. The reflected image of the grating is thus superposed upon the direct image, and two systems of concentric rings are seen upon the surface of the grating. At the point of contact, C, and around it, are seen the rings of thin plates described by NEWTON, and increasing in size with the radius of the surface MN. Around and concentric with these as shown at *ab*, fig. 16, is seen a beautiful system of serrated rings formed upon the paragenic spectra, as in fig. 16, the number of rings upon the second spectrum being double those on the first, as before, and becoming narrower and closer as they recede from the centre. When the first and second spectra are close to one another, as in fig. 17, the rings upon entering the second spectrum are doubled, as shown at *mmm*. These rings are seen only when the grooves are inclined to the plane of reflexion. By increasing the inclination, they become smaller and more distinct, their size being a minimum, and their distinctness a maximum, when the azimuth of the grooves is 90°. When the azimuth is 0°, or when the grooves are turned into the plane of reflexion, the rings open, as at fig. 18, and when turned into azimuth 1° or 2°, those on the side *ab*, fig. 18, go back to the left, and those on the side *cd* bend into a ring, as shown in fig. 19. When the rings are again formed, they increase as the angle of incidence diminishes.

When the rings are increasing or diminishing, or passing from one spectrum to another, their centres are sometimes white, and at other times so black as to eclipse the rings of NEWTON. Their colour is very variable, sometimes black, with colourless intervals, and sometimes richly coloured with the tints of the spectra on which they are seen. When the grating is pressed upon the convex surface, or raised slightly from it, the rings exhibit the same phenomena as those of thin plates.

When the ray RR′ (fig. 19) from the bar of light, reaches the eye at E, the grooves being slightly inclined to the plane of reflexion, the hyperbolic bands are seen, as in fig. 12, and when the ray *rr′* reaches the eye at *e*, the hyperbolic bands are seen as shown in fig. 13, and when the eye receives all the rays between R′ and *r′*, the direct and inverted systems of hyperbolas are seen, as in fig. 20. If, when these are seen, we look at the surface of the grating, we shall see the system of concentric rings produced by the union of the two systems of hyperbolas.

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